SSCL-N-853

# Phase-Space Dilution Effect of Quadrupole Component in C-type Dipoles of LEB–MEB Transfer Line

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April 1994

*Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

# Phase-Space Dilution Effect of Quadrupole Component in C-type Dipoles of LEB–MEB Transfer Line

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**Abstract**

The C-type dipoles of the LEB–MEB transfer line at the Superconducting Super Collider Laboratory have a strong quadrupole component. The effect of this component on the amplitude functions ($\beta$ and $\alpha$) mismatch, dispersion functions ($\eta$ and $\eta'$) mismatch, and phase-space dilution is studied, and some measures to correct the effect are suggested.

## 1.0 INTRODUCTION

The LEB–MEB transfer line[1] at the Superconducting Super Collider Laboratory transports 12 GeV/c proton beam from the Low Energy Booster (LEB) to the Medium Energy Booster (MEB). Most of the dipoles used in the transfer line are of the symmetrical H-type. In order to prevent dipoles in the transfer line from interfering with MEB magnets, vertical dipoles SEPA and SEPB (Figure 1) have to be of the C-type with openings facing down. According to the optics of the transfer line, the bending angles and field strengths of dipoles SEPA, SEPB, and SEPC are the same. To reduce the cost, only one multiple power supply is used for these three dipoles. Therefore, dipole SEPC has to be of the same C-type, and there are three C-type dipoles in the transfer line.

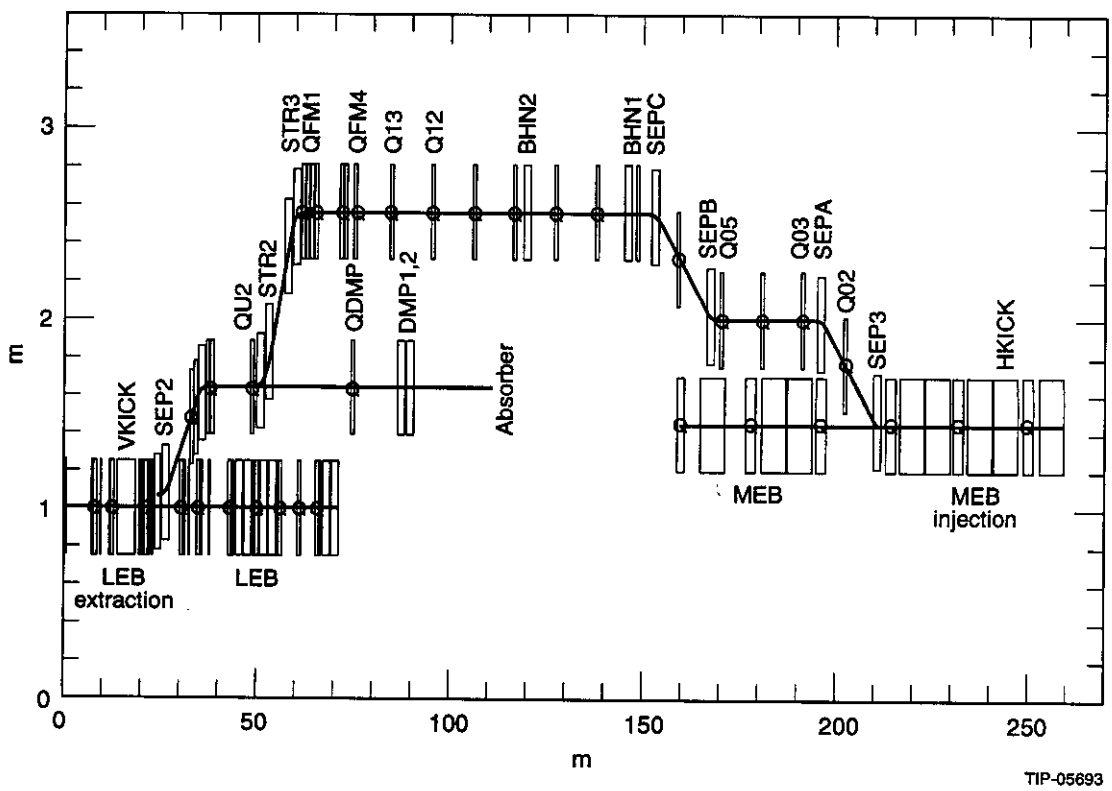


**Figure 1. LEB–MEB Transfer Line and Absorber Line (Elevation View, LEB630 and LEB917).**

The design result of the C-type dipoles indicates that the field of these dipoles has a strong quadrupole component. The field distribution[2] in the middle plane is shown in Figure 2. The field strength falls down, mainly linearly, by about $\Delta B/B_0 = 0.85*10^{-3}$ over a range of 20 mm from the beam center ($x = 0$ mm at magnet entrance and exit boundaries) toward the opening of the dipole. The dimensionless $n$-value, expressing the quadrupole component in a dipole, is defined by the equation

$$B = B_0 (1 - \frac{nx}{\rho}) \quad ,$$

where ρ is the bending radius. Substituting the field deviation data mentioned above and ρ = 52.725 m into the equation, we have $n = -2.24$ for dipole SEPC, which bends the beam down and has an opening facing down. The $n$-value is to be used to specify the parameters of dipole SEPC for beam optics calculation. Table 1 lists the $n$-values for these three C-type dipoles in different bending and opening facing conditions.

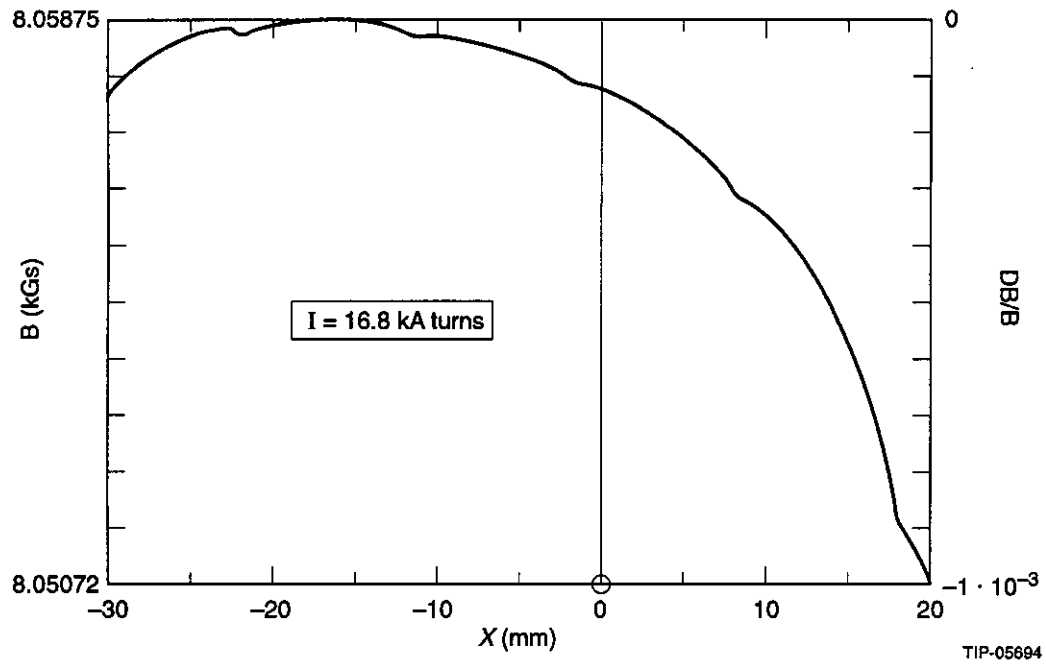


**Figure 2. Field Distribution in the Middle Plane of a C-type Dipole.**

**Table 1. *n*-values.**

| Dipole | SEPC | SEPB | SEPA |
|---|---|---|---|
| Bending | down | up | down |
| Opening facing | down up | down up | down up |
| *n*-value | −2.24 +2.24 | +2.24 −2.24 | −2.24 +2.24 |

## 2.0 AMPLITUDE FUNCTION MISMATCH AND CORRECTION

The amplitude function mismatch at the MEB injection point, caused by the quadrupole components in the C-type dipoles, is calculated with code TRANSPORT.[3] The $n$-value in the input data of TRANSPORT is before accomplishing the coordinate rotation about the $z$-axis. A list of the mismatch calculation results is shown in Table 2. The corresponding phase-space dilution factor is

$$F = 1 + \frac{1}{2}\frac{\beta_2}{\beta_1}\left[\left(1 - \frac{\beta_1}{\beta_2}\right)^2 + \left(\frac{\beta_1}{\beta_2}\alpha_2 - \alpha_1\right)^2\right] \quad ,$$

where subscripts 1 and 2 denote the situations with and without quadrupole component errors, respectively.

Line 0.0 in Table 2 lists the amplitude functions without quadrupole component errors. The amplitude functions with quadrupole component error for each C-type dipole are listed in lines 1.1–1.4. It can be seen that the phase-space dilution factor $F$ due to the amplitude function mismatch increases by 0.3%. The total quadrupole component effect of the three dipoles is $\Delta F = 0.8\%$ (line 1.5), if all dipoles have their openings facing down.

There are two measures to correct the amplitude function mismatch and reduce the phase-space dilution.

## 2.1 Dipole SEPC Opening Facing Up

Dipoles SEPA and SEPB must have their openings facing down because of the interference. But dipole SEPC may choose its opening facing either up or down. From the point of view of phase-space dilution, dipole SEPC should have its opening facing up for the following reason.

According to the lattice optics, the transformation matrix from dipole SEPC to dipole SEPA is –I. Therefore, the transformation matrices $R_1$ and $R_2$ from a certain point upstream to the exits of SEPC and SEPA, respectively, satisfy the equation

$$R_2 = -IR_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} R_1 \quad .$$

If there are quadrupole component errors in the two dipoles and these components may be expressed by two thin quadrupoles located at the exit of each dipole, the transformation matrix $R_2{}^*$ at the exit of dipole SEPA now satisfies

$$R_2{}^* = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} R_1 = \begin{bmatrix} -1 & 0 \\ \frac{1}{f_1} + \frac{1}{f_2} & -1 \end{bmatrix} R_1 \quad ,$$

where $f_1$ and $f_2$ are the focal lengths of the two thin quadrupoles. If dipole SEPC has its opening facing up, opposite SEPA, we have $f_2 = -f_1$ and

$$R_2{}^* = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} R_1 = R_2 \quad .$$

It means that the quadrupole component effects of dipoles SEPC and SEPA are compensated for each other.

Line 1.6 in Table 2 lists the TRANSPORT calculation result on quadrupole component effect. Here dipole SEPC has its opening facing up, dipoles SEPB and SEPA facing down. The result shows that the dilution factor increment reduces from 0.8% to 0.3%, consistent with the analysis.

TABLE 2. BETA, ALPHA MISMATCHING AND PHASE SPACE DILUTION ANALYSIS

LEB642E, BETA MISMATCH & DILUTION CAUSED BY Q-COMPONENT OF C-DIPOLE, JULY 13,93

AT THE END OF THE BEAM LINE

| LINE | RUN | ALPHAX1 (rad) | BETAX1 (m) | ALPHAY1 (rad) | BETAY1 (m) |
|---|---|---|---|---|---|
| 0.0 | LEB630E | -2.09590 | 51.09230 | 0.73110 | 14.40940 |

| LINE | RUN | ALPHAX2 (rad) | BETAX2 (m) | ALPHAY2 (rad) | BETAY2 (m) | Fx | Fy | |
|---|---|---|---|---|---|---|---|---|
| 1.1 | LEB642E01 | -1.93750 | 48.56960 | 0.73820 | 14.61910 | 1.00287 | 1.00011 | (SEPC: n=-2.2408, OPENING FACING DOWN) |
| 1.2 | LEB642E04 | -2.27040 | 54.10900 | 0.72430 | 14.20350 | 1.00286 | 1.00011 | (SEPC: n=+2.2408, OPENING FACING UP) |
| 1.3 | LEB642E06 | -2.16220 | 50.81920 | 0.75800 | 14.43300 | 1.00303 | 1.00033 | (SEPB: n=+2.2408, OPENING FACING DOWN) |
| 1.4 | LEB642E02 | -1.93420 | 48.54670 | 0.73820 | 14.61910 | 1.00303 | 1.00011 | (SEPA: n=-2.2408, OPENING FACING DOWN) |
| 1.5 | LEB642E07 | -1.82560 | 45.37430 | 0.77190 | 14.84720 | 1.00777 | 1.00062 | (SEPC/B/A: n=-/+/-2.2408, OPENING FACING DOWN/D/D) |
| 1.6 | LEB642E08 | -2.14750 | 50.64170 | 0.75820 | 14.42880 | 1.00252 | 1.00034 | (SEPC/B/A: n=+/+/-2.2408, OPENING FACING UP/DOWN/D) |
| 1.7 | LEB642E23 | -2.10210 | 51.56480 | 0.73230 | 14.31690 | 1.00013 | 1.00004 | (SEPC/B/A: n=+/+/-2.2408, OPENING FACING UP/DOWN/D) (Q05: DG=-0.12906 T/m, B0 REDUCING TO 2.8145 kG) |
| 2.1 | LEB642E09 | -2.01470 | 49.76910 | 0.73460 | 14.51380 | 1.00072 | 1.00003 | (SEPC: n=-1.1204, OPENING FACING DOWN) |
| 2.2 | LEB642E21 | -2.18110 | 52.53900 | 0.72760 | 14.30600 | 1.00071 | 1.00003 | (SEPC: n=+1.1204, OPENING FACING UP) |
| 2.3 | LEB642E10 | -2.12580 | 50.87870 | 0.74440 | 14.41880 | 1.00076 | 1.00008 | (SEPB: n=+1.1204, OPENING FACING DOWN) |
| 2.4 | LEB642E11 | -2.01290 | 49.75370 | 0.73460 | 14.51380 | 1.00076 | 1.00003 | (SEPA: n=-1.1204, OPENING FACING DOWN) |
| 2.5 | LEB642E15 | -1.95750 | 48.12380 | 0.75140 | 14.62630 | 1.00194 | 1.00015 | (SEPC/B/A: n=-/+/-1.1204, OPENING FACING DOWN/D/D) |
| 2.6 | LEB642E16 | -2.12130 | 50.82850 | 0.74450 | 14.41770 | 1.00067 | 1.00008 | (SEPC/B/A: n=+/+/-1.1204, OPENING FACING UP/DOWN/D) |
| 3.1 | LEB642E12 | -2.07930 | 50.81780 | 0.73180 | 14.43020 | 1.00003 | 1.00000 | (SEPC: n=-0.22408, OPENING FACING DOWN) |
| 3.2 | LEB642E22 | -2.11260 | 51.37180 | 0.73040 | 14.38870 | 1.00003 | 1.00000 | (SEPC: n=+0.22408, OPENING FACING UP) |
| 3.3 | LEB642E13 | -2.10130 | 51.03720 | 0.73370 | 14.41090 | 1.00003 | 1.00000 | (SEPB: n=+0.22408, OPENING FACING DOWN) |
| 3.4 | LEB642E14 | -2.07890 | 50.81410 | 0.73180 | 14.43020 | 1.00003 | 1.00000 | (SEPA: n=-0.22408, OPENING FACING DOWN) |
| 3.5 | LEB642E17 | -2.06770 | 50.48080 | 0.73510 | 14.45250 | 1.00008 | 1.00001 | (SEPC/B/A: n=-/+/-0.22408, OPENING FACING DOWN/D/D) |
| 3.6 | LEB642E18 | -2.10080 | 51.03220 | 0.73370 | 14.41090 | 1.00003 | 1.00000 | (SEPC/B/A: n=+/+/-0.22408, OPENING FACING UP/DOWN/D) |

## 2.2 Adjusting the Gradient of Quadrupole Q05

The residual amplitude function mismatch and phase-space dilution increment of 0.3% when dipole SEPC faces up is caused by the quadrupole component in dipole SEPB. According to the lattice optics of the transfer line, there is a pure quadrupole (Q05) downstream of SEPB and near the exit of SEPB. If the integral gradient error in SEPB is $\Delta G_1 L_1$ and we adjust the gradient of quadrupole Q05 by an amount of $\Delta G_2$, which satisfies the equation $\Delta G_2 L_2 = -\Delta G_1 L_1$, the effect of quadrupole component in SEPB should be compensated in the first order.

Substituting $\Delta G_1 = 0.85*10^{-3}*0.75918$ T/0.02 m = 0.032265 T/m, SEPB length $L_1 = 2$ m, and Q05 length $L_2 = 0.5$ m into the equation $\Delta G_2 L_2 = -\Delta G_1 L_1$, we have $\Delta G_2 = -0.12906$ T/m. The TRANSPORT numerical calculation shows that the phase-space dilution increment is reduced from 0.3% to 0.01% (line 1.7, Table 2), if the gradient of quadrupole Q05 is adjusted by an amount of $\Delta G_2$.

Obviously, the phase-space dilution can be reduced by comprehensively using these two measures. On the other hand, if the quadrupole components in the C-type dipoles could be reduced, say, by a factor of 2 or 10, the phase-space dilution factors would be reduced also, as shown in lines 2.1–2.6 and 3.1–3.6 of Table 2.

# 3.0 DISPERSION FUNCTION MISMATCH

The dispersion function mismatch ($\Delta\eta$ and $\Delta\eta'$) caused by the quadrupole components in the C-type dipole at the MEB injection point is calculated and listed in Table 3. The phase-space dilution factor corresponding to the mismatch is

$$F = 1 + \frac{1}{2}\left[\frac{\Delta\eta_{eq}\sigma_p}{\sigma_0}\right]^2 ,$$

where $\sigma_p$ is the momentum spread, $\sigma_0$ is the half beam size, and

$$\Delta\eta_{eq} \equiv \sqrt{\Delta\eta^2 + (\beta\Delta\eta' + \alpha\Delta\eta)^2} .$$

The calculated phase-space dilution factors due to dispersion function mismatch are also listed in Table 3. The results show that the dilution is negligible.

Table 3. ETA AND ETAp MISMATCHING AND PHASE SPACE DILUTION ANALYSIS

LEB642E, BETA MISMATCH & DILUTION CAUSED BY Q-COMPONENT OF C-DIPOLE, JULY 13,93

AT THE END OF THE BEAM LINE

| LINE | RUN | ETAx (m) | ETAxp | ETAy (m) | ETAyp | ALPHAx (rad) | BETAx (m) | ALPHAy (rad) | BETAy (m) | SIGMAx (mm) | SIGMAy (mm) | DP |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | LEB630E | -1.06293 | -0.04862 | 0.00000 | 0.00000 | -2.09590 | 51.09230 | 0.73110 | 14.40940 | 1.54821 | 0.82219 | 0.00100 |

| LINE | RUN | ETAx (m) | ETAxp | ETAy (m) | ETAyp | Dxeq (m) | Dyeq (m) | Fx | Fy |
|---|---|---|---|---|---|---|---|---|---|
| 1.1 | LEB642E01 | -1.11711 | -0.05040 | 0.00015 | -0.00002 | 0.05871 | 0.00023 | 1.00072 | 1.00000 |
| 1.2 | LEB642E04 | -1.00881 | -0.04685 | -0.00026 | 0.00002 | 0.05880 | 0.00028 | 1.00072 | 1.00000 |
| 1.3 | LEB642E06 | -1.01045 | -0.04639 | -0.00690 | 0.00034 | 0.05263 | 0.00690 | 1.00058 | 1.00004 |
| 1.4 | LEB642E02 | -1.11880 | -0.05046 | 0.00404 | -0.00043 | 0.06045 | 0.00518 | 1.00076 | 1.00002 |
| 1.5 | LEB642E07 | -1.12022 | -0.04998 | -0.00265 | -0.00010 | 0.07643 | 0.00429 | 1.00122 | 1.00001 |
| 1.6 | LEB642E08 | -1.01588 | -0.04660 | -0.00307 | -0.00006 | 0.04727 | 0.00437 | 1.00047 | 1.00001 |
| 1.7 | LEB642E23 | -1.07130 | -0.04902 | 0.00190 | -0.00026 | 0.00886 | 0.00303 | 1.00002 | 1.00001 |
| 2.1 | LEB642E09 | -1.09001 | -0.04951 | 0.00005 | -0.00001 | 0.02934 | 0.00012 | 1.00018 | 1.00000 |
| 2.2 | LEB642E21 | -1.03586 | -0.04774 | -0.00016 | 0.00001 | 0.02952 | 0.00016 | 1.00018 | 1.00000 |
| 2.3 | LEB642E10 | -1.03668 | -0.04751 | -0.00348 | 0.00017 | 0.02630 | 0.00348 | 1.00014 | 1.00001 |
| 2.4 | LEB642E11 | -1.09086 | -0.04954 | 0.00199 | -0.00021 | 0.03022 | 0.00254 | 1.00019 | 1.00000 |
| 2.5 | LEB642E15 | -1.09161 | -0.04931 | -0.00134 | -0.00005 | 0.03795 | 0.00216 | 1.00030 | 1.00000 |
| 2.6 | LEB642E16 | -1.03847 | -0.04758 | -0.00155 | -0.00003 | 0.02453 | 0.00220 | 1.00013 | 1.00000 |
| 3.1 | LEB642E12 | -1.06834 | -0.04880 | -0.00003 | 0.00000 | 0.00582 | 0.00004 | 1.00001 | 1.00000 |
| 3.2 | LEB642E22 | -1.05751 | -0.04844 | -0.00007 | 0.00001 | 0.00584 | 0.00012 | 1.00001 | 1.00000 |
| 3.3 | LEB642E13 | -1.05768 | -0.04840 | -0.00074 | 0.00004 | 0.00526 | 0.00074 | 1.00001 | 1.00000 |
| 3.4 | LEB642E14 | -1.06851 | -0.04881 | 0.00036 | -0.00004 | 0.00592 | 0.00048 | 1.00001 | 1.00000 |
| 3.5 | LEB642E17 | -1.06867 | -0.04876 | -0.00031 | -0.00001 | 0.00753 | 0.00048 | 1.00001 | 1.00000 |
| 3.6 | LEB642E18 | -1.05789 | -0.04841 | -0.00035 | 0.00000 | 0.00504 | 0.00043 | 1.00001 | 1.00000 |

# REFERENCES


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