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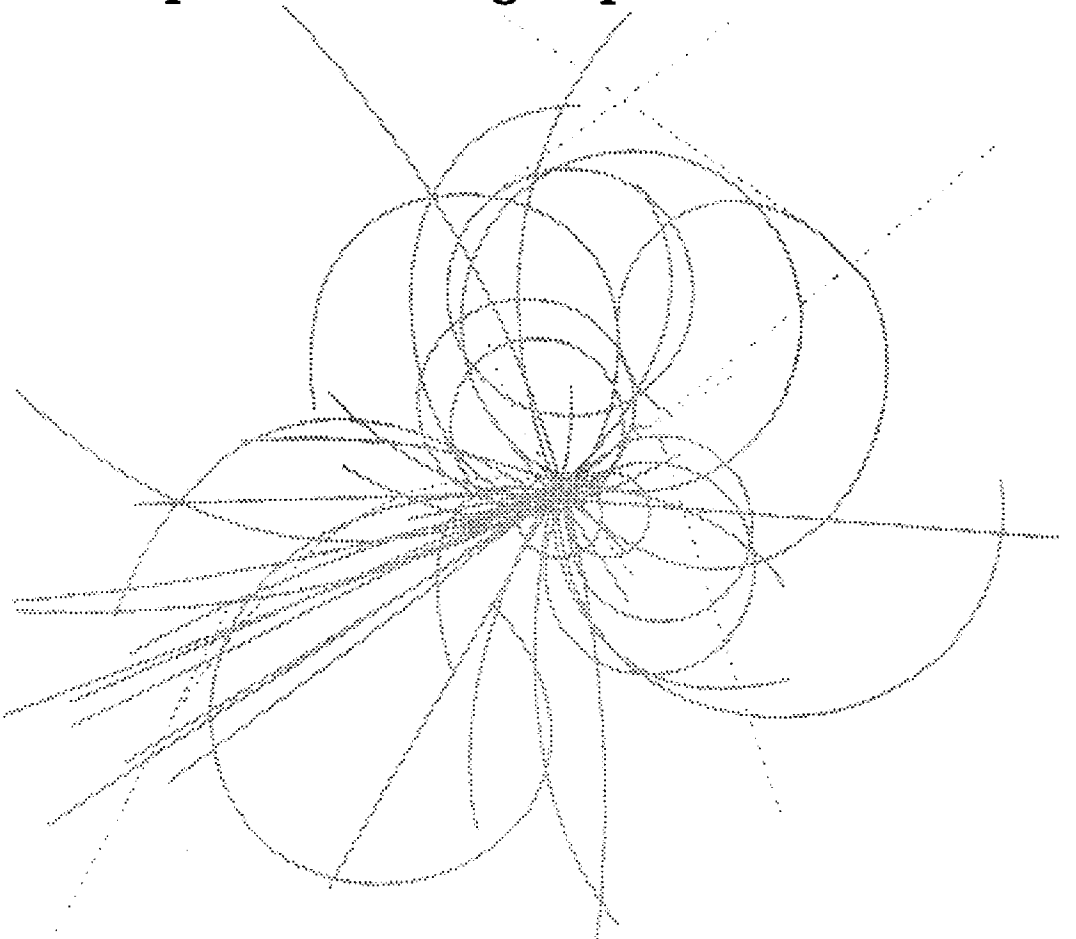

Divergent Quench Velocity Expression

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Superconducting Super Collider Laboratory*

2550 Beckleymeade Avenue

Dallas, TX 75237-3946

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Abstract

A quench velocity expression which has a divergent behavior close to the critical current density is reported here. This quench velocity has the qualitative behavior presented by the quench velocity measurements made so far on the 17-meter-long Superconducting Super Collider (SSC) Research and Development (R&D) dipole magnets, which show a clear depart from the expected theoretical adiabatic expression.

1. INTRODUCTION

When a normal zone, with dimension larger than the Minimum Propagation Zone (MPZ), is formed in a superconducting (s.c.) *cable*,[1] it propagates along the conductor with a so-called "axial quench velocity" and the phenomenon itself is called "quench". When the s.c. cable is used to form the coil of a superconducting magnet, the store energy of the magnet is dissipated in the coil in the form of heat generated by the normal zone of the s.c. cable. Special care must be taken to protect the magnet against overheating or major damage may occur in the event of a quench.

The heat speed propagation gives us information about the temperature in the coil and the peak voltage reached between normal zone and s.c. zone. If the speed is slow, energy will concentrate in a small region of the coil being able to raise the temperature of the hot-spot (where the first quench appears) to a very dangerous value (reaching the melting point of one of the components of the cable, degrading the critical current density, or producing a fail on the Kapton *insulation*[2]). If the speed is high, the energy will spread in an extended region of the coil decreasing the hot-spot temperature. The peak voltage depends on how fast the current decays and this one, in turn, depends on how fast the heat is spread in the coil which depends meanly on the quench velocity. The quench velocity is one of the must important parameters to be considered in computer simulation of the quench behavior of a magnet.

The quench velocity measured so far in the coil (inner-upper turns) of the SSC R&D 4-cm-dipole magnets has a very high value when the operating current is close to the short sample limit (consequently, the hot-spot temperature measured indicates that is not *high*[3]), the values are higher than could be expected from the theoretical point of view. This same quench velocity behavior was found in the Hadron-Elektron-Ring-Anlage (HERA) superconducting dipole *magnet*[4] and up to this time there has not been a clear explanation for this phenomenon.

It has been proposed the thermal hydraulic quench-back *mechanism*[5] as a possible explanation to these very high quench velocity values. However, in this report, the information indicates it is possible to obtain a qualitative explanation for this phenomenon using the Fourier conduction mechanism.

2. ADIABATIC QUENCH VELOCITY EXPRESSION

Most of the quench velocity expressions are equivalent to the next *one*[6]

$$v = \frac{J_o}{(\delta c)_s}\sqrt{\frac{\rho k}{\theta_s - \theta_o}}\left[1 - \frac{2(\theta_s - \theta_o)}{\theta_1 - \theta_o}\right] \Big/ \sqrt{1 - \frac{\theta_s - \theta_o}{\theta_1 - \theta_o}}\,, \tag{1}$$

where J_o is the current density flowing in the copper matrix; δ is the density; c is the specific heat; the quantity (δc) represents the average taken all over the conductor elements and evaluated at the temperature θ_s; k is the thermal conductivity; θ_o is the batch temperature; the temperature θ_1 depends on θ_o, the resistivity ρ, J_o, the cross section area A, the perimeter P, and the heat transfer coefficient h, as it follows

$$\theta_1 = \theta_o + \frac{\rho J_o^2}{hP}\,; \tag{2}$$

the temperature θ_s depends on the critical temperature, θ_c (at zero current density), and the generation temperature θ_g as

$$\theta_s = (\theta_g + \theta_c)/2\,. \tag{3}$$

The generation temperature θ_g is calculated from Figure 1 and it depends on the operating current density J_o and the critical current density, J_{co} (at the operating temperature) in the following form

$$\theta_g = \theta_c - (\theta_c - \theta_o)\frac{J_o}{J_{co}}\,. \tag{4}$$

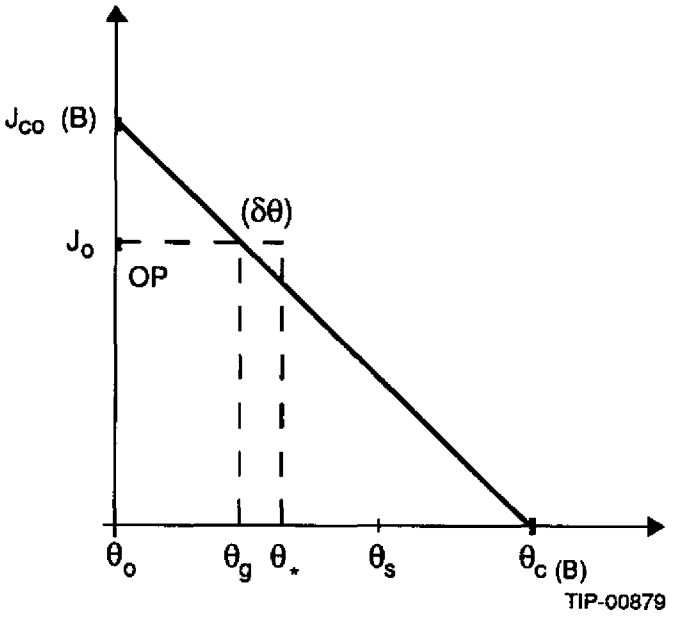

Figure 1. Critical current density as a function of the temperature for a constant magnetic field B showing the operating point of the conductor OP the temperature at which the ohmic heat generation starts θ_g and the temperature at which the normal zone propagation initiates θ_*.

The heat transfer term corresponds to a very fast transient process having almost non-effect in quench simulations, so it is common to make $h = 0$ in Eq. (1) to obtain the adiabatic expression

$$v_{as} = \frac{J_o}{(\delta c)_s}\sqrt{\frac{L_o\theta_s}{\theta_s - \theta_o}}\,, \tag{5}$$

where $L_o = \rho k/\theta_s = 2.45 \times 10^{-8} W\Omega K^{-2}$ is the Lorentz number. This quench velocity expression depends on the magnetic field through the critical temperature θ_c and the critical current density J_{co} (it is higher for higher magnetic field). However, this expression does not express the experimental singular behavior of the quench velocity when the operational current approaches to the critical current *density*.[5] In fact, using the relations (3) and (4), the term $\theta_s - \theta_o$ can be

written as

$$\theta_s - \theta_o = (\theta_c - \theta_o)(2 - \frac{J_o}{J_{co}})/2 \tag{6}$$

which shows a singularity for Eq. (5) when $J_o = 2J_{co}$ (we must note that operating at a current density higher than J_{co} has no physical meaning).

3. SINGULAR QUENCH VELOCITY

Consider one-dimensional quench propagation and assume that the quench appears at the origin $z = 0$, of the reference system S, inside the conductor at time $t = 0$. The conductor is embedded in a magnetic field B and drives a current density J_o. The normal zone is propagating on the positive and negative direction with a speed V which divides the conductor in a normal zone (I) and a s.c. zone (II) regions, as it is shown in Figure 2. The temperature of the normal zone is described by the following equation

$$(\delta c)\frac{\partial \theta}{\partial t} = k\frac{\partial^2 \theta}{\partial z^2} + \rho J_o^2 - \frac{hP(\theta - \theta_o)}{A} , \tag{7a}$$

and the temperature of the s.c. zone is described by the equation

$$(\delta c)\frac{\partial \theta}{\partial t} = k\frac{\partial^2 \theta}{\partial z^2} - \frac{hP(\theta - \theta_o)}{A} , \tag{7b}$$

where the parameters appearing here have the same meaning as described above.

Change the variables x and t by ξ and τ defined as

$$\xi = x - Vt \tag{8a}$$

and

$$\tau = t , \tag{8b}$$

which are the coordinates of a reference system $\tilde{S}$ traveling to the right at the speed V. Assuming a stationary state in this reference system (the temperature

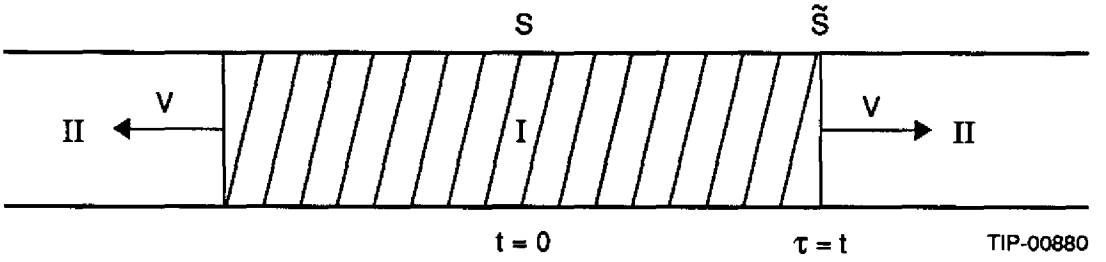

Figure 2. Normal and superconducting zones during a quench event.

does not change with time) and using the variable U defined by

$$U = \theta - \theta_o \ , \tag{9}$$

the equations for the temperature in the normal and s.c. zones are given by

$$k\frac{d^2U_{\rm I}}{d\xi^2} + (\delta c)V\frac{dU_{\rm I}}{d\xi} - \frac{hPU_{\rm I}}{A} + \rho J_o^2 = 0 \tag{10a}$$

and

$$k\frac{d^2U_{\rm II}}{d\xi^2} + (\delta c)V\frac{dU_{\rm II}}{d\xi} - \frac{hPU_{\rm II}}{A} = 0 \ . \tag{10b}$$

The solutions of these equations are, respectively,

$$U_{\rm I}(\xi) = \rho A J_o^2/Ph + a_1 \exp(q_1\xi) + a_2 \exp(q_2\xi) \tag{11a}$$

and

$$U_{\rm II}(\xi) = b_1 \exp(q_1 \ xi) + b_2 \exp(q_2\xi) \ , \tag{11b}$$

where q_1 and q_2 are the characteristic roots of Eq. (10) given by

$$q_1 = \frac{(\delta c)V}{2k}\left[\sqrt{1 + (hP/Ak)(\frac{2k}{(\delta c)V})^2} - 1\right] \tag{12a}$$

and

$$q_2 = \frac{(\delta c)V}{2k}\left[\sqrt{1+(hP/Ak)(\frac{2k}{(\delta c)V})^2}+1\right] . \tag{12a}$$

Now consider the following conditions in solutions (11a) and (11b):

The heating temperature is located on the far left-hand side of our reference system $\tilde{S}$, that is,

$$\lim_{\xi\to-\infty} U_{\mathrm{I}}(\xi) = U_1 = \rho A J_o^2/hP . \tag{13}$$

The batch temperature θ_o is at the far right-hand side, that is,

$$\lim_{\xi\to+\infty} U_{\mathrm{II}}(\xi) = 0 . \tag{14}$$

Solutions (11a) and (11b) are matched at the front ($\xi = 0$) where the boundary of the normal and s.c. zone is, to the Trigger temperature, θ_*, which takes into account the MPZ required to have a quench, as explained in the Appendix (see also Figure 1). This condition is expressed by

$$U_{\mathrm{I}}(0) = U_{\mathrm{II}}(0) = U_* . \tag{15}$$

Using the above conditions in Eq. (11), the solutions become of the following form

$$U_{\mathrm{I}}(\xi) = U_1 + (U_* - U_1)\exp(q_1\xi) \tag{16a}$$

and

$$U_{\mathrm{II}}(\xi) = U_* \exp(q_2\xi) . \tag{16b}$$

Finally, applying the condition of continuity of heat flux at the boundary $\xi = 0$, expressed by

$$-k\left(\frac{dU_{\mathrm{I}}}{d\xi}\right)_{\xi=0} = -k\left(\frac{dU_{\mathrm{II}}}{d\xi}\right)_{\xi=0} \tag{17}$$

and after making some arrangements, we obtain finally,

$$V_{\rm g} = \frac{J_o}{(\delta c)_*}\sqrt{\frac{\rho k}{\theta_* - \theta_o}}\frac{\left|\frac{2hP(\theta_*-\theta_o)}{A\rho J_o^2} - 1\right|}{\sqrt{1 - \frac{hP(\theta_*-\theta_o)}{A\rho J_o^2}}} \,. \tag{18}$$

The square root in the denominator of Eq. (18) is well defined since the condition of heat propagation implies the following condition is satisfied

$$\rho J_o^2 > \frac{hP(\theta_* - \theta_o)}{A} \,. \tag{19}$$

Using relation (4) and the following approximation

$$\frac{\rho k}{\theta_c - \theta_o} = L_o \,, \tag{20}$$

we can express the quench velocity in Eq. (18) in terms of the fraction of the critical current density,

$$q = \frac{J_o}{J_{co}} \,, \tag{21a}$$

and the copper-to-s.c. ratio,

$$\gamma = cu : sc \,, \tag{21b}$$

in the following way

$$V_{\rm g} = \frac{J_{co}\sqrt{L_o}}{\gamma(\delta c)_*}\frac{F(h)q}{\sqrt{1 + \frac{\delta\theta}{\theta_c-\theta_o} - q}} \,, \tag{22}$$

where $\delta\theta$ is given in the Appendix and the function $F(h)$ is defined by

$$F(h) = \frac{\left|\frac{2hP(\theta_*-\theta_o)}{A\rho J_o^2} - 1\right|}{\sqrt{1 - \frac{hP(\theta_*-\theta_o)}{A\rho J_o^2}}} \,. \tag{23}$$

The adiabatic quench velocity ($h = 0$) is then given by

$$V_{\mathrm{ag}} = \frac{J_{co}\sqrt{L_o}}{\gamma(\delta c)_*} \frac{q}{\sqrt{1 + \frac{\delta\theta}{\theta_c - \theta_o} - q}} \tag{24}$$

which shows explicitly a singularity at the point $q = 1 + \delta\theta/(\theta_c - \theta_o)$. For q lower than 1, the value obtained with Eq. (24) approaches to that obtained with Eq. (5).*

4. INNER COIL OF THE SSC R&D DIPOLE MAGNETS

In order to make the calculation of the adiabatic quench velocity as a function of q, variation of the magnetic field in the conductors as a function of the operating current must be known. Turn 16 approximation is given by the following *relation*[3]

$$B = 0.7505 + 0.947 \times 10^{-3}\mathbf{I}\,, \tag{25}$$

where the current $\mathbf{I}$ is given in amperes and the magnetic field B is given in teslas. The critical current density of the conductor, at the bath temperature $\theta_o = 4.35$ K, can be deduced from the *expression*[7]

$$J_{co}(B) = P_1\left[1. - \frac{0.040608}{1 + (B-5)P_5}\right]\frac{1 + (B-5)P_6}{1 + (B-5)P_7 + (B-5)^2 P_8}\,, \tag{26}$$

where the coefficients P_1, P_5, P_6, P_7, and P_8 are given by

$$P_1 = 2.475 \times 10^9 \ \mathrm{A/m^2}\,, \tag{27a}$$

$$P_5 = -0.16309\,, \tag{27b}$$

$$P_6 = -0.23174\,, \tag{27c}$$

$$P_7 = -0.02125\,, \tag{27d}$$

and

* It is possible to choose approximation $\rho k/\theta_* = L_o$ instead of Eq. (20) to obtain a singular quench velocity expression also, but this equation has a different limit for small q.

$$P_8 = -0.02042\,. \tag{27e}$$

The critical temperature at zero current density can be approached by the following *expression*[7]

$$\theta_c(B) = 9.2(1 - B/14.5)^{0.59}\,. \tag{28}$$

Figure 3 shows the result of the calculations comparing the values of both expressions (5) and (24), as well as some experimental results. The length L used to fit the experimental data is between 10 μm and 15 μm (these values are consistent with the MPZ length condition) which gives us a $\delta\theta$ of the order of 10^{-6} K.

5. CONCLUSIONS

Using the Fourier conduction mechanism, it was possible to obtain an adiabatic quench velocity expression (24) which reproduces the quench behavior of the SSC R&D dipole magnets with respect to the fraction of critical current. This is an important parameter for confidence of the quench simulation studies, in order to find the appropriate quench protection system for these magnets. The determination, of the quench velocity for the others turns in the coil, requires a more elaborate computer program now in progress. Without the term $\delta\theta$, expression (24) is singular exactly at $J_o = J_{co}$ and reproduces only qualitatively, the experimental data. The term $\delta\theta$ is used to fit the experimental data which, in turn, gives us information about the MPZ length needed to trigger a quench.

Although, the Fourier conduction mechanism gives a possible explanation of the behavior that the quench velocity has with respect to the fraction of the short sample current limit. More detailed experiments are required to rule out or to accept it as the responsibility of very high quench velocities observed close to the short sample limit.

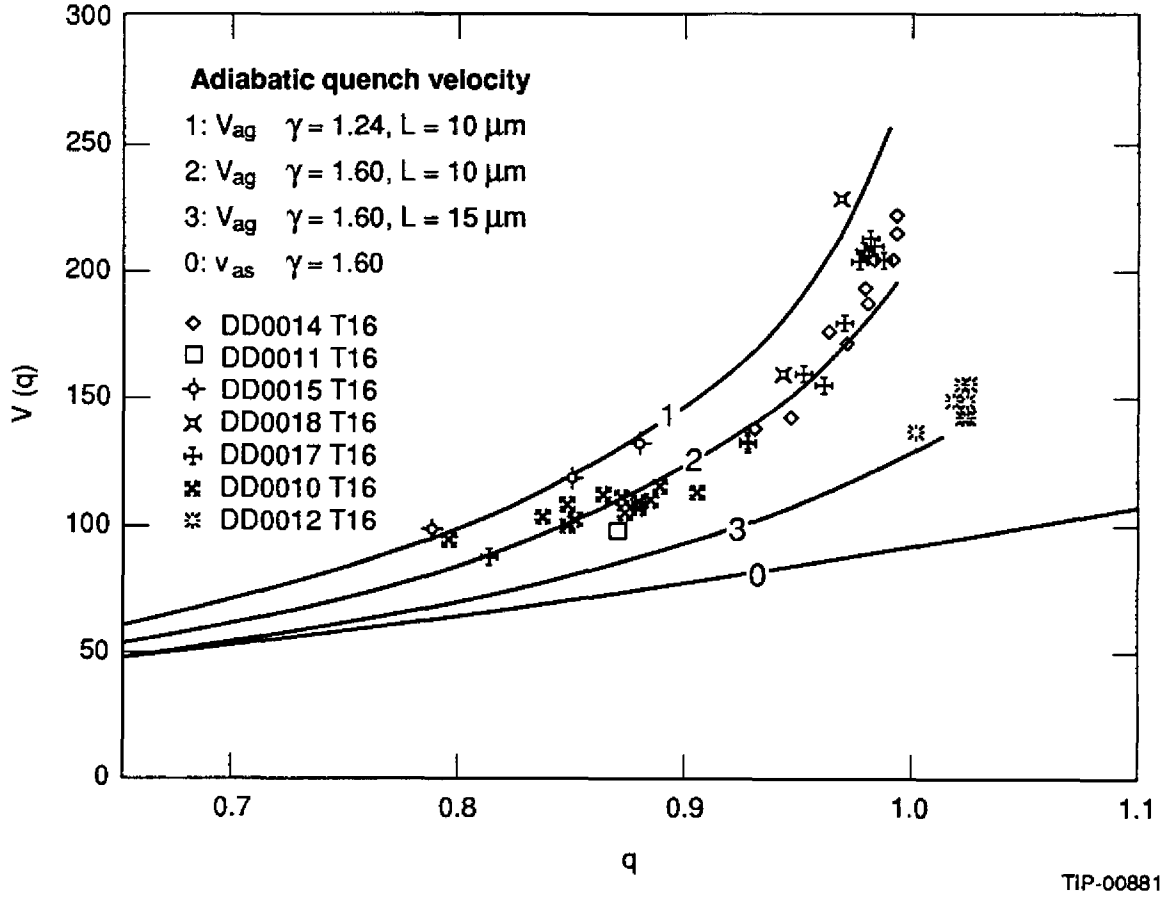

Figure 3. Measured propagation velocities compared with both theoretical expressions (5) and (24). The length of Minimum Propagation Normal Zone needed to fit the data is 10 μm or 15 μm.

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APPENDIX

The power generated in the normal zone of the conductor is given by

$$P_g = L(\rho_{cu}\mathbf{I}_1{}^2 + \rho_{sc}\mathbf{I}_2{}^2) \ , \qquad (A.1)$$

where L is the length of the normal zone, $\mathbf{I}_1$ is the current flowing in the copper matrix, $\mathbf{I}_2$ is the current flowing in the s.c. filaments, and ρ is the resistivity of the components. Because the voltage across the normal zone is the same for both components, the relation between their currents is given by

$$\mathbf{I}_1 = \gamma\rho_{sc}\mathbf{I}_2/\rho_{cu} \ , \qquad (A.2)$$

where γ is the ratio of copper to s.c. areas. Using Eqs. $(A.1)$ and $(A.2)$, the power generated can be approximated by

$$P_g = [\gamma^2\rho_{sc}/\rho_{cu} + 1]\rho_{sc}\mathbf{I}_2^2 L \simeq \gamma^2\rho_{sc}^2/\rho_{cu}\mathbf{I}_2^2 L \ . \qquad (A.3)$$

The heat is conducted along the wire out where there is a variation $\delta\theta$ in the temperature with respect to the generating temperature θ_g. The heat lost is given by

$$P_{lost} = 2k_{cu}(1+\gamma)a_{sc}(\delta\theta)/L \ , \qquad (A.4)$$

where k_{cu} is the thermal conductivity of the copper. Equaling Eqs. $(A.4)$ and $(A.3)$, and making some arrangements, obtaining

$$(\delta\theta) = \gamma^2\rho_{sc}^2 a_{sc}L^2 J_o^2/2k_{cu}(1+\gamma)\rho_{cu} \ , \qquad (A.5)$$

where $J_o = \mathbf{I}_2/a_{sc}$ is the operating current density.

The temperature at which the normal zone starts propagating along the cable will be given by

$$\theta_* = \theta_g + (\delta\theta) \qquad (A.6)$$

which depends on the length of the normal zone. This length must be higher than the MPZ (the MPZ-length is about 1 μm).