SSCL-N-827

A Simple Model for Normal Sextupole Components b_2 in HEB Dipoles from 200 GeV to 2 TeV

S. Ohnuma

Superconducting Super Collider Laboratory*
2550 Beckleymeade Ave.
Dallas, TX 75237

July 1993

*Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

A Simple Model for Normal Sextupole Components b_2 in HEB Dipoles from 200 GeV to 2 TeV

S. Ohnuma

University of Houston

1. Introduction

The purpose of this note is to present a simple but plausible model of the normal sextupole component b_2 in HEB dipoles (which do not yet exist) between the injection flat at 200 GeV and the extraction flattop at 2 TeV. The particular numerical values for various quantities given here should not be taken as something one expects to have in the final HEB dipoles. At present, there are too many uncertainties surrounding the characteristics of (yet to be designed and built) HEB dipoles that even a semi-quantitative estimate may be premature. It is nevertheless necessary to have a qualitative picture of the behavior of b_2 as a function of the ramping dipole current from beginning to end in order to prepare believable correction schemes. By far the most important of the correction elements is the chromaticity correcting sextupoles together with steering magnets for closed orbits and correction quadrupoles for tune adjustments. According to the available data on dipoles built for the main collider, there will be a large variation from dipole to dipole in various multipole components, including b_2, and one must look into the necessity of harmonic correction systems. Particularly troublesome is the skew quadrupole component a_1 at low excitation. It may be necessary to revive an old idea of magnet sorting based on a_1 and b_2 if one is to avoid elaborate correction systems. Questions related to harmonic corrections are, however, not addressed in this note except to emphasize the importance of further studies by the HEB Group.

Various sources of the information used in preparing this note are:

1. M. Wake(SSC/KEK), "Harmonic Field Estimation for HEB Operation", MD-MKA-0-93-007(SSCL), March 31 1993.

2. A. Chao, et al., "Dynamic Aperture & Extraction Studies for the SSC High-Energy Booster", SSCL-296, September 1990. (See p. 5.)

3. D. Johnson, "Control of b_2 at 150 GeV and During TeV Parabola", Fermilab EXP-160, August 22 1988.

4. S. Ohnuma, "Sextupole Component b_2 of Doubler Dipoles at Low Fields", Fermilab UPC No. 172, December 6 1983.

5. Private communications from the following people:

   a) M. Wake, SSC/KEK

   b) T. Ogitsu, SSC/MTL

   c) D. Herrup, Fermilab

   d) R. Hanft, Fermilab

   e) N. Gelfand, Fermilab

   f) W. Xie, SSC/Accelerator Systems

Any misrepresentations and errors which may have seeped into this note are all purely unintentional. Anyone with serious reservations and doubts on data used here should notify the author so that, if warranted, corrections can be made promptly. Matters of judgments (and not of factual data) should be discussed in a separate memo or report. It is always advisable to present editorial opinions clearly labeled as such in order not to confuse the issues which are already complex for everyone concerned.

2. Ramping of Main Dipoles

Even this basic design item is not yet firmly established at present. One outstanding question is how to switch magnet polarity, every other batch to fill two collider rings alternately or to switch only once after one ring is completely filled. It may be easier for the HEB power supplies to adopt the latter option but the former is by no means excluded if the stability of reversed ramping is assured. The ramping used here is only for one polarity, ignoring the possible complication of reversing the polarity with some gymnastics.

1. Injection flat: $0 < t < t_1$ $(t_1 \equiv T_{in})$

2. Parabola: $t_1 < t < t_2$ $(t_2 - t_1 \equiv T_P)$

3. Linear part: $t_2 < t < t_3$ $(t_3 - t_2 \equiv T_L)$

4. Parabola: $t_3 < t < t_4$ (identical to 2.)

5. Extraction flattop: $t > t_4$

During injection flat, I (dipole current) = I_{in} (fixed) with B_{in} (in T)= T.F.(in T/kA) x I_{in} (in kA), B_{in}= $B\rho$ in T-m/ρ, B_ρ = 200/0.299793, ρ = (number of dipoles)*(magnet length)/(2π) = (12.288m x 512/(2π) = 1001.32 m.

The transfer function T.F. is yet to be determined but it is expected to be close to 1.02 which should be compared with 1.0452 for the collider dipoles. The magnet length is taken to be 12.288m (14.856 m for the collider) and the total number of dipoles for the current version of HEB lattice is 512.

Before fixing other ramp parameters, it is necessary to estimate the final current I_4 (for $t > t_4$) at the flattop. Although the final beam momentum is fixed at 2 TeV, there is a question of saturation so that

$$I_4 = (\text{Saturation Factor}) \times B_f/\text{T.F. with } B_f = 6.6625\text{T at 2 TeV.}$$

The saturation is assumed to be 2.5% (Saturation Factor = 1/0.975) at the maximum current, rising quadratically from I = 4 kA.

During the first parabola,

$$I(t) = I_{in} + \alpha(\Delta t)^2;\ \Delta t = \text{time measured from the start of parabola} \quad (1)$$

$$\alpha =(1/2)(I_4 - I_{in})/[T_P\,(T_P + T_L)]$$

In the linear rise between t_2 and t_3, dI/dt is constant:

$$\dot{I}_\circ = dI/dt\ (t_2 < t < t_3) = 2\alpha T_P \quad (2)$$

During the second parabola, which is assumed to be identical in shape to the first parabola,

$$I(t) = I_4 - \alpha(\Delta t)^2;\ \Delta t = t - t_4 \quad (3)$$

Once I is found as a function of time, the momentum of the beam is determined from the resulting $B\rho$ with the saturation effect taken into account beyond 4 kA (the value assumed in this note).

3. Scaling

Since it is necessary to rely on the data from Fermilab Tevatron dipoles until more data are accumulated on HEB dipoles (or at least on collider dipoles), scaling for multipole components is needed. According to ref. 2, the most comprehensive report on this is

> H.E. Fisk, et al., "Preliminary Report of the Magnetic-Errors Working Group of the SSC Aperture Workshop", SSC-7 (1985).

One starts by defining the effective coil radius r_e:

$$r_e = (\text{dipole aperture})/2 + 1 \text{ cm} \tag{4}$$

That is, for Tevatron dipoles with aperture = 3", $r_e = 4.81$ cm and for HEB dipoles with aperture = 5 cm, $r_e = 3.5$ cm. The systematic sextupole component scales as $(r_e)^{-3}$ so that

$$b_2 \text{ (HEB)} = b_2 \text{ (Tevatron)} \times 2.60$$

Keep in mind, though, that b_2 (Tevatron) is in $10^{-4}/\text{in}^2$ while b_2 (HEB) is in $10^{-4}/\text{cm}^2$. Conversion for b_2 from Tevatron to HEB dipoles is (including inch to cm conversion)

$$b_2 \text{ (HEB) in } 10^{-4}/\text{cm}^2 = 0.40 \times b_2 \text{ (Tevatron) in } 10^{-4}/\text{in}^2$$

Conversion such as this will of course become obsolete as the data on real HEB dipoles are accumulated in the (near? distant?) future.

4. Geometrical and Persistent Current Sextupoles

In order to compensate for a large negative contribution from two ends, the Tevatron dipole was designed to have a positive sextupole in the body of magnet. This involves a three-dimensional code such as TOSCA. As the HEB dipole is much longer (more than 12 m *vs.* 6 m for Tevatron), the end contribution will be less significant. Contribution arising from saturation may also play a role in the design of geometrical sextupole value.

Contribution from the persistent current may be dominant in the low field excitation. This gradually decreases due to the decrease in the critical current of the superconductor as well as the increasing dipole field. Note that b_2 is the normal sextupole field in dipoles <u>relative to</u> the main bend field.

In this note, it is assumed that b at 200 GeV is -2.25 units which decreases to -0.03 units at 2 TeV. The change in this part of b is taken to be proportional to $\sqrt{I(t) - I_{in}}$. This is mostly a matter of convenience without much data to support, but the change should be more rapid at low field than at high field.

5. "Drift" and "Snapback"

Ever since the phenomenon of "drift in b_2" has been discovered at Fermilab in the Tevatron operation, there have been many discussions on this subject. It is well known by now that the amount of drift during the injection flat depends significantly on the history of magnet excitation but the dependence on time seems to be logarithmic rather than exponential. For details, see for example data in ref. 3. The particular dependence is of course not important for the HEB operation since the injection lasts only for 20 seconds or so (compared with many minutes for the Tevatron) during which logarithmic and exponential are indistinguishable.

In this model, one specifies the amount of drift from $t = 0$ to $t = t_1$ (end of injection flat) together with the time constant if the dependence is exponential. If the dependence is logarithmic, one must specify the amount of drift from, say, $t = 0.1$s to $t = t_1$ without any specific time constant. For the example given in this note, logarithmic dependence is assumed with the amount of drift in b_2 during 20 seconds of injection flat to be 0.35 units. It is important to emphasize once more that values such as this must be up- dated as more data become available. None of them should be regarded as "final".

As soon as the ramp goes into the first parabola region, $t > t_1$, the drifting sextupole component "snaps back" to the pre-drifting value at $t = 0$. The duration in which this happens is not determined by time but by the change in the excitation current I. According to Wake, the change happens during the first 30A while Ogitsu believes it will happen in less than 10A. This must be studied further with the real HEB dipole. It is assumed here, for the sake of certainty, that the snapback will be in the first 10A and the dependence is quadratic in the increase in current, the rate of change taking the maximum value at the start of parabola and zero at $\Delta I = I(t)-I_{in} = +10$A. Since the duration in time is expected to be less than a second, the precise current dependence should not be significant.

6. Eddy Current Sextupole

Wake and Ogitsu believe that the eddy current in the dipole cable depends primarily on the contact resistance between strands which can be estimated from the measured ac loss. The type of insulation (Ebanol, Zebra, Tiger, ...) certainly determine the value of contact resistance r_c but even with the same type, the variation from magnet to magnet is very large. Furthermore, as the sextupole component due to the eddy current is quite sensitive to its distribution (variation from turn to turn in a magnet, for example), it is possible to have a large variation in b_2 from magnet to magnet. Wake suggests b_2 arising from eddy current to be distributed from 0.5 to 4.5 units at the injection field. Although this may be an overestimate, one must be prepared for the necessity of harmonic sextupole correction. Again, it is too early to make a definite plan without reliable data. Most (but not all) measurements have been done with dI/dt = 4 A/s (expected in the collider), and data are then extrapolated to the HEB value of dI/dt = 60 A/s.

Ogitsu has suggested a simple model based on a uniform contact resistance r_c in a magnet. In this model, the sextupole field B_2 (eddy) obeys the equation

$$B_2 + \tau(dB/dt) = k\,(dI/dt) \tag{5}$$

where τ is the time constant (in second). The parameter k, in T/(A/s), specifies the amount of sextupole field. The driving term dI/dt is constant in the linear region of the ramp but it varies linearly with time in the parabola region. The differential equation must be solved with the initial condition (at $t = t_1$, the start of parabola) $B_2 = 0$. Subsequently, at $t = t_2$, t_3 and t_4, it must be connected smoothly.

It is easy to see that B_2 will be proportional to $2\alpha k$ where α is the parameter defined in Eqs.(1) and (2). Ogitsu uses a quantity B_{2c} in 10^{-4} T$\mu\Omega$/(A/s) together with r_c to specify the eddy current characteristics. The time constant τ is determined from the relation

$$\tau(\text{in s}) \sim 10/r_c; \;\; r_c \text{ in } \mu\Omega, \tag{6}$$

while B_{2c} is related to k by

$$k = (B_{2c}/r_c) \tag{7}$$

so that $2\alpha k = (B_{2c}/r_c)(\dot{I}_o/T_P)$ with $\dot{I}_o$ = dI/dt in the linear part of the ramp and $T_P = t_2 - t_1$ = parabola dulation in seconds.

A messy but straightforward calculation yields the following form for B_2 in various regions of the ramp:

i) Injection flat. $B = 0$.

ii) First parabola. $(t_1 < t < t_2)$

$$B_2(t) = 2\alpha k \{\Delta t - \tau E(\Delta t)\} \tag{8}$$

with $\Delta t = t - t_1$ and $E(\Delta t) = 1 - \exp(-\Delta t/\tau)$.

iii) Linear rise. $(t_2 < t < t_3)$

$$B_2(t) = 2\alpha k \{T_P - E(T_P)\exp(-\Delta t/\tau)\} \tag{9}$$

with $\Delta t = t - t_2$ and $E(T_P) = 1 - \exp(-T_P/\tau)$.

iv) Second parabola $(t_3 < t < t_4)$

$$B_2(t) = 2\alpha k \{T_P + \tau - (\Delta t) - \tau E(T_P, T_L)\exp(-\Delta t/\tau)\} \tag{10}$$

with $\Delta t = t - t_3$ and

$E(T_P, T_L) = 1 + [1 - \exp(-T_P/\tau)] \exp(-T_L/\tau)$.

v) Flattop. $(t > t_4)$

$$B_2(t) = B_2(t_4)\exp(-\Delta t/\tau) \text{ with } \Delta t = t - t_4 \tag{11}$$

7. Natural Chromaticity and Chromaticity Due to b_2.

These have been found (with some hand calculations) from a DIMAD output supplied to me by Mingyang Li.

$$\xi_x = -52 + 86\, b_2; \quad \xi_y = -53 - 72\, b_2 \tag{12}$$

where b_2 is in the standard unit of $10^{-4}/\text{cm}^2$. In estimating two coefficients, +86 and -72, the linear lattice parameters in each dipole are averaged over the length of dipole.

8. Required Current of Correction Sextupoles

Based on the design by Peilei Zhang, one can find the required strength of correction sextupoles (assume here to have the same value in all F-type sextupoles and in all D-type sextupoles, on two independent power supplies). In order to create compensating chromaticities $\Delta\xi_x$ and $\Delta\xi_y$, the required strengths are

$$S_F = (4.75 \times 10^{-4})(\Delta\xi_x) + (0.89 \times 10^{-4})(\Delta\xi_y),$$

$$S_D = -(1.80 \times 10^{-4})(\Delta\xi_x) - (9.44 \times 10^{-4})(\Delta\xi_y) \qquad (13)$$

where $S_{F,D} = (B''L/B\rho)$ of each sextupole with length L, in units of $(\text{meter})^{-2}$.

The corresponding current in each sextupole is estimated from the design by Wanfen Xie ("Ordered-Wound" type):

$$(1/2)(B''x^2 L) \text{ at } x = 1 \text{ cm, in T-m} = 0.00243\ I(A) - 0.61 \times 10^{-5}\ I^2 \qquad (14)$$

in which the second term representing saturation is not important for I < 40A.

One specific example is given in Appendix, showing among other things that

a) Snapback does not pose any difficulty for correction sextupole power supply.

b) The maximum current is 10A.

Again it is important to keep in mind that almost all parameters used in this example are still very preliminary and the HEB group must be aware of any development at the MTL. A close collaboration between magnet group and accelerator group is absolutely essential for the project to be successful.

Appendix: An Example of Chromaticity Correction Requirements

T.F. = 1.02, T_{in} = 20s, T_P = 1.5s, T_L = 100s, b_2 = -2.25 at 200 GeV to -0.03 at 2 TeV, amount of drift = 0.35 in 20s, snapback in 10A, Ogitsu's parameters B_{2c} = 0.45, r_c = 18($\mu\Omega$)

Saturation at full current is 2.5% starting from 4kA

****

time(s)	current(kA)	field(T)	p(GeV/c)	b2_eddy	b2	SF(A)	SD(A)
1. Injection							
2.0	0.653	0.666	200.0	0.000	-2.169	1.44	0.74
4.0	0.653	0.666	200.0	0.000	-2.088	1.40	0.68
6.0	0.653	0.666	200.0	0.000	-2.041	1.38	0.65
8.0	0.653	0.666	200.0	0.000	-2.007	1.36	0.62
10.0	0.653	0.666	200.0	0.000	-1.981	1.35	0.61
12.0	0.653	0.666	200.0	0.000	-1.960	1.34	0.59
14.0	0.653	0.666	200.0	0.000	-1.942	1.33	0.58
16.0	0.653	0.666	200.0	0.000	-1.926	1.32	0.57
18.0	0.653	0.666	200.0	0.000	-1.912	1.32	0.56
20.0	0.653	0.666	200.0	0.000	-1.900	1.31	0.55
2. Parabola							
20.1	0.653	0.666	200.1	0.013	-1.901	1.31	0.55
20.2	0.654	0.667	200.2	0.048	-1.906	1.31	0.55
20.3	0.655	0.668	200.5	0.101	-1.913	1.32	0.56
20.4	0.656	0.669	201.0	0.170	-1.917	1.32	0.56
20.5	0.658	0.671	201.5	0.252	-1.909	1.32	0.56
20.6	0.660	0.674	202.2	0.344	-1.878	1.31	0.54
20.7	0.663	0.676	203.0	0.443	-1.806	1.28	0.49
(end of snap back)							
20.8	0.666	0.679	203.9	0.550	-1.599	1.19	0.34
20.9	0.669	0.683	204.9	0.661	-1.475	1.13	0.25
21.0	0.673	0.687	206.1	0.776	-1.347	1.08	0.16
21.1	0.677	0.691	207.4	0.893	-1.217	1.02	0.06
21.2	0.682	0.695	208.8	1.012	-1.086	0.96	-0.04
21.3	0.687	0.700	210.3	1.131	-0.954	0.90	-0.13
21.4	0.692	0.706	211.9	1.250	-0.821	0.84	-0.24
21.5	0.698	0.712	213.7	1.369	-0.690	0.78	-0.34

3. Linear rise

21.6	0.704	0.718	215.5	1.476	-0.571	0.73	-0.43
21.7	0.710	0.724	217.3	1.561	-0.474	0.68	-0.51
21.8	0.716	0.730	219.2	1.629	-0.395	0.65	-0.58
21.9	0.722	0.736	221.0	1.683	-0.331	0.62	-0.63
22.0	0.728	0.742	222.8	1.725	-0.279	0.60	-0.68
22.1	0.734	0.748	224.6	1.757	-0.237	0.58	-0.72
22.2	0.740	0.754	226.4	1.781	-0.204	0.56	-0.75
22.3	0.746	0.760	228.3	1.798	-0.178	0.55	-0.78
22.4	0.751	0.766	230.1	1.810	-0.157	0.55	-0.80
22.5	0.757	0.773	231.9	1.817	-0.141	0.54	-0.82
22.6	0.763	0.779	233.7	1.821	-0.129	0.54	-0.84
22.7	0.769	0.785	235.6	1.822	-0.120	0.54	-0.85
22.8	0.775	0.791	237.4	1.820	-0.114	0.54	-0.86
22.9	0.781	0.797	239.2	1.817	-0.110	0.54	-0.87
23.0	0.787	0.803	241.0	1.812	-0.108	0.55	-0.88
23.5	0.817	0.833	250.2	1.770	-0.114	0.57	-0.91
24.5	0.877	0.894	268.4	1.570	-0.253	0.70	-0.84
25.5	0.936	0.955	286.6	1.545	-0.225	0.73	-0.93
26.5	0.996	1.016	304.9	1.464	-0.258	0.80	-0.95
27.5	1.055	1.076	323.1	1.383	-0.294	0.88	-0.97
28.5	1.115	1.137	341.4	1.310	-0.327	0.95	-0.98
29.5	1.174	1.198	359.6	1.243	-0.355	1.03	-1.00
30.5	1.234	1.259	377.8	1.183	-0.379	1.10	-1.02
35.5	1.532	1.562	469.0	0.953	-0.451	1.45	-1.14
40.5	1.830	1.866	560.2	0.798	-0.473	1.76	-1.32
45.5	2.127	2.170	651.4	0.686	-0.468	2.04	-1.55
50.5	2.425	2.474	742.6	0.602	-0.446	2.29	-1.83
55.5	2.723	2.778	833.8	0.536	-0.415	2.51	-2.15
60.5	3.021	3.081	925.0	0.483	-0.377	2.70	-2.51
65.5	3.319	3.385	1016.2	0.440	-0.336	2.87	-2.91
70.5	3.617	3.689	1107.4	0.404	-0.292	3.01	-3.35
75.5	3.915	3.993	1198.6	0.373	-0.247	3.13	-3.82
80.5	4.212	4.296	1289.6	0.347	-0.200	3.22	-4.34
85.5	4.510	4.596	1379.8	0.324	-0.153	3.29	-4.88
90.5	4.808	4.893	1468.9	0.304	-0.105	3.34	-5.45
95.5	5.106	5.186	1556.8	0.287	-0.058	3.36	-6.06
100.5	5.404	5.475	1643.4	0.272	-0.010	3.36	-6.69
105.5	5.702	5.758	1728.4	0.259	0.037	3.34	-7.34
110.5	5.999	6.035	1811.8	0.247	0.084	3.29	-8.02
115.5	6.297	6.307	1893.3	0.236	0.131	3.23	-8.71
120.5	6.595	6.572	1972.7	0.227	0.177	3.14	-9.43

121.5	6.655	6.624	1988.3	0.225	0.187	3.12	-9.57
4. Parabola							
121.6	6.660	6.629	1989.8	0.223	0.186	3.13	-9.58
121.7	6.666	6.633	1991.2	0.220	0.184	3.14	-9.56
121.8	6.671	6.638	1992.5	0.214	0.179	3.17	-9.54
121.9	6.675	6.642	1993.7	0.207	0.173	3.20	-9.50
122.0	6.679	6.645	1994.8	0.199	0.165	3.24	-9.44
122.1	6.683	6.649	1995.8	0.189	0.156	3.28	-9.38
122.2	6.687	6.651	1996.7	0.179	0.146	3.33	-9.32
122.3	6.690	6.654	1997.5	0.168	0.136	3.38	-9.24
122.4	6.692	6.656	1998.1	0.156	0.125	3.44	-9.16
122.5	6.694	6.658	1998.7	0.144	0.113	3.50	-9.07
122.6	6.696	6.660	1999.2	0.131	0.100	3.56	-8.98
122.7	6.698	6.661	1999.5	0.118	0.088	3.62	-8.89
122.8	6.699	6.662	1999.8	0.105	0.074	3.68	-8.79
122.9	6.699	6.662	1999.9	0.091	0.061	3.75	-8.69
123.0	6.699	6.662	2000.0	0.077	0.047	3.82	-8.59
5. Flattop							
124.0	6.699	6.662	2000.0	0.065	0.035	3.88	-8.50
125.0	6.699	6.662	2000.0	0.054	0.024	3.93	-8.42
126.0	6.699	6.662	2000.0	0.045	0.015	3.97	-8.35
127.0	6.699	6.662	2000.0	0.038	0.008	4.01	-8.30
128.0	6.699	6.662	2000.0	0.031	0.001	4.04	-8.25
129.0	6.699	6.662	2000.0	0.026	-0.004	4.06	-8.21