SSCL-N-828

The Interactive Beam Dynamics Laboratory

B. Cole* and L. Singleton

Superconducting Super Collider Laboratory†
2550 Beckleymeade Avenue
Dallas, TX 75237

July 1993

* Also at Physics Department, University of North Texas, Denton, Texas.

† Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

The Interactive Beam Dynamics Laboratory

Ben Cole†, Len Singleton
Machine Simulation and Correction Group,
Superconducting Super Collider Laboratory, Dallas, Texas.
† *also at* Physics Department, University of North Texas, Denton, Texas

1. Introduction

The dynamics of particles in phase space is central to the study of particle motion in synchrotrons. Traditionally, such studies are conducted either by examining a plot consisting of a single particle recorded at a number of different times a la Poincare, or by studying separate plots containing the positions of a number of particles at a single time step. Advances in graphics workstation technology are providing physicists with new ways in which to study these phenomena. In particular, this paper describes an application developed on a Silicon Graphics Crimson workstation that provides the scientist with the ability to interactively manipulate a simple model of an accelerator as it is running and observe the effects of these manipulations on the particle motion. The system may be used to demonstrate such effects as decoherence, slow extraction, resonance islands, coupling resonances, and synchro-betatron coupling.

The simple model has three degrees of freedom, with a variable linear coupling between the horizontal and longitudinal motion only (i.e., no linear coupling in the transverse plane). At startup, the particles are read in from a file, and the code opens three windows on the workstation screen: A plot window for displaying phase space information during the simulation, a data window for displaying current values of the parameters describing the model, and a 'beam current' window to display a record of the number of particles remaining in the machine as a function of turn number. The system then begins to iterate the particles, re-plotting the phase space at every turn. The ability to rep-lot this information quickly even for large numbers of particles ($\sim 10^4$) is the key feature enabling the interactive operation of such a program. The user may observe any of the three phase spaces available, or physical space in the two transverse dimensions.

2. Description of the Model

We begin with the expression for a full transfer matrix for a beamline with no RF cavity present and no linear transverse coupling[1-2]:

$$\mathbf{M_{t1}} = \begin{bmatrix} \cos\phi_x & \sin\phi_x & 0 & 0 & 0 & D_x \\ -\sin\phi_x & \cos\phi_x & 0 & 0 & 0 & D_{px} \\ 0 & 0 & \cos\phi_y & \sin\phi_y & 0 & 0 \\ 0 & 0 & -\sin\phi_y & \cos\phi_y & 0 & 0 \\ F1 & F2 & 0 & 0 & 1 & F3-\alpha \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where we assume $\beta_x = \beta_y = 1$ and $\alpha_x = \alpha_y = 0$ at both ends of the transfer. ϕ_x and ϕ_y are the horizontal and vertical phase advances. $-\alpha$ is change in the arrival time per unit change in the momentum in the absence of any dispersive effects (i.e. for uncoupled synchrotron motion.) We restrict ourselves to the case in which the contribution of the energy variation to the tranverse momentum (p_x) is 0 at both ends of the transfer. In this case the expressions for the quantities related to dispersion are:

$$\begin{aligned} D_x &= \eta_{x2} - \cos\phi_x \eta_{x1} \\ D_{px} &= \sin\phi_x \eta_{x1} \\ F1 &= -\sin\phi_x \eta_{x2} \\ F2 &= \cos\phi_x \eta_{x2} - \eta_{x1} \\ F3 &= \sin\phi_x \eta_{x1} \eta_{x2} \end{aligned} \tag{2}$$

The transfer matrix for a thin rf cavity (in the linear approximation) is trivially:

$$\mathbf{M_{rf}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & V & 1 \end{bmatrix}$$

where V is the energy change per unit longitudinal displacement.

We now construct an elementary 'lattice' as follows: The nonlinearity is at the end of the lattice, where there is nonzero dispersion η_x. The particles first transfer through betatron phases ϕ_x and ϕ_y to an RF cavity located at a location where $\eta_x = 0$, then through an RF cavity. Finally, the particles are transferred through a subsection with

$\phi_x = \phi_y = 0$ from the dispersion-free location back to the end of the lattice where the nonlinearity is located. In principle, the phase advances in the second transfer need not be zero, but for the present purposes it simplifies the results.

The simplified forms of Eq. (2) are:

$$\begin{aligned} D_x &= -\cos\phi_x \eta_x \\ D_{px} &= \sin\phi_x \eta_x \\ F1 &= 0 \\ F2 &= -\eta_x \\ F3 &= 0 \end{aligned} \tag{3}$$

The composite matrix for the first transfer and the RF cavity is the product $M_{rf} M_{t1}$:

$$\mathrm{M_{comp}} = \begin{bmatrix} \cos\phi_x & \sin\phi_x - \eta_x^2 V & 0 & 0 & 0 & -\eta_x \cos\phi_x \\ -\sin\phi_x & \cos\phi_x & 0 & 0 & 0 & \eta_x \sin\phi_x \\ 0 & 0 & \cos\phi_y & \sin\phi_y & 0 & 0 \\ 0 & 0 & -\sin\phi_y & \cos\phi_y & 0 & 0 \\ 0 & -\eta_x & 0 & 0 & 1 & \alpha \\ 0 & -V\eta_x & 0 & 0 & V & 1 + V\alpha \end{bmatrix}$$

For the second transfer, the relations corresponding to Eq. (2) are:

$$\begin{aligned} D_x &= \eta_x \\ D_{px} &= 0 \\ F1 &= 0 \\ F2 &= \eta_x \\ F3 &= 0 \end{aligned} \tag{4}$$

and the corresponding transfer matrix is then:

$$\mathrm{M_{t2}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \eta_x \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \eta_x & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The total linear transfer matrix is :

$$\mathbf{M_{total}} = \begin{bmatrix} \cos\phi_x & \sin\phi_x - \eta_x^2 V & 0 & 0 & \eta_x V & \eta_x\left(1 + V\alpha - \cos\phi_x\right) \\ -\sin\phi_x & \cos\phi_x & 0 & 0 & 0 & \eta_x \sin\phi_x \\ 0 & 0 & \cos\phi_y & \sin\phi_y & 0 & 0 \\ 0 & 0 & -\sin\phi_y & \cos\phi_y & 0 & 0 \\ -\eta_x \sin\phi_x & -\eta\left(1 - \cos\phi_x\right) & 0 & 0 & 1 & \alpha + \eta_x^2 \sin\phi_x \\ 0 & -\eta_x V & 0 & 0 & V & 1 + V\alpha \end{bmatrix}$$

At the end of the lattice is a single sextupole which applies a kick according to:

$$\Delta p_x = b\left(y^2 - x^2\right)$$

$$\Delta p_y = 2bxy$$

The table below lists the parameters of the model:

Table 1: Parameters for simple model.

Parameter	Units	Description
q_x	none	Horizontal Tune
q_y	none	Vertical Tune
η	meters	Horizontal Displacement per fractional energy deviation
α	meters	Longitudinal Displacement per fractional energy deviation
V	meters^{-1}	Fractional Energy change per longitudinal displacement
b	meters^{-2}	angular kick of sextupole

3. User's Guide

The code is invoked by typing

```
bdl <particle file name> <number of particles to iterate>
```

where the file contains the initial conditions of the particles to be iterated. The number of particles is specified because the user may wish to run fewer particles than are in the file to increase the iteration speed of the model at the expense of less detail in the phase space. Fig. 2 shows the three windows generated by the code. The Main window displays the plots on a turn by turn basis. The legends in the lower left and right corners provide the current turn number and number of particles currently in the machine. A particle is lost if it exceeds unit amplitude in either s or y. The numbers along the left and bottom axes show the scale of the plot. They are the values of their respective coordinates at the leftmost tick mark.

The Data window shows the current settings of the model. The numbers in the parentheses to the left of the parameters are the corresponding dial numbers used for setting the parameter in question. The longitudinal tune is given by

$$q_s = \sqrt{|\alpha V|}. \tag{5}$$

The viewing option selected is also displayed: Horizontal Plane, Vertical Plane, Longitudinal Plane, or X-Y Plane.

The "perf" window provides a perfmeter-style running histogram of the number of particles in the machine vs. turn number. Currently, the system is set for a maximum number of turns of 10000. The current run is displayed as a filled red area, while the result of the previous run is typically displayed as a yellow line in order to make comparisons easily. This line may be erased at any time by hitting the erase button.

The system is controlled by the dial and button box located to the left of the console. Fig. 1 shows the location of the pertinent buttons. The following is a brief description of the functions of each button.

- ☐ Quit
- ☐ Pause: stops the motion of the system if running, or resumes motion of the system if stopped. The button lights to denote the stopped condition.
- ☐ Erase: Causes the blue line showing the results of the previous run in the perf window to be erased the next time the perf window is updated.
- ☐ Reinject: Causes the particle file to be reread and the turn number set to 0.
- ☐ Zero buttons: sets the corresponding parameter to 0.

4. Activities

The fundamental purpose of laboratory exercises is the education of students, and it is in this spirit that the Beam Dynamics Laboratory is intended. The following is an exercise which a student might undertake immediately after his/her first introduction to nonlinear resonances. The student logs into the guest area on the Silicon graphics machine and changes to the `bdl` directory.

For the first two experiments use a collection of particles having a gaussian distribution in x and x' with a standard deviation of .2, and 0 in all other coordinates. This distribution is located in the file `d1bunch.dat` located in the `bunches` subdirectory. Immediately after starting the program, the horizontal phase space view should be selected. (For this particular lab, only the sextupole strength and the horizontal tune are adjusted.)

The effect of the third integer resonance from a sextupole is the first example of a nonlinear resonance typically encountered, and an excellent first choice for the laboratory as well. The tune should be set somewhat above the third integer, say $q_x = .345$. Then, the sextupole strength should be increased to a value of about .2. Note that there is already some change in the particle dynamics seen in the phase space monitor. The next step is to slowly adjust the horizontal tune towards the third integer resonance (.333) and observe the formation of the standard triangular phase space and loss of particles, as shown in Fig. 2

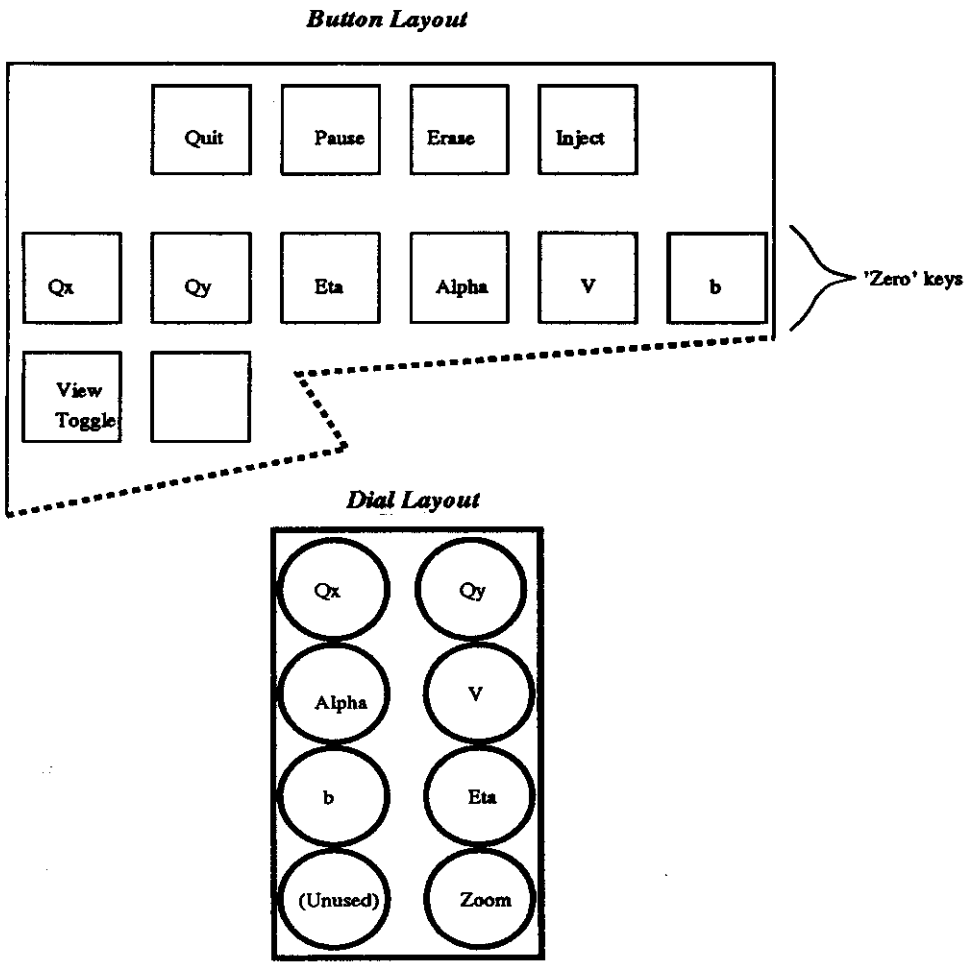

Figure 1: Layout of Buttons and Dials for BDL

The second experiment is to observe and populate the resonance islands associated with the 2/5 resonance. The tune is again set slightly above the resonant tune, such as .410. Next, begin slowly increasing the sextupole strength (and moving the resonant amplitude inwards) while observing the phase space monitor. As the resonant amplitude reaches the populated region of phase space, the islands will become visible as 'holes' (non-populated regions) in the phase space when the sextupole strength is slightly larger than 1. This is because slow variation of the machine shifts the separatrices associated with the resonance islands adiabatically, and the particles, which are initially outside the islands, remain outside. Continue increasing the sextupole strength until the holes are into the well-populated portion of the phase space, as shown in Fig. 3. At this point, perturb the

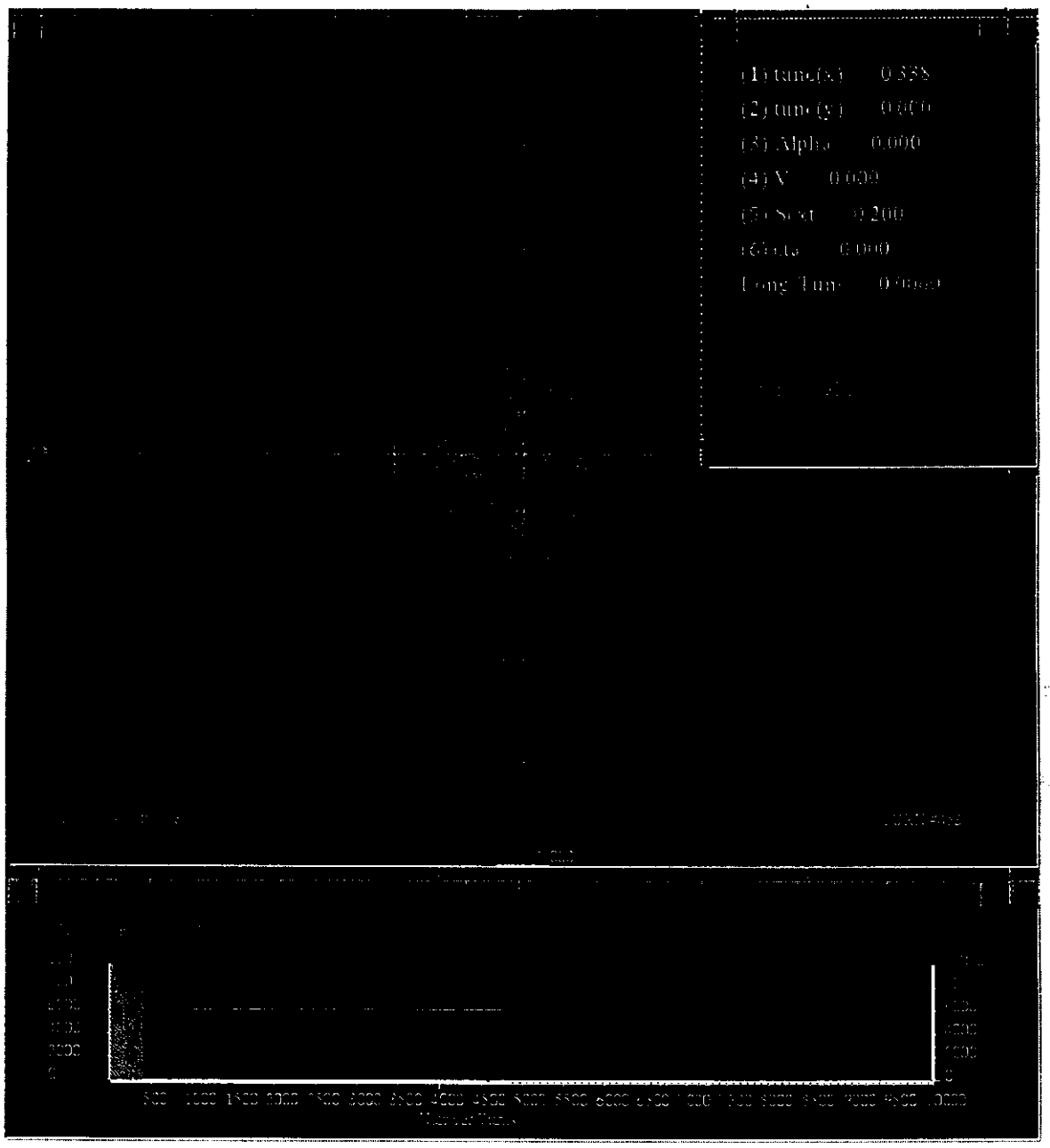

Figure 2: Phase space near third integer resonance

tune of the machine downwards slightly (.002), shifting the islands suddenly inwards and capturing particles *inside* the shifted islands. Observe the existence of an "island tune" for particle motion inside the islands, and observe decoherence within the islands gradually causing the islands to become invisible against the main portion of the beam. Finally,

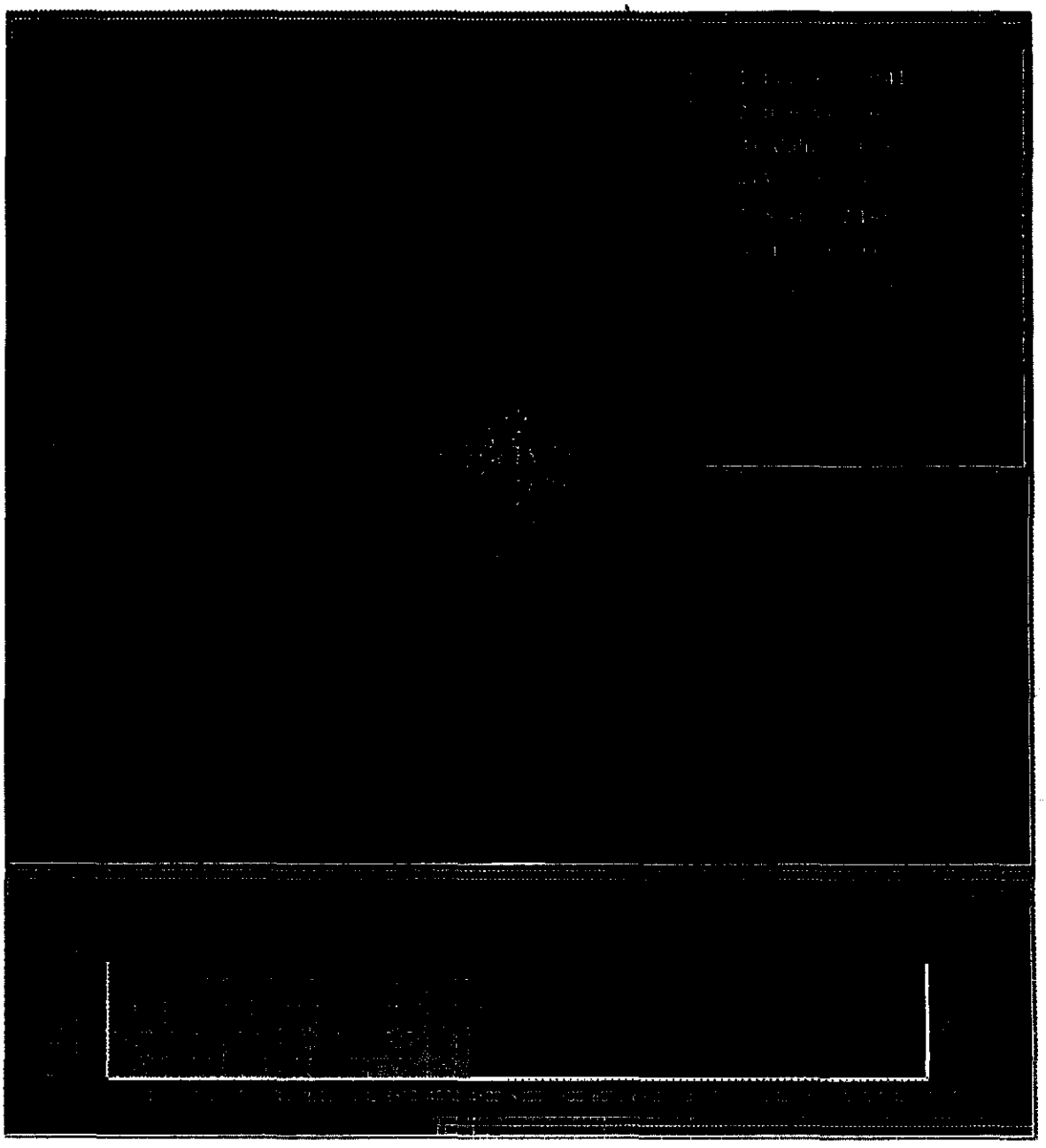

Figure 3: Phase monitor showing unpopulated islands against the particle distribution

slowly reduce the sextupole strength, gradually moving the now-populated islands to large amplitudes, away from the center of the bunch, as shown in Fig. 4. Finally, adjust the tune slightly again and observe the decoherence of the islands as the resonance amplitude is suddenly shifted away from the particles.

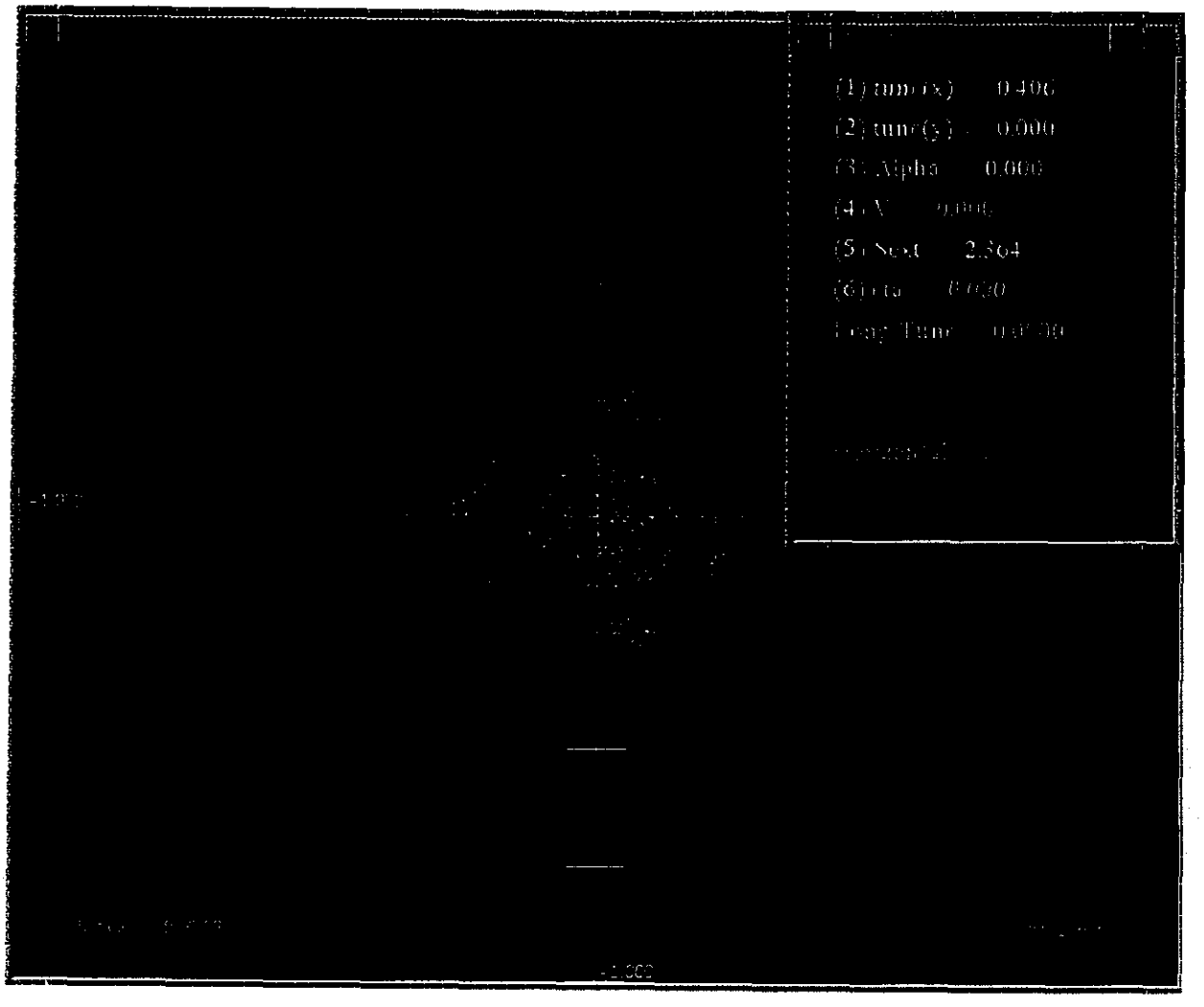

Figure 4: Phase monitor showing filled islands extracted from the population.

Islands can also be populated by injecting a distribution of particles directly into an already-created island. (or said differently, injection into an already nonlinear machine.) This may be observed by running the code on the distribution found in `dloffset.dat`. This is a more compact distribution (standard deviation of .1), and is offset by .5 in x'. Setting the tune to .410 and the sextupole strength to 1.25 and re-injecting the bunch, *observe the population of a single island.* Note that the particles 'hop' from island to island. Because beam monitors record the location of the centroid of the beam, this stable motion will result in a persistent signal. This experiment is the numerical analogue of the Fermilab E778 experiment[3].

Next the behavior of particles in the presence of a two-dimensional resonance may be studied. For this experiment, the particle distribution is gaussian in x, x', y, and y', again with a standard deviaton of .2, and is found in the file **d2bunch.dat**. The horizontal and vertical tunes are set to $q_x = .23$, $q_y = .383$, tunes which are not themselves resonant. Next, the sextupole strength is set to .05, a small value. Continued observation of the particle monitor shows that a significant portion of the beam is lost, however, on the time scale of 1000 turns. (see Fig. 5) Observe the horizontal phase space monitor while this loss takes place and notice the halo formed around the central core of the beam. Once the particle loss stops, increase the vertical tune to .384, and observe whether further particle loss takes place? Repeat this experiment for .383, and finally .385. Next, return the vertical tune to .380 and increase the sextupole strength to .1. Re-inject the bunch and observe the particle loss. By using the number of particles surviving 1000 turns after injection as a figure of merit, the width of the resonance in q_x and q_y may be mapped out.

The final experiment concerns the effect of longitudinal motion on resonances. For this experiment, you will use the three dimensional bunch found in d3bunch.dat, which has particles distributed as a gaussian with a variance of .2 in all phase space variables. First, set up a similar situation to the previous experiment: $q_x = .220$, $q_y = .385$, $b = .1$, and notice a small particle loss from the nearby resonance. Now, adjust $\alpha = -.001$, $V = .001$. Toggle the view to the longitudinal plane and note the rotation of the bunch in longitudinal phase space. At this point, while the two transverse planes are coupled together, the longitudinal motion is not coupled to either. Now introduce such coupling by increasing the value of η to .2, Re-inject the bunch and allow it to run for 10^4 turns. Notice onset of slow loss near 1000 turns (Fig. 6). Because the variation of momentum in this scaled system is much larger than for real machines (here $\sigma_p/p = .2$), the small value for eta is deceptive. The product of the dispersion and the rms momentum here is .004, while for the Tevatron $\bar{\eta} = 3$, $\sigma_p/p = .0002$. This was done purposefully in order to compress the slow particle loss into 10^4 turns.

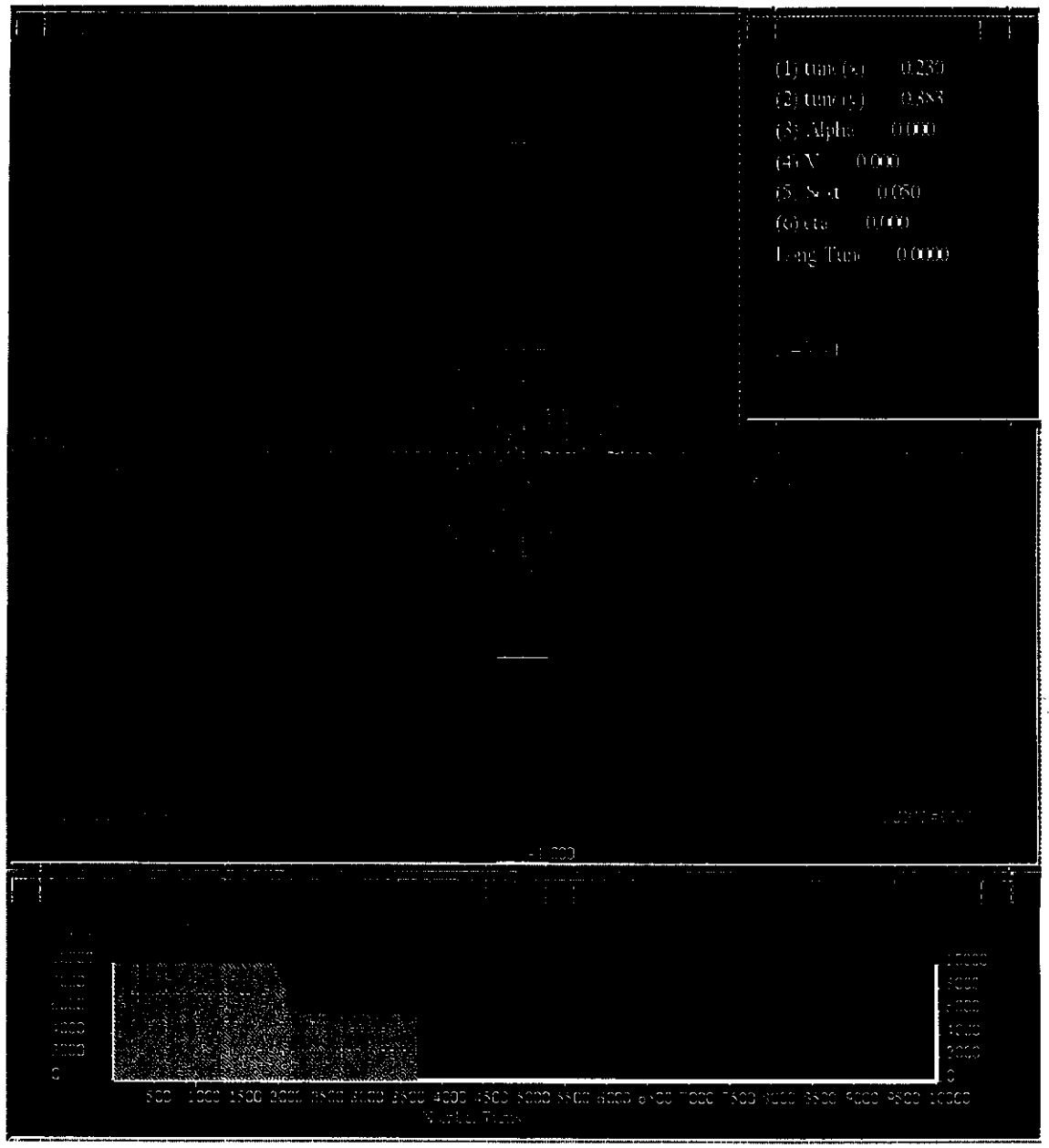

Figure 5: Configuration space view of two-dimensional resonance and particle loss

5. Conclusion

The Beam Dynamics Laboratory is a useful tool for students to become accustomed to manipulating beams in accelerators. Students may gain intuition and experience about some aspects of the behavior of beams without requiring access to actual accelerators.

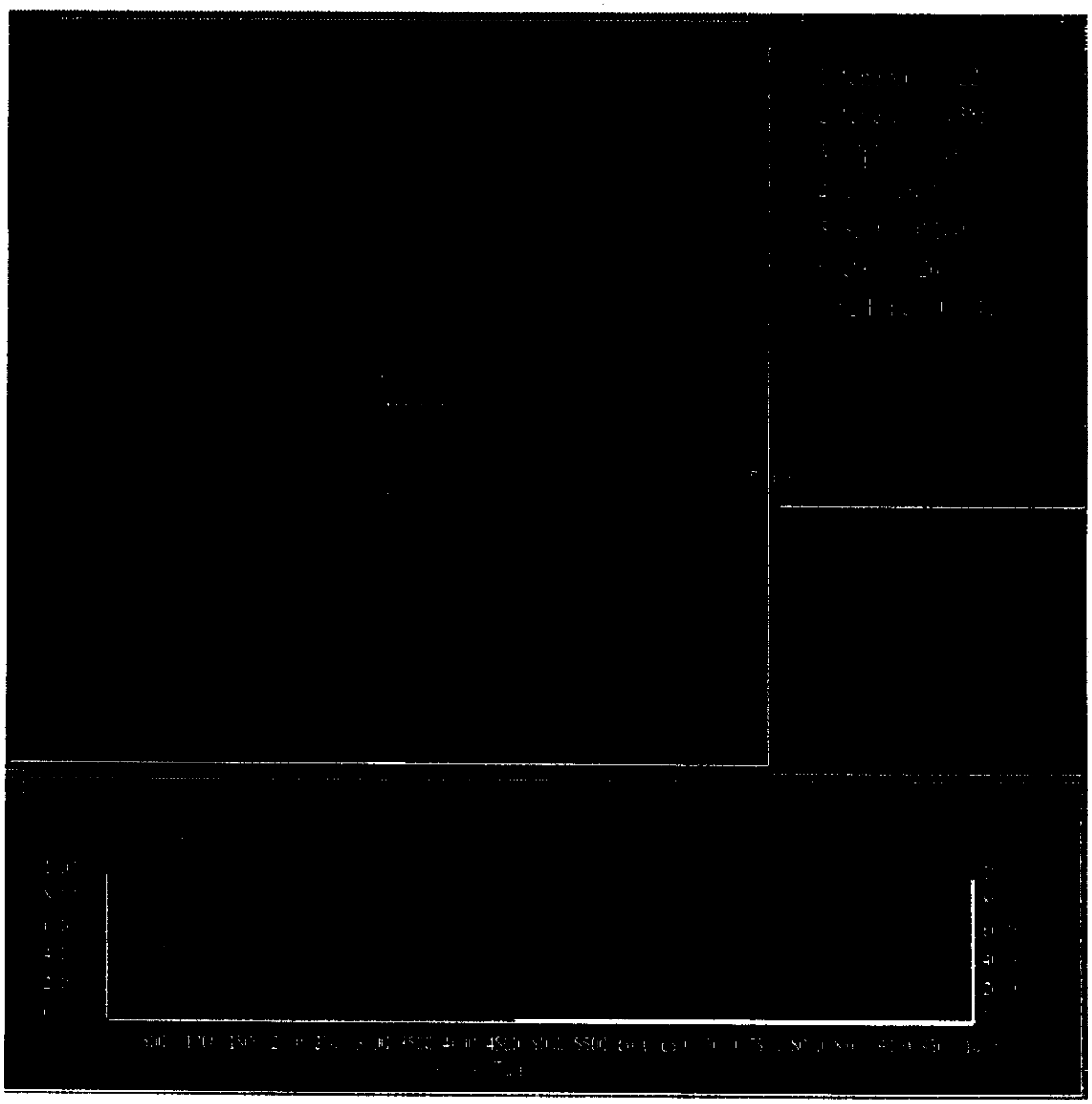

Figure 6: Results of run with three degrees of freedom

Other types of nonlinearities (octupole, weak-strong beam-beam) or other simple effects (tune modulation, gas scattering, dipole ripple, etc.) may be reasonably added, enabling the study of their effects in different operating scenarios. Possible display enhancements include emittance plots and loss characterization display (horizontal vs. vertical). Similarly, while the current (admittedly elementary) representation of the accelerator presents few

opportunities for the researcher, the BDL may be used as a testbed for steered simulation. Certainly, if "maps with knobs" are ever clearly developed they could be implemented in a similar manner. As the complexity of the map is increased, the iteration speed for a given number of particles will decrease, but these issues are in principle tractable.

Readers interested in trying the system out are welcome to contact B. Cole. We wish to thank George Bourianoff and Garry Trahern for helpful comments and criticisms. Further criticisms and suggestions are welcomed.

Bibliography

1. Cole, B. Ph. D. Thesis, In preparation..
2. H.-J. Shih, J. Ellison, B. Newberger, and R. Cogburn, Longitudinal Beam Dynamics with RF Noise, SSCL-578.
3. Chao, A. et. al. Phys. Rev. Lett. 61, 2752-5.