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Circulation of Heat Loads in Design

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CALCULATION OF HEAT LOADS IN DESIGN

Recommendations for designers
of the Institute of Nuclear Physics,
Siberian Department of the Academy of Sciences of the USSR
[INP, SD AS USSR]

Author B. Z. Persov

This book begins the series "Recommendations for designers," which is devoted to various aspects of the design of electrophysical installations.

This paper contains basic data for calculation of the elements of constructions that operate under conditions of heat loads. Methods of calculation of heat release for various forms of heat sources and heat removal under various conditions of cooling are given. Special characteristics of calculation for pulsed heat effects are discussed, as well as calculation of elements that operate in a nonsteady-state heat regime. All of the calculation formulas are given in a form that is convenient for design calculations and their use is illustrated by detailed examples of calculation.

Introduction

The serviceability of any device, i.e., the performance by a device of its functions, is determined first by properly completed design calculation. In the process of this calculation the strength of the parts of the device, the accuracy of performance of assigned movements or retention of assigned geometry, the capacity

of the device to complete assigned functions in the course of a required period of time, and so forth, and analyzed.

However, even a completely properly completed calculation of all these parameters for normal conditions still does not assure serviceability of a device, if the release of some amount of heat occurs in it during operation.

The fact is that in the process of the heating of parts the limits of strength and flow and the modulus of elasticity of materials become lower. Allowances for stresses decrease by comparison with those calculated for normal conditions, while strains of parts increase. In addition, because of the temperature-related changes of the dimensions of parts gaps in joints change, which may cause, for example, individual parts to become jammed or to stick.

Moreover, if the heating of individual parts is not uniform through the cross section, in these parts additional temperature stresses and strains arise.

Many electrophysical devices also make specific demands on the heatup of parts during operation. Thus, parts that are located in a vacuum must not heat up above a certain temperature, if such heatup will cause an unacceptable increase of gas emission. Excessive heatup of parts in a high-voltage device may lead to electrical breakdown. It is quite obvious that to assure the serviceability of a device all of these factors must be taken into account by the designer in designing it.

Necessary conditions for solving this problem are:

1) ability to find the actual sources of heat in a design and to evaluate amounts of heat being produced;

2) ability to calculate the process of heat removal so as to maintain working temperatures at a permissible level according to calculation results.

Basic questions connected with calculation of heat release and heat removal in the elements of constructions are discussed in this handbook.

1. Heat release in parts

Sources of heat can be quite varied: passage of an electrical current through parts, radiation on the part of some heated body, bombardment by charged particles, and so forth. Let us consider the main sources of heat, in order to understand the basic principles of calculation of these factors.

1.1 Passage of a current through a conductor

1.1.1 Continuous regime

It is well known that the power generated in a conductor when a constant electrical current passes through it is

$$P = I^2R$$

$$\text{or} \qquad P = U^2/R \tag{1}$$

where I is the current flowing through the conductor, A;

U is the voltage across the ends of the conductor, B;

R is the resistance of the conductor, ohm;

P is the power generated in the conductor, W.

However, the situation is this simple only when a direct current passes through the conductor. If the current is alternating, expression (1) becomes:

$$P = I^2_{eff}R$$

$$\text{or} \qquad P = U^2_{eff}/R \tag{2}$$

where I_{eff} is the effective value of current, i.e., the value of direct current which, flowing through the conductor, would lead to the generation of the same power P as it is generated in the passage of the given alternating current;

U_{eff} is the effective value of voltage, i.e., the value of direct voltage, which, being applied to the conductor, would lead

to the generation of the same power P as it is generated in the passage of the given alternating current.

The effective values of currents and voltages for the case of periodically varying current $I(\tau)$ and voltage $U(\tau)$ are given by the expressions:

$$\left.\begin{aligned} I_{eff} &= \sqrt{\frac{1}{T}\int_0^T I^2(\tau)\,d\tau} \\ U_{eff} &= \sqrt{\frac{1}{T}\int_0^T U^2(\tau)\,d\tau} \end{aligned}\right\} \quad (3)$$

where $I(\tau)$ is the dependence of the change of current on time;

$U(\tau)$ is the dependence of the change of voltage on time;

T is the period of change of the current and voltage (Figure 1).

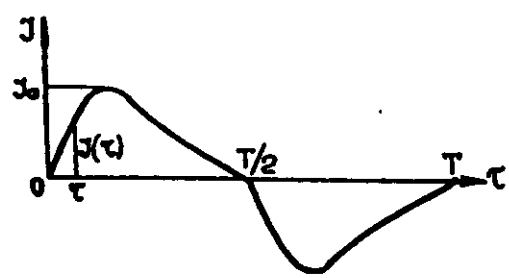


Figure 1

For a sinusoidally varying current it is easy to find that

$$I_{eff} = \frac{I_a}{\sqrt{2}} \text{ and } U_{eff} = \frac{U_a}{\sqrt{2}}$$

from whence follow the well-known expressions:

$$\left.\begin{aligned} P_{sin} &= \frac{I_a^2 R}{2} \\ P_{sin} &= \frac{U_a^2}{2R} \end{aligned}\right\} \quad (4)$$

In the case of an alternating current a different approach is also needed with regard to the resistance of the conductor.

The interaction of an alternating current flowing through a conductor with the magnetic field created by this current leads to the so-called skin effect, which consists of the displacement of current onto the surface of the conductor. As a result, with distance from the surface of the conductor deeper into the material of the conductor the current decreases exponentially. For calculation of the power generated in the conductor one can use expressions (2), if one provisionally considers that a direct current with magnitude I_{eff} flows through some surface layer of the conductor, which is called the skin layer.

The magnitude of the skin layer is determined by the expression:

$$\delta_{sk} = \sqrt{\frac{\rho}{\pi \mu_0 \mu' f}}, \text{ m} \qquad (5)$$

where ρ is the resistivity of the material, ohm·m;

$\mu_0 = 0.4\ \pi \times 10^{-6}$ H/m is the absolute magnetic permeability of vacuum;

μ' is the relative magnetic permeability of the conductor material;

f is the frequency of the change of the current, Hz

Naturally, the skin effect leads to an increase of the resistance of the conductor and to an increase of the power generated in the conductor.

Thus, for copper at f = 50 Hz, $\delta_{sk} \approx 9.4$ mm, while at f = 100,000 Hz δ_{sk} is only 0.21 mm.

Thus, if the diameter of the conductor is 15 mm, one can consider that at a frequency of 50 Hz current flows through the entire cross section of the conductor and then the resistance of a conductor 1 m long will be

$$R_{50} = \rho \frac{L}{S} = 1{,}75 \cdot 10^{-8} \frac{1}{176 \cdot 10^{-6}} = 10^{-4} \text{ ohm}$$

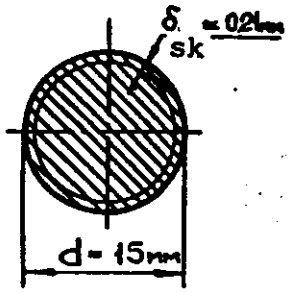


Figure 2

On the other hand, at a frequency of 100,000 Hz current flows only in a surface layer 0.21 mm thick, i.e., through an area $S \approx 10^{-5}$ m^2 (10 mm^2). In this case, the resistance of the conductor and, therefore, the power generated in it at a frequency of 100,000 Hz will be ≈18 times more than at a frequency of 50 Hz.

One further special property must be discussed.

Figure 1 shows a dependence of current on time where the processes of the increase and decrease of current have different slopes. In this case one cannot calculate power in the conductor from the skin layer that corresponds to the frequency of the increase or frequency of the decrease of current. The function $I(\tau)$ must be expanded into a Fourier series, the skin layer and the power must be determined from the frequency and amplitudes of the individual harmonics and the resulting powers added together. In some specific cases this laborious, although more accurate, method can be replaced by an analysis of some sine wave that is similar to the actual function $I(\tau)$.

1.1.2 Pulsed regime

All that we have discussed relates to currents flowing through conductors continuously. However, in a large number of problems we must deal with pulsed processes, where the current passes through

the conductor in pulses of duration τ_p, which repeat with some frequency f_p (Figure 3).

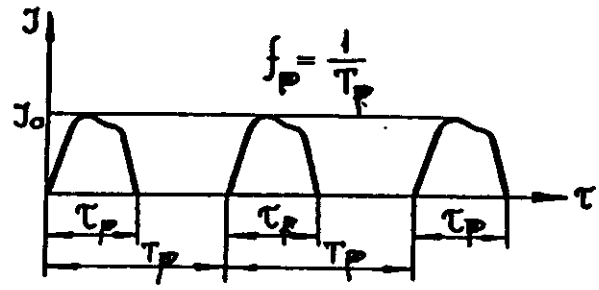


Figure 3

If the law of the change of current through time τ_p is known, $I(\tau)$, then the power evolved in the conductor during this time can be found from equation (2). Heat removal, however, must be calculated from the average power, which is equal to

$$P_f = \frac{P}{\nu} \qquad (6)$$

where $\nu = T_p/\tau_p$ is the duty factor*.

Two points here are important.

First, the depth of the skin layer in this case is calculated not from frequency f_p, but rather from the frequency that corresponds to the change of current during time τ_p.

Second, when there are solitary current pulses (i.e., when $\tau_p << T_p$) the energy produced in the conductor during time τ_p proves to be smaller than during the same time for pulses that continuously follow one another. In particular, for sinusoidal pulses this decrease is ≈1.5 (E. A. Abramyan and S. B. Vasserman, "Magnet of a air-core accelerator with opposing beams," IPN SD AS USSR Report No. 250, 1963). Therefore, when finding the power in the conductor from I_{eff} and δ_{sk} one can provisionally consider that

*[Translator's note: the Russian word for duty factor is used here, although the equation is that of the reciprocal duty factor.]

in the case of single sinusoidal pulses the depth of the skin layer δ_{sk} will be ≈1.5 times greater than when there is a continuous succession of pulses.

1.2 Conductor in an alternating [variable] magnetic field

We will consider the generation of heat in a tube of a conducting material placed in a magnetic field which changes according to the law

$$H = H_0 \sin \omega \tau$$

and which is directed along the axis of the tube (Figure 4).

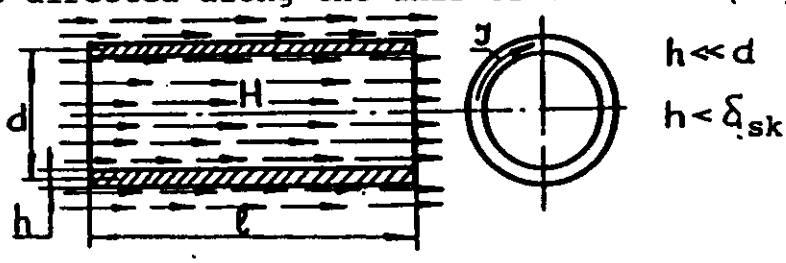


Figure 4

As in any turn of a coil penetrated by a variable field, an emf equal to

$$U = -\frac{d\Phi}{d\tau}$$

will be induced in the tube. The magnetic flux will be

$$\Phi = BS = \mu H S = \frac{\pi d^2}{4} \mu H_0 \sin \omega \tau \qquad \text{(here } \mu = \mu_0 \cdot \mu' \text{)}$$

Then

$$U = -\frac{d\Phi}{d\tau} = -\frac{\pi d^2 \mu H_0 \omega}{4} \cos \omega \tau$$

In correspondence with expression (4), since the field changes sinusoidally, we have:

$$P = \frac{U_a^2}{2R}, \text{ where } U_a = -\frac{\pi d^2 \mu H_0 \omega}{4} \text{ and } R = \rho\frac{L}{S} = \frac{\pi \rho d}{\ell h}$$

Finally, we obtain the power in a tube that is in an axial sinusoidally varying magnetic field in the form:

$$P \simeq 0{,}1\,\frac{\mu^2 H_0^2 \omega^2 d^5 \ell h}{\rho} \qquad (7)$$

If this field H is also pulsed, i.e., it varies in correspondence with Figure 5, then the denominator of expression (7) must also contain the duty factor $\nu = T_p/\tau_p$.

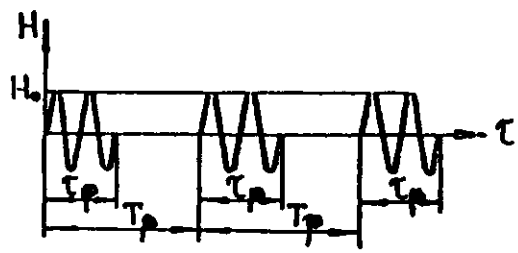


Figure 5

We have considered the case where the field changes sinusoidally. However, if the relationship is different, then the power must be determined from expression (2) and U_{eff} from expression (3).

1.3 Bombardment of a body by charged particles [1,2,3]

When a body is bombarded by a beam of charged particles, the kinetic energy of the particle beam or a share of this energy is released in the form of heat. How does one evaluate the amount of heat that is released in the bombarded body? Here two questions must be answered:

1) What is the power of the particle beam?
2) What part of this power heats the bombarded body?

1.3.1 Power of beam of charged particles

If each particle in the beam has charge q (C) and kinetic energy T (J), then if the current in the beam is I (A), the power of this beam can be found from the expression:

$$P_b = TI/q\nu, \ W \qquad (8)$$

where ν is the duty factor.

However, as a rule, we do not know the kinetic energy of the particles either in joules or electron volts. In this case the power in the beam is

$$P_b = TI/\nu, \ W \qquad (8')$$

1.3.2 Power going to heat the body

For this reason there arises the question, is the power that heats the bombarded body not always equal to the power of the beam?

The fact is that the particles, upon striking a solid body, do not immediately give up their energy to this body, but rather lose it gradually, penetrating to a certain depth into the body and encountering particles of matter in this path. Dependences of the loss of energy on the path traversed by a particle in the body differ for different particles, and the character is connected with the nature and parameters of the particles.

In particular, for electrons with kinetic energies above several hundreds of keV one can consider with accuracy that is sufficient for thermal design calculations that an electron, on passing through a substance, loses energy uniformly over the entire path.

Having given up all of its kinetic energy in collisions, the electron stops and "hangs up" in the body. However, if the bombarded body is a wall of finite thickness, then it is possible

for the electron to reach the second surface of this wall without having given up all of its store of kinetic energy and to leave the body with the remaining energy. In this case the energy given up to the body by the particle (and going to heat the body) proves to be less than the kinetic energy of the particle, i.e.,

$$Q = Q_b \cdot K, \text{ where } K \leq 1 \qquad (9)$$

The path in which a particle penetrating a material gives up all of its kinetic energy to the material, is called the <u>penetration depth</u>.

The penetration depth is dependent on the type of the particle, its kinetic energy and on the material of the body.

Here one should keep in mind that the particle does not move in the body on a linear trajectory, but rather on a complex trajectory that is determined by random collisions with the particles of the material (Figure 6).

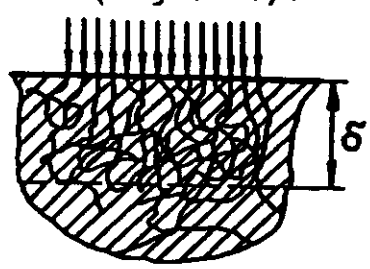


Figure 6

In connection with this, the penetration depth is understood to be some statistically average depth δ, to which a sufficiently large number of particles penetrate into the material. Of course, formulas that describe these processes are approximate. However, their accuracy is quite satisfactory for design calculations.

Returning to electrons, for which, as was already noted, the losses of energy are proportional to the path traversed in the material, we can say that the coefficient in expression (9) will be

$$K = h/\delta \qquad (10)$$

where h is the thickness of the part penetrated by the beam in the direction of particle flight;

δ is the penetration depth of electrons with given kinetic energy into a given material.

(Expression (10) is valid for $h \leq \delta$, while $h > \delta$ K is always equal to 1).

If thickness h is a design parameter, known in each specific case, then the penetration depth of electrons into a material should be determined for each case.

Thus, for electrons with kinetic energy

$$500 \leq T \text{ (keV)} \leq 3000$$

the penetration depth is described well by the following expression:

$$\delta = \frac{10^{-4} T^{1,5}}{\gamma} \qquad \text{(II)}$$

where γ is the density of the material, kg/m^3;

T is kinetic energy, keV;

δ is the penetration depth, m.

Here we will make one comment:

For electrons with T = 2 MeV = 2000 keV and copper with $\gamma = 8.9 \times 10^3$ kg/m^3 we obtain

$$\delta = \frac{10^{-4} \cdot 2000^{1,5}}{8,9 \cdot 10^{3}} = 10^{-3} \text{ м} = 1 \text{ мм}$$

A "tablet" of copper with area 1 cm^2 and thickness 1 mm (Figure 7) weighs 0.89 g. As we saw, the entire electron beam at T = 2 MeV will "settle" in this thickness. However, the electrons give up their energy in proportion to the path traversed in the material. Therefore, if this path is measured in terms of the weight of

material "traversed" by an electron, for an area of 1 cm^2, we obtain the rate of loss of energy by electrons in this case

$$\frac{\Delta T}{\Delta G} = \frac{2\ \text{МэВ}}{0{,}89\ \text{g}} = 2{,}24\ \frac{\text{MeV}}{\text{g}}$$

It turns out that at energies above 500 keV electrons in all materials give up energy at a rate approximately equal to 2 MeV/g.

This figure then can be used in preliminary estimates.

For example, if there is a wall of stainless steel (γ = 7.85 x 10^3 kg/m^3 = 7.85 g/cm^3) 2 mm thick bombarded by electrons, there will "remain" in it an energy

$$\Delta T = 7.85 \times 0.2 \times 2 \times 1 = 3.14\ \text{MeV}$$

Thus, particles with energy of ≈3 MeV will give up all of their energy to the body, heating throughout its entire thickness. Particles with energy above 3 MeV will pass through the wall, "leaving" in it a power proportional to an energy of 3 MeV. On the other hand, particles with energy less than 3 MeV will "stick" in the wall without reaching the opposite surface, and will heat only that part of the thickness of the wall equal to δ.

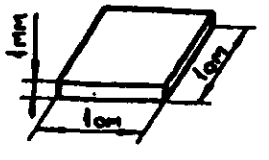


Figure 7

2. Heat removal and distribution of temperatures

Thus, we have found--basically--what sources of heat there are in constructions and how to evaluate the released heat power in various cases.

What then happens with this power?

Next this heat is spontaneously transferred in the direction of decreasing temperature. This transfer is accomplished in correspondence with the laws of heat transfer, which, generally speaking, is a very complex process.

This process is usually analyzed by separating it into three elementary methods of the transfer of heat--heat conduction, convection and heat radiation.

HEAT CONDUCTION is the transfer of heat by direct contact of bodies (or parts of the same body) that have different temperatures.

The phenomenon of **CONVECTION** is observed in moving liquids or gases. The transfer of heat here occurs simply because of the migration of material in space.

HEAT RADIATION is the phenomenon of the transfer of heat in the form of electromagnetic waves with initial conversion of heat energy to radiant energy and then back from radiant energy to heat energy.

Although each of these forms of heat exchange in pure form is rarely encountered, it is convenient to study them separately, after first establishing a number of necessary concepts.

A **TEMPERATURE FIELD** is the set of values of temperature at all points of a body at a given moment of time.

Mathematically, a temperature field is described by the equation

$$t = t(x, y, z, \tau) \qquad (12)$$

If temperature is dependent on time τ, the temperature field is **UNSTEADY-STATE**, while if it is not dependent on time it is steady state. In space a temperature field can be one-, two- or three-dimensional. Equation (12) is the equation of a three-dimensional field. The simplest is a one-dimensional steady-state temperature field, which is described by the expression

$$t = t(x) \qquad (12')$$

TEMPERATURE GRADIENT. In a body for any temperature field there are points that have the same temperature. Their geometric

locus is called an isothermal surface. A change of temperature in a body is observed only in directions that intersect isothermal surfaces (for example, x in Figure 8). Temperature changes the most sharply in the direction normal to an isothermal surface

$$\lim_{\Delta n \to 0} \frac{\Delta t}{\Delta n} = \frac{dt}{dn} = \operatorname{grad} t \qquad \text{(I3)}$$

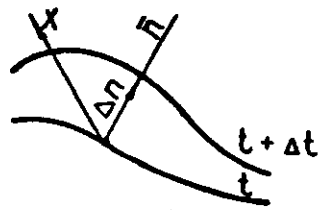


Figure 8

A **TEMPERATURE GRADIENT** is a vector directed along the normal to an isothermal surface in the direction of increasing temperature.

HEAT FLUX. Heat is transferred spontaneously in the direction of decreasing temperature. The amount of heat that is transferred across some surface per unit of time is called the **HEAT FLUX** P (W).

The heat flux through a unit of surface is called the **SPECIFIC** [or **UNIT**] **HEAT FLUX**, or the **HEAT FLUX DENSITY**, or the **HEAT LOAD OF A SURFACE** q (W/m^2).

The specific heat flux is a vector, the direction of which coincides with the direction of the spread of heat at a given point and is opposite to the temperature gradient vector.

2.1 Heat conduction [4,5]

This is the transfer of heat within a solid body from points at higher temperature to points at lower temperature. The mechanism of heat conduction consists of the transfer of energy due to thermal motion and energetic interaction among the molecules, atoms and electrons of which the body consists.

Fourier's law

In studying the process of heat conduction in solid bodies, Fourier established that the amount of heat that is transferred is proportional to the decrease of temperature, the time and the cross-sectional area perpendicular to the direction of propagation of heat.

Mathematically, this looks like:

$$\vec{q} = -\lambda \text{grad } t \qquad (14)$$

This equation is the Fourier law — the basic law of heat conduction.

Coefficient of heat conduction

The proportionality factor in equation (14) is called the **HEAT CONDUCTION COEFFICIENT**. This is a physical property of a material, the meaning of which can be understood from the equation:

$$\lambda = \left| -\frac{q}{\text{grad } t} \right| = \frac{Q}{F\tau \frac{\Delta t}{l}} = \frac{P}{F \frac{\Delta t}{l}} \quad (\text{W/m}\cdot\text{deg}) \quad (15)$$

The value of the heat conduction coefficient is the amount of heat transmitted through a unit of area per unit of time at a temperature gradient equal to unity.

The solution of all specific problems of heat conduction consists of writing and solving the Fourier equation for the given case. Let us look at a few examples of how this is done.

2.1.1 Heat conduction of a flat wall

2.1.1.1 Concentrated power (Figure 9)

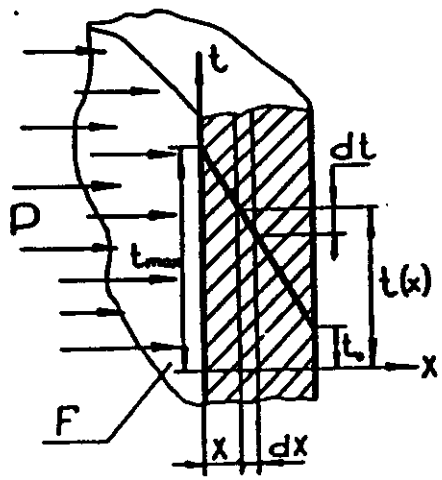


Figure 9

$$q=\frac{P}{F};\ grad\ t=\frac{dt(x)}{dx}$$

From the Fourier equation we have:

$$\frac{P}{F}=-\lambda\frac{dt(x)}{dx}$$

$$dt(x)=-\frac{P}{\lambda F}dx$$

$$t(x)=-\frac{P}{\lambda F}x+C$$

If $x = h$, $t = t_0$, from which

$$C=t_0+\frac{Ph}{\lambda F}$$

$$\left.\begin{aligned} &t(x)=t_0+\frac{P}{\lambda F}(h-x)=t_0+\frac{Ph}{\lambda F}\left(1-\frac{x}{h}\right);\ t_{max}=t(0)=t_0+\frac{Ph}{\lambda F}\\ &\Delta t(x)=t(x)-t_0=\frac{Ph}{\lambda F}\left(1-\frac{x}{h}\right);\ \Delta t_{max}=t(0)-t_0=\frac{Ph}{\lambda F}\end{aligned}\right\}\ (16)$$

The notion of heat resistance is very convenient for practical calculations; this can be illustrated using an example of electricity-heat analogy (Figure 10).

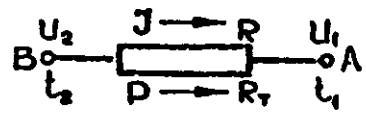


Figure 10

If current I passes through the segment of the circuit with resistance R, in this segment there will be a drop of voltage $\Delta U = I*R$. If the potential of point A is U_1, then the potential of point B will be

$$U_2 = U_1 + I*R$$

This is the universally known Ohm's law for an electrical circuit. By analogy one can write this same law for a heat circuit, making the following substitutions:

$$U \rightarrow t$$
$$I \rightarrow P \qquad (17)$$
$$R \rightarrow R_T$$

We find:

$$t_2 = t_1 + PR_T$$
$$\qquad (18)$$
$$\Delta t = t_2 - t_1 = PR_T$$

Comparing (18) and (16), we see that the heat resistance of a flat wall for concentrated heat power P is:

$$R_T = h/\lambda F, \text{ deg/W} \qquad (19)$$

If one knows the heat resistance R_T, one can always easily obtain the drop of temperature through the wall by multiplying the power passing through it by R_T.

2.1.1.2 Distributed power

A possible case is where the heat does not arrive at the plate from an external source on one side, but rather is released within the plate itself. For example, if a direct current passes through

the plate, heat will be released in it uniformly throughout its volume with intensity q_V (W/m^3).

In correspondence with Figure 9 the heat power released in a layer of thickness x will be:

$$P(x) = q_v F x$$

$$q = \frac{P(x)}{F} = q_v x; \quad grad\ t = \frac{dt(x)}{dx}$$

From the Fourier equation:

$$q_v x = -\lambda \frac{dt(x)}{dx}$$

$$dt(x) = -\frac{q_v}{\lambda} x dx; \quad t(x) = -\frac{q_v x^2}{2\lambda} + C$$

If x = h, t = t_0, from which $C = t_0 + \frac{q_v h^2}{2\lambda}$

$$\left.\begin{aligned} t(x) &= t_0 + \frac{q_v h^2}{2\lambda}\left[1 - \left(\frac{x}{h}\right)^2\right]; \quad t_{max} = t_0 + \frac{q_v h^2}{2\lambda} \\ \Delta t(x) &= t(x) - t_0 = \frac{q_v h^2}{2\lambda}\left[1 - \left(\frac{x}{h}\right)^2\right]; \quad \Delta t_{max} = \frac{q_v h^2}{2\lambda} \end{aligned}\right\} \quad (20)$$

We will transform the last expression somewhat:

$$\Delta t_{max} = \frac{q_v h^2}{2\lambda} \cdot \frac{F}{F} = \frac{q_v h F \cdot h}{2\lambda F} = \frac{P h}{2\lambda F}$$

In correspondence with (18), we see that the heat resistance of a plate for distributed power is

$$R_T = h/2\lambda F \qquad (21)$$

i.e., it is half that of the resistance when there is concentrated power.

2.1.2 Heat conduction of a cylindrical wall (Figure 11)

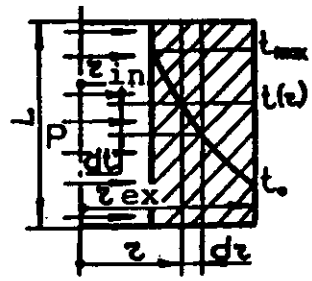


Figure 11

$$q=\frac{P}{2\pi r L};\ grad\,t=\frac{dt(r)}{dr}$$

$$\frac{P}{2\pi r L}=-\lambda\frac{dt(r)}{dr};\ dt(r)=-\frac{P\,dr}{2\pi\lambda L r}$$

$$t(r)=-\frac{P}{2\pi\lambda L}\ln r+C$$

If $r = r_{ex}$, $t = t_0$, from which

$$C=t_0+\frac{P\ln r_{ex}}{2\pi\lambda L}$$

Making substitutions, we have:

$$\left.\begin{aligned} t(r)&=t_0+\frac{P}{2\pi\lambda L}\ln\frac{r_{ex}}{r}\\ t_{max}&=t_0+\frac{P}{2\pi\lambda L}\ln\frac{r_{ex}}{r_{in}}\\ \Delta t(r)&=\frac{P}{2\pi\lambda L}\ln\frac{r_{ex}}{r}\\ \Delta t_{max}&=\frac{P}{2\pi\lambda L}\ln\frac{r_{ex}}{r_{in}}\end{aligned}\right\}\quad(22)$$

In correspondence with (18) we find the heat resistance of a cylindrical wall for concentrated power:

$$R_T=\frac{\ln r_{ex}/r_{in}}{2\pi\lambda L}\qquad(22')$$

One should note that the temperature parameters (expression (22) and heat resistance of a cylindrical wall are dependent on the ratio of radii r_{ex} and r_{in}, but not on their absolute values.

2.1.3 Contact heat conduction [5]

A very common case is the case of heat transfer across the contact of two surfaces. In the general case this contact has the form shown in Figure 12.

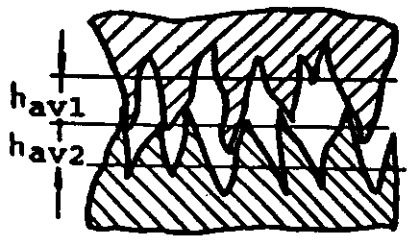


Figure 12

In correspondence with this, the transfer of heat is accomplished both across surface roughnesses and through the medium between roughnesses.

The question of contact heat exchange is very complicated, a great deal of research has been devoted to it, and there are many varying recommendations for calculation of this process.

The following is a convenient method of calculation. The heat resistances of the contact and the interlayer ("medium") are calculated separately by the formulas:

$$\left.\begin{aligned} R_{TK} &= \frac{3\sigma_B \cdot 10^{-4}}{2{,}1\, N\, \bar{\lambda}} \\ R_{TC} &= \frac{h_{av1} + h_{av2}}{2\lambda_c S_{nom}} \end{aligned}\right\} \quad (23)$$

where R_{TK} is the heat resistance of the contact, deg/W;

R_{TC} is the heat resistance of the interlayer between surfaces, deg/W;

h_{av1} and h_{av2} are the average values of the heights of the microroughnesses on the contacting surfaces, which are determined by the roughness class of the surfaces, m;

S_{nom} is the nominal contact surface area, m^2;

σ_B is the tensile strength of the less plastic material of the contacting parts, kg/m^2 (σ_B can be used in kg/cm^2 and then one can omit the factor 10^{-4});

N is the total force at the contact, kg;

$\lambda = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$ is the reduced coefficient of heat conduction of the contact, W/(m·deg) (λ_1 and λ_2 are the coefficients of heat conduction of each of the contacting materials);

λ_c is the coefficient of heat conduction of the medium in the interlayer, W/(m·deg).

Since these two resistances are "connected" in parallel, then one obtains not the resistances, but rather the inverse quantities--conductances. Then the total heat resistance of the contact R_T can be found from the expression:

$$\frac{1}{R_T} = \frac{1}{R_{TK}} + \frac{1}{R_{Tc}} = \frac{2{,}1N\bar{\lambda}}{3\sigma_B \cdot 10^{-4}} + \frac{2\lambda_c S_{nom}}{h_{av1} + h_{av2}} \qquad (24)$$

This method gives values of R_T that are somewhat too low in the case of the contact of large surfaces that have significant waviness. The "guilty party" in this is the second term of expression (24), since for wavy parts the value of the interlayer in fact proves to be greater than $(h_{av1} + h_{av2})/2$. The way out here is either to consider the actual waviness in the calculation or to assure that the flatness error of the parts does not exceed h_{av}.

It should be noted that even in this case the first term of the formula gives a quite reliable result, in spite of waviness. The fact is that the contact of the surfaces is actually accomplished at individual points — contact spots that form as the ridges of the microroughnesses are compressed under the effect of force N. Here the actual area of contact, which then determines its resistance, is always determined by the ratio:

$S_{act} = N/\sigma_{com}$ (σ_{com} is the strength of material in compression)

If a part is wavy, there will be fewer contact points, but then each of them will be "mashed" to a greater area and the total actual area of contact will again prove to be equal to N/σ_{com}. Incidentally, this also explains the fact that the heat resistance of the metal part of the contact is not dependent on the nominal contact surface S_{nom}, as can be seen from (23) and (24).

Very often one must have some heat contact between parts in a vacuum, where $\lambda_c = 0$. In this case only the first term of expression (24) remains and

$$R_T = R_{TK} = \frac{36_{\text{в}} \cdot 10^{-4}}{2{,}1 N \lambda} \quad \text{deg/W} \qquad (25)$$

In completing this section, a number of comments must be made.

First, in books that are devoted to contact heat exchange, the dimension of R_T is assumed to be not deg/W, but rather m^2.deg/W. In correspondence with this, the nominal surface of contact S_{nom} will appear in the numerator of the first expressions (23) and will vanish in the denominator of the second of these expressions. The fact is that in the literature the discussion concerns the specific heat resistance of the contact (i.e., the heat resistance of a unit of area of contact). Here, however, we give expressions for the total heat resistance of contact, which is more convenient for use in practice. Returning to the electricity-heat analogy, we see that according to Ohm's law

$$U = I \cdot R$$

where R is the total electrical resistance.

Using the notion of total electrical resistance, one can use a completely analogous Ohm's law for a heat circuit (with substitutions as in expression (17)):

$$\Delta t = PR_T$$

If, however, one uses the notion of the specific heat resistance, in the last expression one would have to introduce the term S_{nom} again, which seems less convenient.

Second, expression (25) can also be used for evaluation of the electrical resistance of a flat contact. For this one need only replace the quantity $\bar{\lambda}$ in it by

$$\frac{1}{\rho} = \frac{2}{\rho_1 + \rho_2}$$

where ρ_1 and ρ_2 are the resistivities of the materials of the contacting parts, ohm·m.

2.2 Convective heat exchange [4,6]

This is the transfer of heat between the surface of a solid body and a liquid or gaseous medium.

The amount of transferred heat P is proportional to the surface area of heat exchange F and the difference of temperatures of the wall and liquid ($t_w - t_{lq}$) in correspondence with Newton's law:

$$P = \alpha F(t_w - t_{lq}) \qquad (26)$$

The proportionality factor is α, which is called **COEFFICIENT OF CONVECTIVE HEAT TRANSFER** and is equal to the amount of heat transmitted through a unit of surface for a one degree difference of temperatures between the surface and the liquid:

$$\alpha = \frac{P}{F(t_w - t_{lq})} \ W/m^2 \cdot deg \qquad (27)$$

The heat resistance in the process of convective heat exchange is determined by the expression

$$R_T = 1/\alpha F \qquad (28)$$

The process of convective heat transfer is linked with the conditions of motion of the liquid. As is well known, there are two basic types of motion of a liquid: laminar (stream-like, quiescent motion) and turbulent (vortex-like, unordered motion). Processes of heat removal occur differently in these forms of flow and, naturally, there immediately arises the question: when is a flow which kind?

Reynolds established, as a result of special studies that an indicator of one or another flow regime is a particular dimensionless complex

$$wl/\nu$$

where w is the velocity of motion of the liquid, m/sec; ν is the kinematic viscosity of the liquid, m^2/sec; l is a characteristic linear dimension of the swept body, m. This complex was later named the **REYNOLD'S CRITERION or REYNOLD'S NUMBER**

$$Re = \frac{wl}{\nu} = \frac{wl\gamma}{\mu g} \qquad (29)$$

where γ is the density of the liquid, kg/m^3;

$g = 9.81$ m/sec^2 is gravitational acceleration;

μ is the coefficient of dynamic viscosity of the liquid, kg·sec/m^2.

The transition of laminar flow to turbulent flow occurs at some value of Re, which is called the critical value (Re_{cr}). If $Re < Re_{cr}$ the flow is laminar, while if $Re > Re_{cr}$ it is turbulent. The value of Re_{cr} is dependent on specific conditions. Thus, for the flow of a liquid or gas in tubes and channels

$$Re_{cr} \approx 2300$$

The characteristic dimension of the swept body l is also dependent on the specific conditions of the process. For example, for the flow of a liquid in tubes and channels this dimension is the so-called **HYDRAULIC DIAMETER**, which is defined by the expression:

$$d_h = 4F/U$$

where F is the cross-sectional area of the channel, m^2;

U is the parameter of the channel, m.

It is easy to see that for a round tube $d_h = d$.

A second serious question that arises in the use of expression (26) is the question of finding the coefficient of convective heat transfer α.

The process of convective heat transfer is a very complex process, and α is a function of a very large number of quantities that characterize this process: the shape and dimensions of the swept body, its surface temperature, the temperature of the liquid, the velocity of motion and the physical properties of the liquid, and so forth. A rigorous mathematical description of this process is very difficult. In connection with this, **SIMILARITY THEORY** is used to determine the coefficient of convective heat transfer.

The essence of this theory is that under similar conditions similar processes proceed in a similar way. This means that to determine some quantity ϕ_A in process A it is sufficient to take the same quantity in process B, ϕ_B, and multiply it by a "similarity factor" C:

$$\phi_A = \phi_B \cdot C$$

The similarity factors for different quantities in similar processes cannot be arbitrarily prescribed, but rather must be quite specific. They are complexes of certain specific physical quantities and are called **SIMILARITY NUMBERS**. They are usually named for scientists who did successful work in this field: Reynolds (Re), Nusselt (Nu), Fourier (Fo), and so forth. A characteristic property of similarity numbers is their dimensionlessness.

The existence of similarity numbers makes it possible to simplify the calculation of heat processes substantially. Thus, the intensity of convective heat exchange is always characterized by the **NUSSELT NUMBER**.

$$Nu = \alpha l/\lambda \qquad (30)$$

where l is a characteristic linear dimension, m;

λ is the coefficient of heat conduction of the heat carrier [heat transfer agent], W/(m·deg);

α is the coefficient of convective heat transfer, W/(m^2·deg).

Expression (30) is the basic expression for determination of the coefficient of convective heat transfer α:

$$\alpha = (Nu \cdot \lambda)/l \qquad (30')$$

The value of the Nu number is determined as a function of the specific process of heat exchange being analyzed. Thus, for the motion of a liquid in tubes and channels for the turbulent regime the following criterional expression is recommended for the determination of Nu

$$Nu_{lq} \approx 0.023 Re^{0.8} Pr^{0.4} \epsilon_R \qquad (31)$$

where Nu_{lq} is the Nusselt number for determination of the average coefficient of convective heat transfer over the length; the coefficient of convective heat exchange over the length is used because the liquid gradually becomes heated as it moves along the body being cooled, the properties of the liquid change and α changes;

Re_{lq} and Pr_{lq} are the Reynolds and Prandtl numbers for the average temperature of the liquid (this is indicated by subscript "lq");

ϵ_R is a coefficient that takes into consideration the curvature of the tube and is equal to

$$\epsilon_R = 1 + 1.77 d/R \qquad (32)$$

where d is the diameter of the tube and R is the average radius of curvature.

Formulas for calculation of similarity numbers and coefficients of convective heat transfer under various conditions of convective heat exchange are given in Table I.

If heat is being removed with water, the coefficient of convective heat transfer can be determined from the graphs in **Figure 13.**

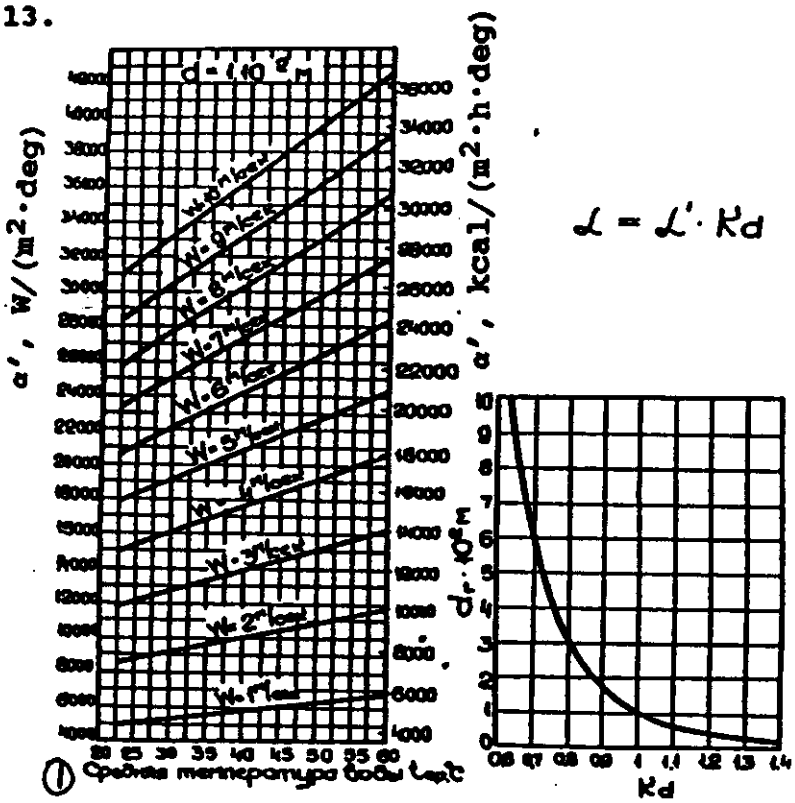


Figure 13. Coefficient of convective heat transfer for the removal of heat with water (corresponds to the relationship

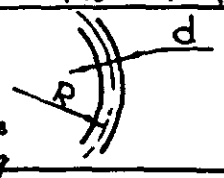
$$\alpha = 0.023 \frac{\lambda}{d^{0.2}} \left(\frac{Wr}{Mq}\right)^{0.8} \cdot P_r^{0.4}]$$

Key:
1 - Average temperature of water t_{av}, °C

Table I

(1) КОНВЕКТИВНЫЙ ТЕПЛООБМЕН

(2) Передаваемая мощность $P=\alpha F(t_w-t)_{ж}$	Коэффициент теплоотдачи (3) (ℓ – определяющий размер) $\alpha = Nu\frac{\lambda}{\ell}$

Процесс (4)		(6) Критериальное уравнение	Определяющий размер (7)	Определяющая температура (8)
Течение в трубах и каналах (d – диаметр трубы; $\ell_{тр}$ – длина) (9)	Ламинарный $Re<2300$	$\frac{\ell_{тр}}{d}>0{,}067Re_{ж}\cdot Pr_{ж}^{5/6}$ $Nu\simeq 4\left[Pr_{ж}/Pr_{w}\right]^{0,25}$ (11)	Круглые трубы – диаметр d	Для параметров с индексом „ж" („жидкость") – средняя температура теплоносителя;
		$\frac{\ell_{тр}}{d}<0{,}067Re_{ж}\cdot Pr_{ж}^{5/6}$ $Nu\simeq 1{,}4\left[Re_{ж}\cdot\frac{d}{\ell_{тр}}\right]^{0,4}\cdot Pr_{ж}^{0,33}\cdot\left[Pr_{ж}/Pr_{w}\right]^{0,25}$	Некруглые каналы – гидравлический диаметр $d_h=\frac{4F}{U}$	
	Переходный $Re>2300$ (10)	Жидкости (12): $Nu_{ж}=0{,}023Re_{ж}^{0,8}Pr_{ж}^{0,4}\varepsilon_R$, $\varepsilon_R=1+1{,}77\frac{d}{R}$ (13) Воздух: $Nu_{ж}=0{,}018Re_{ж}^{0,8}$ 	F – площадь сечения канала; U – периметр канала (14)	Для параметров с индексом „с" („стенка") – температура охлаждаемой поверхности (15)
Естественная конвекция (16)	–	$Nu=C\left[Pr\cdot Gr\right]^{n}$ (см. таблицу ниже)	Шар и горизонтальная труба (17) – диаметр; вертикальная труба и вертикальная плита – высота охлаждаемого (нагреваемого) участка; горизонтальная плита – меньший размер; влияние (горяча сторона сверху – „с" увеличить на 30%, горяча сторона снизу – „с" уменьшить на 30%)	$\frac{1}{2}(t_{ж}+t_{w})$
Теплопровод в прослойках (18)	–	Рассчитывается теплопроводность с эквивалентным коэффициентом теплопроводности $\lambda_{эк}=\varepsilon_к\lambda$ (19) $\varepsilon_к=\begin{cases}1 & Pr\cdot Gr\le 1000\\ 0{,}18\left[Pr\cdot Gr\right]^{0,25} & Pr\cdot Gr>1000\end{cases}$ λ – коэффициент теплопроводности среды при определяющей температуре	(20) Толщина прослойки	(21) Среднеарифметическое температур стенок прослойки $\frac{1}{2}\left[t_{w1}+t_{w2}\right]$

$\left[Pr\cdot Gr\right]$	$<10^{-3}$	$10^{-3}\div 5\cdot 10^{2}$	$5\cdot 10^{2}\div 2\cdot 10^{7}$	$>2\cdot 10^{7}$
C	0,45	1,18	0,54	0,135
n	0	1/8	1/4	1/3

(22) Критерии подобия

(24) Наименование	Формула (25)	Что определяет (26)
Рейнольдса (27)	$Re=\frac{W\ell}{\nu}=\frac{W\ell\gamma}{\mu g}$	Гидродинамический режим течения теплоносителя (32)
Нуссельта (28)	$Nu=\frac{\alpha\ell}{\lambda}$	Интенсивность конвективного теплообмена (33)
Грасгофа (29)	$Gr=g\beta\frac{\ell^3}{\nu^2}\Delta t$	Относительную эффективность свободного конвективного теплообмена (34)
Фурье (30)	$Fo=\frac{a\tau}{h^2}$	Нестационарную теплопроводность (35)
Прандтля (31)	$Pr=\frac{\mu C_p g}{\lambda}$	Теплофизическую характеристику теплоносителя (36)

(23) Условные обозначения

ℓ – определяющий линейный размер, м

Δt – характерный перепад температуры, град

τ – время, с (37)

$g=9{,}81\ \frac{м}{с^2}$

α – коэффициент теплоотдачи, $\frac{Вт}{м^2\cdot град}$

Теплоноситель

W – скорость движения, $\frac{м}{с}$

γ – удельный вес, $\frac{кг}{м^3}$

ν – коэффициент кинематической вязкости, $\frac{м^2}{с}$

μ – коэффициент динамической вязкости, $\frac{кг\cdot с}{м^2}$

λ – коэффициент теплопроводности, $\frac{Вт}{м\ град}$

C_p – весовая удельная теплоемкость, $\frac{Вт\cdot с}{кг\cdot град}$

β – коэффициент объемного расширения, $\frac{1}{град}$

$a=\frac{\lambda}{C_p\gamma}$ – коэффициент температуропроводности, $\frac{м^2}{с}$

Key:

1 - Convective heat exchange
2 - Transferred power
3 - Coefficient of convective heat transfer
(l — determining dimension)
4 - Process
5 - Regime
6 - Criterional equation
7 - Determining dimension
8 - Determining temperature
9 - Flow in tubes and channels
(d — diameter of tube; l_m — length)
10 - Turbulent ...
11 - Laminar ...
12 - Liquids:
13 - Air:
14 - Round tubes — diameter d
Nonround channels — hydraulic diameter
$d_h = 4F/U$
F — cross section of channel
U — perimeter of channel
15 - For parameters with subscript "lq" ("liquid") — average temperature of heat carrier; for parameters with subscript "w" ("wall") — temperature of surface being cooled
16 - Natural convection
17 - Sphere and horizontal tube — diameter
Vertical tube and vertical plate — height of cooled (heated) segment
Horizontal plate — smaller dimension in plan view
(hot side up, increase C by 30%;
hot side down, decrease C by 30%)
18 - Heat removal in interlayers
19 - Heat conduction with equivalent coefficient of heat conduction is calculated
λ — coefficient of heat conduction of medium at determining temperature
20 - Thickness of interlayer
21 - Arithmetic mean temperature of walls of interlayer ...
22 - Similarity numbers
23 - Notations
24 - Name
25 - Formula
26 - Which determines
27 - Reynolds
28 - Nusselt
29 - Grashof
30 - Fourier
31 - Prandtl
32 - Hydrodynamic regime of flow of heat carrier
33 - Intensity of convective heat exchange
34 - Relative effective free convective heat exchange

35 - Unsteady-state heat conduction
36 - Thermophysical characteristic of heat carrier
37 - l - determining linear dimension, m
γT - characteristic difference of temperatures, deg
τ - time, sec
$g = 9.81$ m/sec^2
α - coefficient of convective heat transfer, W/(m^2·deg)

HEAT CARRIER

w - velocity of motion, m/sec
γ - density, kg/m^3
ν - coefficient of kinematic viscosity, m^2/sec
μ - coefficient of dynamic viscosity, kg·sec/m^2
λ - coefficient of heat conduction, W/(m·deg)
C_p - specific heat capacity per unit of weight, W·sec/kg·deg
β - coefficient of volumetric expansion, 1/deg
$a = \lambda/C_p\lambda$ - coefficient of thermal diffusivity, m^2/sec

2.3 Heat radiation [4,7,8]

The carriers of the energy of radiation are electromagnetic vibrations in the wavelength range from fractions of a micrometer to several kilometers. In processes of heat transfer we are interested in those forms of radiation that are determined by the temperature and optical properties of bodies, i.e., the visible (light) and heat (infrared) waves with wavelengths from 0.4 to 800 μm.

The nature of light and heat rays are the same and, therefore, the laws of their propagation (refraction, reflection, and so forth) are also the same.

Every body radiates energy into space. Upon striking another body, this energy is partially absorbed, partially reflected, and partially passed through the body. The absorbed part of the energy is converted to heat, while the reflected energy and the energy that is passed through the body reach other bodies and the process repeats.

Thus, every body is continuously radiating and absorbing energy. The result of these conversions of heat energy to radiant energy and back is heat exchange by radiation.

The amount of heat given off or taken up by a body is the difference between the heat absorbed and radiated by it. This difference will exist only when the temperatures of the bodies in a system are different. Bodies with the same temperature form a system that is in thermal equilibrium; in this case all of the bodies radiate and absorb energy, but the incoming energy is equal to the outgoing energy for each body.

2.3.1 Radiant flux

The total radiation that passes per unit of time through surface F is called the **RADIANT FLUX P** (W).

The **RADIANT FLUX DENSITY** $E = dP/dF$ (W/m^2).

If out of the total radiant flux P a part P_A is absorbed by the body, a part P_R is reflected, and a part P_D passes through the body, then

$$P_A + P_R + P_D = P$$

from which

$$\frac{P_A}{P} + \frac{P_R}{P} + \frac{P_D}{P} = 1$$

or:

$$A + R + D = 1 \qquad (33)$$

where A, R and D respectively characterize the absorbing, reflecting and transmitting capacities of the body.

$A = 1$, $R = D = 0$ — all of the energy is absorbed by the body. Such a body is called an **ABSOLUTELY BLACK BODY** (or, simply, a BLACK BODY).

$R = 1$, $A = D = 0$ — all arriving energy is reflected by the body. This is an **ABSOLUTELY WHITE** body;

$D = 1$, $A = R = 0$ — all of the incoming energy passes through the body. This is an **ABSOLUTELY TRANSPARENT** (diathermic) body.

All solid bodies and some liquids (water, alcohols, and so forth) are practically nontransparent for heat rays (are athermic), i.e., D = 0. Then for nontransparent bodies

$$A + R = 1 \qquad (34)$$

From this it follows that if a body absorbs energy well, then it reflects it poorly, and vice versa.

In nature there are no absolutely black or absolutely white bodies. All bodies both radiate and absorb energy, and in this sense they are not completely black, but rather GRAY bodies. The degree of brightness of a body ϵ (also: **"COEFFICIENT OF BLACKNESS", "COEFFICIENT OF GRAYNESS"**) is numerically equal to the absorbing capacity of the body A:

$$\epsilon = A \qquad (35)$$

The degree of blackness [emissivity] is a characteristic of a material that is dependent on the temperature of the body and the status of the surface. Values for various conditions and materials are given in the literature.

2.3.2 Heat exchange by radiation between bodies

According to the Stefan-Boltzman law the radiant flux density of an absolutely blackbody is

$$E_0 = C_0 \left[\frac{T}{100}\right]^4 \ W/m^2 \qquad (36)$$

where T is the temperature of the body, K,

C_0 is the coefficient of radiation of an absolutely blackbody, which is equal to

$$C_0 = 5{,}67 \ W/m^2 \cdot K^4 \qquad (37)$$

For a not absolutely blackbody the radiant flux is

$$P = \varepsilon C_0 \left[\frac{T}{100}\right]^4 F \qquad (38)$$

where ϵ is the degree of blackness of the body and

F is the surface of radiation, m^2.

Radiant flux between two bodies is calculated as

$$P = \varepsilon_{np} C_0 H \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \quad , \text{ W} \qquad (39)$$

where T_1 and T_2 are the temperatures of the bodies, K;

$$\varepsilon_{red} = \frac{1}{1 + \left(\frac{1}{\varepsilon_1} - 1\right)\varphi_{12} + \left(\frac{1}{\varepsilon_2} - 1\right)\varphi_{21}}$$

That is the reduced coefficient of blackness of the system, in which ϵ_1 and ϵ_2 are the degrees of blackness of the bodies, φ_{12} and φ_{21} are the surface-average coefficients of irradiance of body 1 on body 2 and of body 2 on body 1;

$H = \varphi_{12}F_1 = \varphi_{21}F_2$ is the mutual surface of irradiation, m^2 (F_1 and F_2 are the surfaces of radiation of the bodies).

Coefficients of irradiance φ_{12} and φ_{21} are dependent on the shape, dimensions and mutual positions of the bodies and are given in the literature for various cases (for example, [8]).

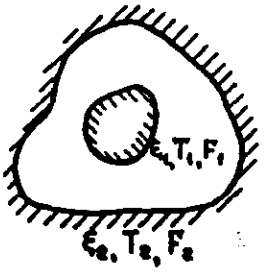
We will consider individual simple, but typical, cases.

2.3.2.1 Two parallel walls at a distance that is small by comparison with their dimensions (Figure 14)

$$\left.\begin{aligned} &\varphi_{12} = \varphi_{21} = 1 \\ &H = F_1 = F_2 \\ &\varepsilon_{red} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \\ &P = \varepsilon_{red} C_0 F_1 \left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \end{aligned}\right\} \qquad (40)$$

Figure 14

2.3.2.2 Two surfaces that form a closed system (Figure 15)



$$\left.\begin{aligned} &\varphi_{12}=1;\ \varphi_{21}=\frac{F_1}{F_2};\ H=F_1 \\ &\varepsilon_{red}=\frac{1}{\frac{1}{\varepsilon_1}+\frac{F_1}{F_2}\left(\frac{1}{\varepsilon_2}-1\right)} \\ &P=\varepsilon_{red}C_0F_1\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right] \end{aligned}\right\} \quad (4I)$$

Figure 15

For $F_2 \gg F_1$

$$\varepsilon_{red} \approx \varepsilon_1 \qquad P=\varepsilon_1 C_0 F_1\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right] \qquad (4I')$$

Equations (41) and (41′) are valid for the case where the inner surface does not have concavities. However, they can also be used if there are concavities, but in this case F_1 should be understood as the surface not of body 1 itself, but of an infinitely thin film surrounding the body (Figure 16).

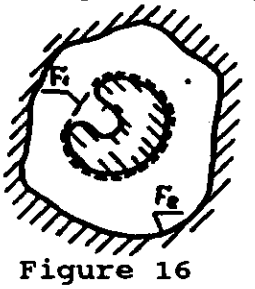


Figure 16

2.3.3 Heat screens

If a screen of a material that is not transparent for heat rays is

placed between two bodies, then the heat flux between the bodies will change.

For simplicity we will consider the case with absolutely black flat surfaces, between which a flat, likewise absolutely black, screen is placed.

Without the screen the density of the heat flux between the bodies was

$$q_1 = C_0(v_1 - v_2) \quad \text{where}$$

$v_1 = \left(\frac{T_1}{100}\right)^4$ and $v_2 = \left(\frac{T_2}{100}\right)^4$ are the temperature factors of bodies 1 and 2. After installing the screen it is only known that heat radiation from body 1 onto the screen is equal to the heat radiation from the screen onto body 2, i.e.,

$$q_2 = C_0(v_1 - v_{scr}) = C_0(v_{scr} - v_2)$$

from which $v_{scr} = \frac{v_1 + v_2}{2}$ and $q_2 = C_0 \frac{v_1 - v_2}{2} = \frac{q_1}{2}$

Thus, the installation of one absolutely black screen reduces the heat flux twofold. It can be shown that two screens will reduce it threefold, three screens fourfold and, in general, n screens reduce heat radiation by a factor of (n + 1) times.

If, however, the screens have high reflecting capacity (i.e., low degree of blackness), their efficiency increases significantly, and the screens become a radical means of reducing the heat flux.

The heat flux of radiation across a closed system of two bodies and n "gray" screens will be:

$$P = \frac{C_0\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_{n+2}}{100}\right)^4\right]}{\frac{1}{\varepsilon_{12}H_{12}} + \frac{1}{\varepsilon_{23}H_{23}} + \ldots + \frac{1}{\varepsilon_{(n+1)(n+2)}H_{(n+1)(n+2)}}} \qquad (42)$$

where F_1, $\epsilon 1$ and T_1 are the surface, degree of blackness and temperature of the radiating body;

F_2, ϵ_2 are the surface and degree of blackness of the first screen;

F_3, ϵ_3 are the surface and degree of blackness of the second screen;

F_{n+1}, ϵ_{n+1} are the surface and degree of blackness of the n-th screen;

F_{n+2}, ϵ_{n+2} and T_{n+2} are the surface, degree of blackness and temperature of the heat receiving body.

ϵ_{12}, $\epsilon 2_3$, ..., $\epsilon_{(n+1)(n+2)}$ are the reduced degrees of blackness in the systems of the first and second, second and third, and so forth up to the (n + 1)-th and (n + 2)-th bodies.

H_{12}, H_{23}, ..., $H_{(n+1)(n+2)}$ are the mutual surfaces of irradiance in these systems of bodies.

2.4 Complex heat transfer

Under real conditions there are always several kinds of heat transfer participating in a process of heat exchange — sometimes in succession, sometimes in parallel.

Thus, for example, in the case of the transfer of heat from one liquid to another through a wall (Figure 17) the heat is first transferred by convection to the wall with coefficient of convective heat transfer α_1, then through the wall with coefficient of heat conduction λ, and, finally, from the wall to the other liquid with coefficient of convective heat transfer α_2.

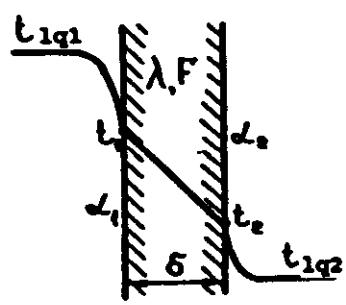


Figure 17

Since the heat fluxes from the first liquid to the wall and from the wall to the second liquid are equal, then:

$$P=\alpha_1 F(t_{1q1}-t_1)$$
$$P=\lambda\frac{F}{\delta}(t_1-t_2)$$
$$P=\alpha_2 F(t_2-t_{1q2})$$

Finding from the second expression

$$t_2=t_1-\frac{P\delta}{\lambda F}$$

and substituting this into the third expression, we find:

$$t_1-t_{1q2}=\frac{P}{\alpha_2 F}+\frac{P\delta}{\lambda F} \qquad t_{1q1}-t_1=\frac{P}{\alpha_1 F}$$

Addition of the last expression gives

$$t_{1q1}-t_{1q2}=P\left[\frac{1}{\alpha_1 F}+\frac{1}{\alpha_2 F}+\frac{\delta}{\lambda F}\right]$$

from which the heat flux from one liquid to the other proves to be equal:

$$P=\frac{t_{1q1}-t_{1q2}}{\frac{1}{\alpha_1 F}+\frac{1}{\alpha_2 F}+\frac{\delta}{\lambda F}}$$

The denominator contains the total overall heat resistance of the system, i.e., the last expression can be rewritten in the form

$$P=\frac{t_{1q1}-t_{1q2}}{R_{T\Sigma}} \qquad (43)$$

This expression is valid for any system, no matter what forms of heat transfer there were in succession "in passing".

This was a case of the successive action of different types of heat transfer. An example of parallel action is, in particular, convective heat transfer from a body to the surrounding medium under ordinary conditions.

As a rule, heat is given up in parallel by convection and by radiation.

The power given up by convection is:

$$P_K = \alpha F(t_1 - t_2)$$

where α is the coefficient of convective heat transfer, $W/(m^2 \cdot deg)$;

F is the surface area of cooling, m^2;

t_1 and t_2 are the temperatures of the body and the surrounding medium, °C.

The power given off by radiation is:

$$P_{rad} = \varepsilon_1 C_0 F\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]$$

where ϵ_1 is the degree of blackness of the body;

$C_0 = 5.67\ W/(m^2 \cdot K^4)$;

F is the surface area of radiation, m^2;

T_1 and T_2 are the temperatures of the body and the medium, K.

The total heat flux from the heated body to the surrounding medium is:

$$P = P_K + P_{rad} = \alpha F(t_1 - t_2) + \varepsilon_1 C_0 F\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] \qquad (44)$$

It is more convenient to write the last expression via the overall coefficient of convective heat transfer α_Σ in a simpler form:

$$\left.\begin{aligned} P &= \alpha_\Sigma F(t_1 - t_2) \\ \text{where } \alpha_\Sigma &= \alpha + \frac{\varepsilon_1 C_0\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{t_1 - t_2} \end{aligned}\right\} \qquad (45)$$

The literature [7,9] gives tables and curves for determination of α_Σ in dependence on t_1 and t_2. The degree of blackness of the body here is assumed to be $\epsilon_1 \approx 0.8$, which is valid for normal exposed surfaces for an average level of dustiness.

On a preliminary basis, in initial estimates one can assume (for bodies with temperature no higher than 100°C)

$$\alpha_{\Sigma} = 10\text{-}20 \ W/(m^2 \cdot deg).$$

3. Unsteady-state processes

We have considered all forms of heat transfer in a steady-state regime, i.e., when the temperature field of the body does not change with time. However, cases where one must analyze the operation of a part or item in the process of heating or cooling, i.e., in an unsteady-state regime, are encountered fairly often in practice.

Here there are usually two possible questions:

1) by what law does the temperature of a body increase or decrease?
2) what is the time that we require to reach a steady-state temperature (what will be the time for "coming up to regime")?

We will discuss using simple examples how the dependence of temperature t on time τ is derived.

3.1 Heating of a body

Let some body of mass m at the initial moment of time ($\tau = 0$) have temperature t_0 equal to the temperature of the surrounding medium. At this moment power P (W), begins to be released in the body uniformly throughout its volume. The specific heat capacity per unit weight of the material is C_p (W·sec/kg·deg).

At time τ the body will have temperature t and in the interval of time $d\tau$ it will change by dt. Let us write the equation of heat balance for this process.

Energy $mC_p dt$ (W·sec), is expended in heating the body by dt.

During time $d\tau$ at temperature t the following energy is given off from the body

$$\frac{t-t_0}{R_T}d\tau \qquad \text{W·sec}$$

(R_T is the heat resistance (deg/W), between the body and the surrounding medium).

However, energy $Pd\tau$ (W·sec), is also released during time $d\tau$, i.e.,

$$Pd\tau = mC_p dt + \frac{t-t_0}{R_T}d\tau$$

$$\frac{d\tau}{mC_pR_T} = \frac{dt}{PR_T-(t-t_0)}; \quad \frac{\tau}{mC_pR_T} = -\ln[PR_T-(t-t_0)]+C$$

For $\tau=0$ $t=t_0$ and $C=\ln(PR_T)$

$$\frac{\tau}{mC_pR_T} = \ln\frac{PR_T}{PR_T-(t-t_0)}; \quad e^{\frac{\tau}{mC_pR_T}} = \frac{PR_T}{PR_T-(t-t_0)}$$

$$t=t_0+PR_T\left(1-e^{-\frac{\tau}{mC_pR_T}}\right) \qquad (46)$$

It is known that $PR_T = t_{max} - t_0$, from which

$$t=t_0+(t_{max}-t_0)\left(1-e^{-\frac{\tau}{mC_pR_T}}\right) \qquad (46')$$

(t_{max} is the temperature of the body in a steady-state regime).

The curve of the change of temperature through time has the form shown in Figure 18.

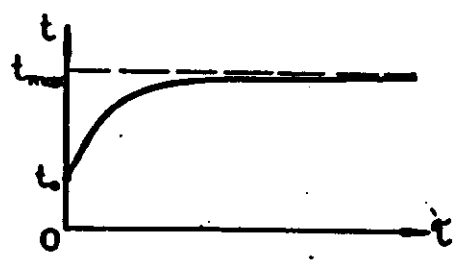


Figure 18

The temperature of the body continuously approaches the steady-state temperature t_{max}, reaching this value at $\tau = \infty$.

3.2 Cooling of a body

A body with mass m and specific heat capacity C_p at the initial moment of time has temperature t_{max} and begins to cool across heat resistance R_T.

At time τ the body has temperature t and in time segment $d\tau$ changes its temperature by dt.

Here the store of energy of the body changes by $-mC_p dt$ and energy

$$\frac{t-t_0}{R_T}d\tau$$

is given off from the body, where t_0 is the temperature of the surrounding medium. Thus, the equation of the heat balance has the form:

$$\frac{t-t_0}{R_T}d\tau = -mC_p dt$$

$$-\frac{d\tau}{mC_pR_T} = \frac{dt}{t-t_0}\ ;\ -\frac{\tau}{mC_pR_T} = \ln(t-t_0)+C$$

For $\tau=0$ $t=t_{max}$ and $C=-\ln(t_{max}-t_0)$

$$-\frac{\tau}{mC_pR_T} = \ln\frac{t-t_0}{t_{max}-t_0}\ ;$$

$$t = t_0 + (t_{max}-t_0)e^{-\frac{\tau}{mC_pR_T}} \qquad (47)$$

The curve corresponding to equation (47) is shown in Figure 19. If $\tau = \infty$ $t = t_0$.

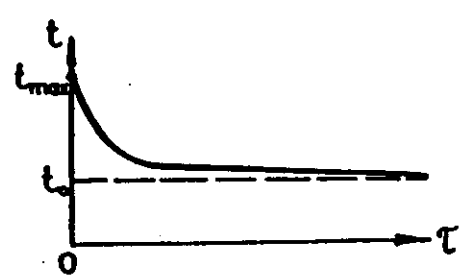


Figure 19

3.3 Time required to come up to regime

It can be seen from equations (46′) and (47) that the quantity mC_pR_T must have the dimension of time. This is indeed so:

$$\frac{kg \cdot W \cdot sec \cdot deg}{kg \cdot deg \cdot W} = sec$$

The segment of time defined by this quantity is called the TIME CONSTANT of the process that is described by equations (46′) and (47).

We will use the notation $mC_pR_T = \tau_0$ and rewrite equations (46′) and (47) in the form:

$$\left.\begin{aligned} t - t_0 &= (t_{max} - t_0)(1 - e^{-\frac{\tau}{\tau_0}}) \\ t - t_0 &= (t_{max} - t_0)\, e^{-\frac{\tau}{\tau_0}} \end{aligned}\right\} \quad (48)$$

It can be seen from equations (48) and Figures 18 and 19 that if $\tau = \tau_0$ the quantity $t_{max} - t(\tau)$ differs from the quantity $(t_{max} - t_0)$ by a factor of e for heating and quantity $t(\tau) - t_0$ differs from $(t_{max} - t_0)$ also by a factor of e during cooling, while if $\tau = 2\tau_0$ these quantities differ by a factor of e^2, if $\tau = 3\tau_0$ they differ by a factor of e^3, and so forth.

However, $e^3 \approx 20$ and the difference of temperature from the steady-state temperature at $\tau = 3\tau_0$ is only about 5%, which in most cases can be neglected (in any case, if $\tau = 4\tau_0$ the disagreement will be ≈1.8%, which is always insignificant).

Thus, with accuracy that is sufficient for design calculations one can consider that the time for the system to come up to regime will be

$$\tau_{regime} \approx (3\text{-}4)\tau_0 = (3\text{-}4)mC_pR_T \quad (49)$$

One more method of calculating the time for the system to reach steady-state regime can be shown.

Let us plot the graph of some process, in which a body of mass m is heated by power P with heat resistance between the body and the surrounding medium R_T. The specific weight heat capacity of the material is C_p and the temperature of the medium, which is equal to the initial temperature of the body, is t_0.

As is known, the temperature of a body in such a process changes according to an exponential law (Figure 20), which is described by the first of equations (48).

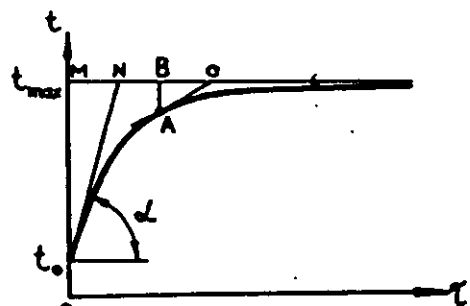


Figure 20

If the length of segment BC is calculated on the scale of time τ, it turns out that, first,

$$BC = \tau_0$$

and, second, quantity BC is not dependent on the coordinate of point A. (BC is a "subtangent" of the exponential curve, the constancy of which is one of the properties of an exponential curve).

But if quantity BC is not dependent on the coordinate of point A, then

$$MN = BC = \tau_0.$$

On the other hand,

$$MN = \frac{t_{max} - t_0}{tg\,\alpha} = \frac{t_{max} - t_0}{\left.\frac{dt}{d\tau}\right|_{\tau=0}} = \frac{t_{max} - t_0}{P/mC_p}$$

After this, we can write that

$$\tau_{peak} \simeq (3...4)\tau_0 = (3...4)\frac{t_{max}-t_0}{P/mC_p} \qquad (50)$$

where $t_{max} - t_0$ is the difference of temperatures of the body and the surrounding medium in a steady-state regime.

3.4 Unsteady-state heat conduction [10]

Often in designing various devices the designer runs up against the need to use parts that experience heating from one side, with propagation of heat into the material. Examples include parts through which high-frequency currents flow (in the skin layer) as well as various collectors, radiation receivers, and so forth.

For all of these parts the temperature field is variable through time, and the heat spreads in them by means of heat conduction, i.e., they operate under conditions of **UNSTEADY-STATE HEAT CONDUCTION.**

How does one calculate the distribution of temperatures in such a part at a given moment of time?

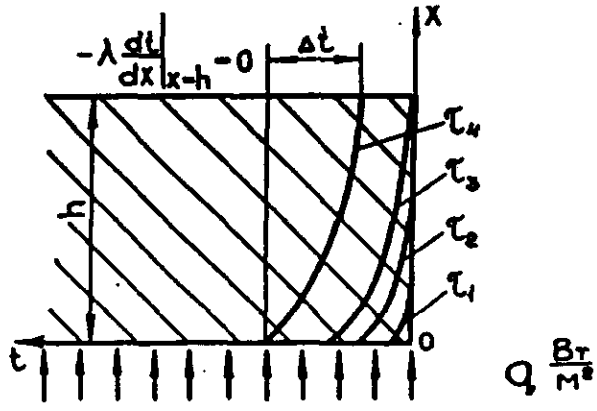
A rigorous mathematical solution of such problems is very difficult and, as a rule, leads to a solution in the form of the sum of the terms of an infinite series, which is not very convenient for practice.

To make practical calculations easier, graphs are plotted using these formulas and these graphs make it possible to do these calculations fairly easily and simply. The solution of a number of such problems in graphic form is given in [10].

In order to become acquainted with the technique of using these solutions we will consider one very typical problem, one—sided heating of an infinite plate.

A plate of thickness h (infinite in area) is heated on one side by a specific heat flux q (W/m^2), which is constant through

time. The initial temperature of the plate is t_0, and heat removal from the "cold" side is absent (Figure 21).



In correspondence with the recommendations of [10] the temperature of any point on such a plate can be determined from the expression:

$$t = t_0 + \theta(qh/\lambda) \qquad (51)$$

where λ is the coefficient of heat conduction of the material, W/m·deg;

θ is a parameter that is a function of the coordinate through the thickness of the material, in which the temperature, properties of the material plate and time of heating are determined.

Parameter θ is determined from graphs, which are given, for example, in [10]. For our problem this graph has the form (Figure 22):

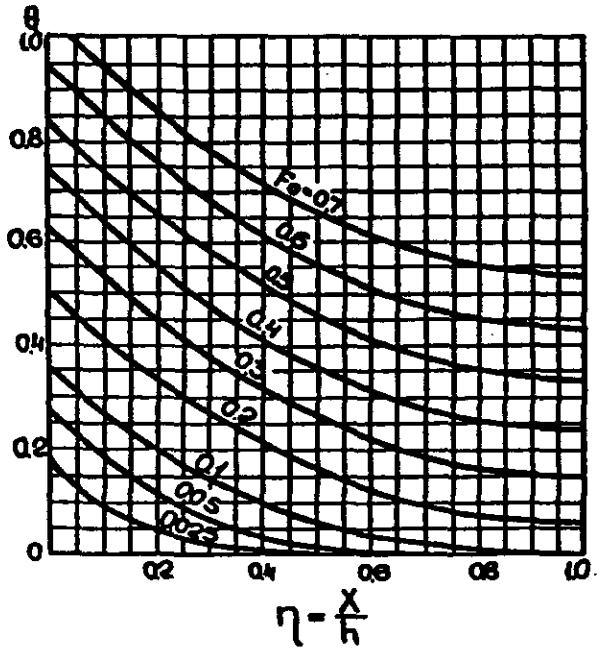


Figure 22

$$\theta = f(Fo, \eta) \qquad Fo = a\tau/h^2$$

where $a = \lambda/C_p\gamma$ is the coefficient of temperature diffusivity of the material, m^2/sec.

For known (given) τ and x one finds Fo and η, and then from them θ, after which the temperature at this point of the plate at time τ is determined from equation (512).

However, besides solving the question of the temperature of a given point on the plate at a given moment of time, this graph makes it possible to answer a number of other interesting questions.

The first question is: in what amount of time will a plate of thickness h be heated over its entire thickness (i.e., how long will it take for the heat front to reach the "cold" surface?)

It can be seen from the graph that this moment corresponds to the quantity

$$Fo = a\tau_{heat}/h^2 = 0.1 \qquad (52)$$

from which the time of heating is:

$$\tau_{heat} = 0.1(h^2/a) \qquad (53)$$

A second question is: to what depth h_{heat} has the heat front penetrated into the body in time τ?

From the same equation (52) we have:

$$h_{heat} = \sqrt{10a\tau} \qquad (54)$$

A third question is: what is the average temperature of the plate at time τ?

The average temperature is determined from the parameter θ= Fo, i.e., it is equal to

$$\bar{t} = t_0 + \bar{\theta}\frac{qh}{\lambda} = t_0 + Fo\frac{qh}{\lambda} \qquad (55)$$

If, however, we wish to find the average temperature of the plate at time τ_{heat} (or the average temperature of the layer heat that has been heated in time τ — naturally for $h_{heat} < h$), then for this case, as can be seen from the graph, Fo = 0.1 and

$$\bar{t}_{heat} = t_0 + 0.1(qh/\lambda) \qquad (56)$$

Finally, a fourth question: how does the temperature of the "hot" surface of the plate differ from the average temperature of the heated layer?

As we already know, for the heated layer

$$Fo = 0.1$$

and the average heating of the heated layer at this time is

$$\bar{t} - t_0 = 0.1(qh_{heat}/\lambda) \qquad (57)$$

It follows from the graph that the heating of the hot surface of the plate at this moment will be

$$t_{hot} - t_0 = 0.35(qh_{heat}/\lambda) \qquad (58)$$

Thus, for one-sided heating of a plate that is uniform over its area the increase of the temperature of its "hot" surface will be 3.5 times higher than the increase of the average temperature of the layer that has been heated in the same time (if $h_{heat} \leq h$).

This simple example clearly shows what broad possibilities are

offered to the developer by ready-made general solutions of problems in graphic form such as are contained in [10].

3.5 Heating of a body by a beam of charged particles [3]

Let us consider the heating of a plate by an electron beam of power P_p (W), which falls uniformly on area F, m^2.

Transforming expression (58), one can obtain the increase of the temperature of the "hot" surface in a time equal to a pulse length τ_p:

$$\Delta t_{surf} = 0{,}35 \frac{P_p}{F} \cdot \frac{\sqrt{10 a \tau_p}}{\lambda} = 0{,}35 \frac{P_p}{F} \cdot 3{,}16 \frac{\sqrt{\lambda \tau_p}}{\sqrt{c_p \gamma} \cdot \lambda} \text{ or}$$

$$\Delta t_{surf} \simeq 1{,}11 \frac{P_p \sqrt{\tau_p}}{F \sqrt{c_p \lambda \gamma}} \qquad (59)$$

However, as experiments show, the temperature of heating of the surface of the body when it is bombarded by electrons actually proves to be lower than that calculated by expression (59). This is due to the fact that the power of the electron beam is released not on the very surface of the plate, but rather in some layer that is equal in thickness to the penetration depth of electrons into the material (expression (11)). This decrease is considered by introducing an additional factor $G(\delta/\sqrt{a\tau_p})$ into expression (59) in the case of heating by an electron beam:

$$\Delta t_{surf} \simeq 1{,}11 \frac{P_p \sqrt{\tau_p}}{F \sqrt{c_p \lambda \gamma}} G\left(\delta / \sqrt{a \tau_p}\right) \qquad (60)$$

The function $G(\delta/\sqrt{a\tau_p})$ is described by complex expressions, but in calculations one can use the following simplified formulas:

$$\left.\begin{array}{ll} \text{for } \dfrac{\delta}{\sqrt{a\tau_p}} \leq 5 & G\left(\dfrac{\delta}{\sqrt{a\tau_p}}\right) \simeq 0{,}222 + 0{,}02\left(5 - \dfrac{\delta}{\sqrt{a\tau_p}}\right)^{2{,}28} \\ \text{for } \dfrac{\delta}{\sqrt{a\tau_p}} \geqslant 5 & G\left(\dfrac{\delta}{\sqrt{a\tau_p}}\right) \simeq \dfrac{1{,}11}{\delta/\sqrt{a\tau_p}} \end{array}\right\} \quad (61)$$

Substituting (61) into (60), we obtain:

$$\left.\begin{array}{ll} \text{for } \dfrac{\delta}{\sqrt{a\tau_p}} \leq 5 & \Delta t_{surf} \simeq \dfrac{0{,}022 P_p \sqrt{\tau_p}}{F\sqrt{\lambda C_p \gamma}} \left[11{,}1 + \left(5 - \dfrac{\delta}{\sqrt{a\tau_p}}\right)^{2{,}28}\right] \\ \text{for } \dfrac{\delta}{\sqrt{a\tau_p}} \geqslant 5 & \Delta t_{surf} \simeq \dfrac{1{,}23 \cdot 10^4 P_N \tau_p}{C_p F T^{1{,}5}} \end{array}\right\} \quad (62)$$

Expressions (61) and (62) make it possible to calculate the heatup temperature of the surface of the body when it is bombarded by electrons. In correspondence with this, if the curve of the distribution of temperatures for ordinary surface heating of a plate has the form of 1 (Figure 23), then when it is heated by a beam of electrons with the same specific heat flux, this curve will have the form of 2.

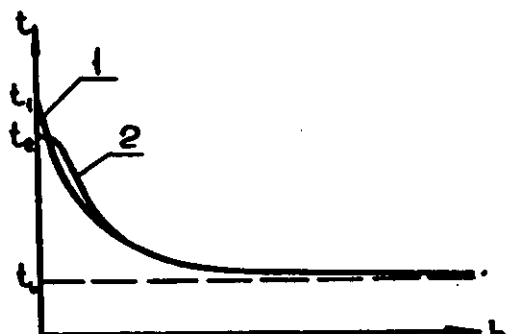


Figure 23

Here the following ratio will be valid:

$$\frac{t_2 - t_0}{t_1 - t_0} = G\left(\frac{\delta}{\sqrt{a\tau_p}}\right)$$

4. Thermoelastic stresses in the elements of constructions

The heating of the parts of constructions, as a rule, causes in them the appearance of mechanical stresses, which in this case are called thermoelastic stresses. Exact calculation of thermoelastic stresses is a very difficult problem, and the engineer in practical reality must always seek reasonable hypotheses that are aimed at simplifying the calculation scheme with the goal of reducing it to one that is amenable to mathematical investigation.

Just to get a sense of the approach to solving such problems, we will discuss very briefly a technique for determining stresses in systems of uniformly heated rods.

The elongation of a rod when heated is, as is well known, defined as

$$\Delta l = \alpha l \Delta t$$

where l is the length, m;

Δt is the temperature of heatup, deg;

α is the coefficient of linear expansion of the material, 1/deg.

If the rod is fixed in place and cannot buckle, then the following stress will arise in it

$$\sigma = -\alpha E \Delta t \qquad (63)$$

where E is the modulus of elasticity of the material (the "-" sign indicates that compressive stresses develop upon heating ($\Delta t > 0$)).

Now we will discuss two rigidly fastened bars of the same length, which are made of different materials (Figure 24).

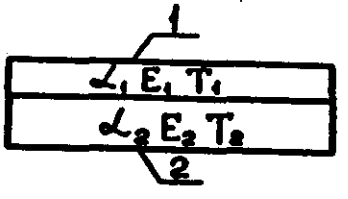


Figure 24

Assume the rods were fastened at temperature T_0, and then one of them has been heated to a temperature of T_1 and the other to T_2. Naturally, there will begin a redistribution of heat in the direction of lower temperature, but in the calculation we will consider that this redistribution does not exist. This is the assumption that immediately makes the problem "doable," with the error going into the calculation allowance.

Since the rods by definition do not bend and do not buckle, then, being rigidly joined to each other, they obtain the same resulting elongations Δl_1 and Δl_2, each of which has thermal and force components. By using expression (63), we can write the equality of these elongations in the form:

$$\alpha_1 \ell (T_1 - T_0) + \frac{\sigma_1 \ell}{E_1} = \alpha_2 \ell (T_2 - T_0) + \frac{\sigma_2 \ell}{E_2}$$

In addition, we have from the static equilibrium the system:

$$\sigma_1 F_1 = -\sigma_2 F_2$$

Simultaneous solution of the last two expressions makes it possible to find the stresses in the form:

$$\left.\begin{aligned} &\sigma_1 = -K\alpha_1 E_1 (T_1 - T_0) \\ &\sigma_2 = -\sigma_1 \frac{F_1}{F_2} \\ &\text{where } K = \frac{1 - \frac{\alpha_2}{\alpha_1} \cdot \frac{T_2 - T_0}{T_1 - T_0}}{1 + \frac{E_1}{E_2} \cdot \frac{F_1}{F_2}} \end{aligned}\right\} \quad (64)$$

If the rods are joined so that some axial shift π of one relative to the other is possible before the beginning of the interaction, then

$$K = \frac{1 - \frac{\alpha_2}{\alpha_1} \cdot \frac{T_2 - T_0}{T_1 - T_0} - \frac{\Delta}{\alpha_1 \ell (T_1 - T_0)}}{1 + \frac{E_1}{E_2} \cdot \frac{F_1}{F_2}} \quad (65)$$

These and an even larger number of other cases have been examined in the corresponding literature [11,12,13,14], but the approach to solving the problem is always the same and is analogous to this one.

We should discuss separately the stresses that arise when bodies are bombarded by charged particles. With ordinary pulsed operation of the beam source, the part is subject to cyclic heating during the time of passage of the beam. This heating causes in the material mechanical stresses, the magnitude of which will be the greater, the greater the amplitude of the pulse temperature. At some temperature these stresses become sufficient to create microfailures in the metal, which, accumulating as the number of cycles increases, lead to failure of the material in the end.

At the same time, for each material there is a critical temperature, at which the stresses that arise in the material are elastic and do not lead to failure for any number of cycles. According to the data of [3], these "safe" temperatures are, for example:

for tungsten $\Delta T_{\delta W} = 843°$

for molybdenum $\Delta T_{\delta Mo} = 595°$

for copper $\Delta T_{\delta Cu} = 110°$

If, however, the heatup temperature exceeds the "safe" temperature, the useful life of the part will be limited to a certain number of heating cycles N, which can be calculated by the formula [3]:

$$N \simeq \frac{U - 50T_0}{16{,}7U} e^{\frac{U}{6(T_0 + \Delta T)}} \qquad (66)$$

where N is the permissible number of cycles of operation of the device;

U is the heat of evaporation of the material, cal/mole;

T_0 is the steady-state temperature before the beginning of the pulse, K;

ΔT is the amplitude of pulsed heating, deg.

We give data on the heat of evaporation for a number of frequently encountered materials (cal/mole), which are difficult to find in the literature:

Tungsten	175,270
Molybdenum	140,460
Aluminum	60,000
Graphite	120,000
Copper	76,320

5. Example of calculation

A copper plate 0.5 m long and 0.1 m wide with thickness h = 0.01 m is heated on one side by a pulsed electron beam that is uniformly distributed over the area and that has current I = 12.5 A at a kinetic electron energy T = 10^6 eV. The current pulse duration $\tau_p = 4 \times 10^{-3}$ sec and the pulse repetition period τ_{rep} = 1 sec.

On the opposite side the plate has a channel for cooling by water with cross-sectional dimensions 0.1 m x 0.01 m; the velocity of the water w = 8 m/sec and the temperature of the water at the inlet t_{in} = 20°C.

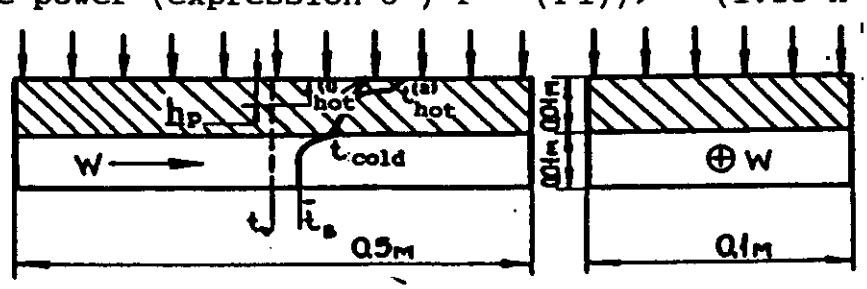
We will define the distribution of temperatures in the plate in a steady-state regime before the next pulse and after it; to determine the time for the system to reach a steady-state regime, and to determine the useful life of the construction.

5.1 Calculation scheme and additional parameters

Duty factor $\nu = \tau_{rep}/\tau_p = 1/(4 \times 10^{-3}) = 250$

Pulse power $P_p = T \cdot I = 1.25 \times 10^7$ W

Average power (expression 8′) $P = (T \cdot I)/\nu = (1.25 \times 10^7)/250 = 5 \times 10^4$ W.



It is necessary to explain precisely what is meant by "distribution of temperatures in a steady-state regime before the next pulse and after it."

A steady-state regime is determined by the average power P, and the distribution of temperatures in the plate will be linear through the thickness with a difference $t^{(1)}_{hot} - t_{cold}$, which is proportional to this power.

However, at the moment of passage of the beam a thin surface layer of the plate (its thickness is the sum of the depth of the penetration of electrons and the depth of heating of the material in τ_p) will be heated to $t^{(2)}_{hot}$. The temperature of the surface after a pulse will exceed its temperature before a pulse by this "surge" $t^{(2)}_{hot} - t^{(1)}_{hot}$. This same quantity $t^{(2)}_{hot} - t^{(1)}_{hot}$ will be then the amplitude of the pulse temperature, which affects the useful life of the item. Here the quantity $t^{(2)}_{hot} - t^{(1)}_{hot}$ is determined by the pulse power of the beam P_p.

5.2 The heatup of water during cooling of the plate

$$\Delta t_{H_2O} = \frac{P}{C_p G} = \frac{5 \times 10^4}{4.18 \times 10^3 \times 8 \times 0.01 \times 0.1 \times 10^3} \simeq 1.5^0$$

(Here G is the flow rate of water, kg/sec)

The average temperature of the water is:

$$t_{H_2O} = t_{in} + 0.5\Delta t_w \approx 21°C$$

The parameters of the water at t = 21°C are:

$$\lambda = 0.6 \text{ W/m·deg}; \ \mu = 10^{-4} \text{ kg·sec/m}^2; \ P_r = 6.87$$

5.3 Temperature of "cold" surface of plate

The "wall-water" difference of temperatures (expression 27) is:

$$\Delta t_{ст-в} = \frac{P}{\alpha F_{вхл}}$$

The temperature of the "cold" surface $t_{cold} = t_{H_2O} + \lambda t_{w-H_2O}$. For determination of the coefficient of convective heat transfer α we find from expression (30′) the Reynolds number Re (expression 29):

$$d_r = \frac{4.0,1.0,01}{2(0,1+0,01)} = 0,018\,\text{м}$$

$$Re = \frac{w d_h \gamma}{\mu g} = \frac{8.\ 0,018.\ 10^3}{10^{-4}\ .\ 9,81} = 1,472\ .\ 10^5$$

$$Nu = 0,023\, Re^{0,8} Pr^{0,4} \varepsilon_l = 0,023.(1,472.10^5)^{0,8}.6,87^{0,4} = 677$$

$$\alpha = \frac{Nu.\lambda}{d_r} = \frac{677.0,6}{1,8.10^{-2}} = 22570$$

W/m^2.deg (according to the graph in Figure 13, $\alpha \approx 23,000$),

after which the difference of temperatures between the wall and water proves to be:

$$\Delta t_{w-H_2O} = \frac{5.10^4}{2,257.10^4.0,5.0,1} = 44^\circ,$$

and the temperature of the "cold" surface of the plate in steady-state regime will be

$$t_{cold} = t_{H_2O} + \Delta t_{w-H_2O} = 21 + 44 = 65°C$$

5.4 Temperature of the "hot" surface of the plate before the next pulse

This temperature, as was already noted, is determined by the difference of temperatures through the thickness of the plate that corresponds to average power P

$$t^{(1)}_{hot} = t_{cold} + \Delta t_{pl}$$

The difference of temperatures through thickness in correspondence with (16) is:

$$\Delta t_{pl} = \frac{Ph}{\lambda F} = \frac{5.10^4.10^{-2}}{400.\ 0{,}05} = 25^0$$

after which $t^{(1)}_{hot} = 65 + 25 = 90°C$.

5.5 Temperature of the "hot" surface of the plate after the next pulse

$$t^{(2)}_{hot} = t^{(1)}_{hot} + \Delta t_{surf}$$

where Δt_{surf} is the amplitude of pulsed heating of the surface by the beam in time τ_p, which is determined by the pulse power of the beam P_p.

To find Δt_{surf} we find from one of expressions (62) the value of parameter $\delta/\sqrt{a\tau_p}$ (δ from expression 11):

$$\frac{\delta}{\sqrt{a\tau_p}} = \frac{10^{-4}\ T^{1,5}}{\gamma\sqrt{a\tau_p}} = \frac{10^{-4}.1000^{1,5}}{8{,}9.10^3\sqrt{1{,}11.10^{-4}.4.10^{-3}}} = 0{,}53$$

Since $\delta/\sqrt{\Delta a\tau_p} < 5$, we obtain from the first of expression (62):

$$\Delta t_{surf} = \frac{0{,}022 P_p\sqrt{\tau_p}}{F\sqrt{\lambda C_p \gamma}}\left[11{,}1 + \left(5 - \frac{\delta}{\sqrt{a\tau_p}}\right)^{2,28}\right] =$$

$$= \frac{0{,}022.1{,}25.10^7\sqrt{4.10^{-3}}.\ 41{,}6}{0{,}05\sqrt{400.389.8{,}9.10^3}} = 388^0$$

After this:

$$t^{(2)}_{hot} = 90 + 388 = 478°C.$$

At what material thickness will this "surge" of temperature begin to be felt?

This thickness is equal to the sum of the penetration depth of electrons and the depth of heating of the material in time τ_p:

$$h_p \simeq \delta + h_{red} = \frac{10^{-4}\, T^{1,5}}{\gamma} + \sqrt{10 a \tau_p} = \frac{10^{-4} \cdot 1000^{1,5}}{8,9 \cdot 10^3} +$$

$$+ \sqrt{10 \cdot 1,11 \cdot 10^{-4} \cdot 4 \cdot 10^{-3}} = 3,55 \cdot 10^{-4} + 2,11 \cdot 10^{-3} \simeq$$

$$\simeq 2,5 \cdot 10^{-3}\ \text{м}$$

5.6 Time to come up to regime

In correspondence with expressions (50) we have:

$$\tau^{5\%}_{regime} = \frac{3(t_{max} - t_0)}{P/mC_p} = \frac{3[t^{(1)}_{hot} - t_0]}{P/mC_p} = \frac{3(90-20)}{\frac{5 \cdot 10^4}{0,5 \cdot 0,1 \cdot 0,01 \cdot 8,9 \cdot 10^3 \cdot 389}} = 7,3\,\text{sec}$$

$$\tau^{2\%}_{regime} = \frac{4[t^{(1)}_{hot} - t_0]}{P/mC_p} \simeq 10\,\text{sec}$$

5.7 Useful life

The useful life in numbers of cycles of heating in correspondence with (66) is:

$$N = \frac{U - 50T_0}{16,7U} e^{\frac{U}{6(T_0 + \Delta T)}} = \frac{76320 - 50(90+273)}{16,7 \cdot 76320} e^{\frac{76320}{6(90+273+388)}} =$$

$$= 1,04 \cdot 10^6\ \text{cycles}$$

If τ_{rep} = 1 sec the time of operation will be ≈290 hours.

It is interesting to evaluate how significant consideration of the penetration depth of electrons into the plate is in this case.

The pulsed heating of the surface Δt_{surf} without consideration of the penetration of particles into the plate is given by expression (59) and in our case is

$$\Delta t_{surf} = 1,11 \frac{P_L \sqrt{\tau_p}}{F\sqrt{\lambda C_p \gamma}} = 1,11\ \frac{1,25 \cdot 10^7 \sqrt{4 \cdot 10^{-3}}}{0,05 \sqrt{400 \cdot 389 \cdot 8,9 \cdot 10^3}} = 472^0$$

The value Δt_{surf} that was obtained exceeds the actual value (i.e., that calculated with consideration of the actual penetration of electrons into the material) only by ≈20% (it was Δt_{surf} = 388°). However, in this case the useful life of the construction falls off sharply, since for Δt_{surf} = 472°C it will be only

$$N = \frac{76320-50(90+273)}{16,7.76320} e^{\frac{76320}{6\cdot(90+273+472)}} \approx 1.88 \times 10^5 \text{ cycles,}$$

i.e., ≈5.5 times less than at Δt_{surf} = 388°C.

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Calculation of Heat Loads in Design

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