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Comparison of Different Planar Channels as Possible Deflectors of Proton Beam*

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Abstract

Proton beam deflection by a bent single crystal is considered. The paper compares properties of different crystal deflectors with different orientations using a model in which the dechanneling length is proportional to the transverse critical energy.

1.0 INTRODUCTION

Charged beam deflection using the effect of planar channeling in a bent single crystal is widely studied both experimentally and theoretically. Capture into a channeling regime is well described by existing models.[1,2] The dependence of dechanneling length L_D on the crystal curvature radius R has been less investigated. It is shown[3] that in recent Fermi National Accelerator Laboratory (FNAL) experiments[4] the measurements on the dechanneling length of a bent crystal are well described in a simple model:

$$L_D \sim pv \cdot E_c\,(pv/R), \tag{1}$$

where pv is the product of momentum and velocity of a channeling particle, and E_c is the depth of a potential well in a bent crystal. This model was first proposed[5] based on a diffusion equation solution for average electron density. Figure 1 shows experimental points and theoretical curves for warm (a) and cooled (b) crystals of silicon. Relation (1) explains also temperature dependence; in this case we should choose the transverse critical distance $x_c(u)$ accurately, u being thermal vibrational amplitude. Analysis of the FNAL data indicates that $x_c = d/2 - 2u$ is a good choice.

The present paper compares properties of different deflectors with different orientations using this model. Because of some uncertainty in the dechanneling length, results presented here (especially in Section 3.0) should be considered as a qualitative illustration rather than as a prediction.

2.0 SCALING TO OTHER CHANNELS

Comparison of dechanneling in different planar channels of different materials gives another confirmation to the model in Relation (1). Taking into account different average electron densities $\langle n_e \rangle$ in these channels in the simplest form, we can rewrite (1) as follows:

$$L_D \sim pv \cdot E_c / \langle n_e \rangle\,. \tag{2}$$

The average electron density could be defined as

$$\langle n_e \rangle = \frac{1}{2x_c} \int_{-x_c}^{x_c} n_e(x)dx = \text{const.} \times U'(x_c)/x_c. \tag{3}$$

Here we assume that inside a channel $n_e(x) = \text{const.} \times U''(x)$. That is, "unpotentiality" of motion is defined by potential itself! Then Relation (2) will be of the form:

$$L_D = \Lambda \cdot pv \frac{E_c}{U'(x_c)} x_c, \tag{4}$$

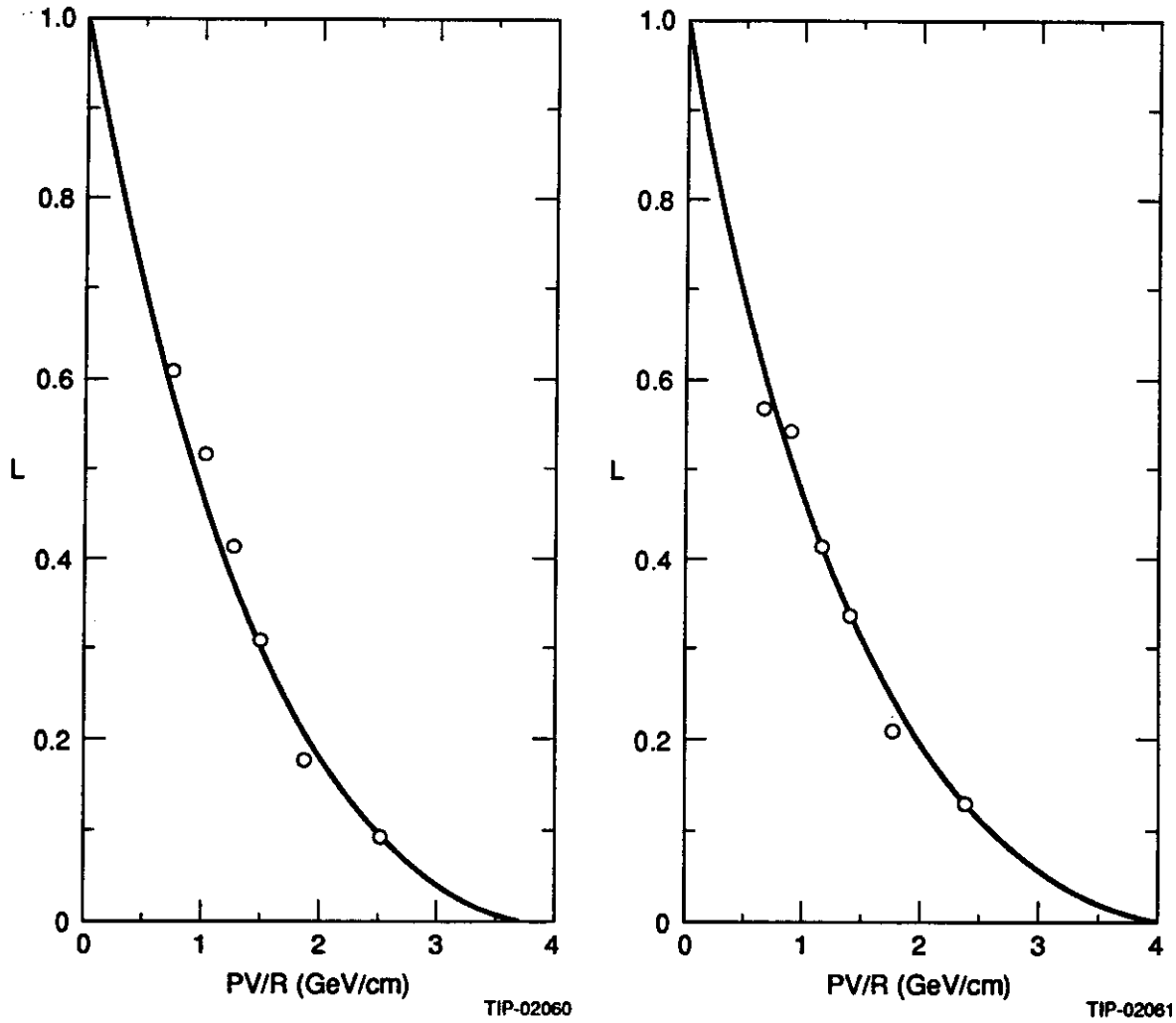

Figure 1. Dependence of Si(110) Dechanneling Length $L_D/\lambda pv$ (λpv is the Dechanneling Length in a Straight Crystal) on Curvature pv/R. (Solid line is Equation (1), points are from FNAL experiment.[4] Here $x_c = d/2 - a_{TF} - u$ is assumed.)
(a) Crystal at 293 K, $\lambda = 0.54\,\mathrm{mm/GeV}$;
(b) crystal at 128 K, $\lambda = 0.65\,\mathrm{mm/GeV}$.

where Λ is some phenomenological constant. Figure 2 plots Equation (4) (with $\Lambda = 2.7\,\mathrm{mm/GeV\,A^2}$) versus experimental data;[6] here we take L_D as for "wide-angular" beam (see Reference 6 for discussion). Five experimental points are in reasonable agreement with Equation (4).

Deflector "strength" is defined by the ratio $\theta_D = L_D/R_T$ (where $R_T = pv/U'(x_c)$), which depends only on crystal properties. In the considered model the ratio

$$\theta_D \sim x_c E_c \tag{5}$$

is proportional to the potential well "volume" in transverse energy-coordinate space. Table 1 shows properties of different planar channels of silicon, germanium, and tungsten.

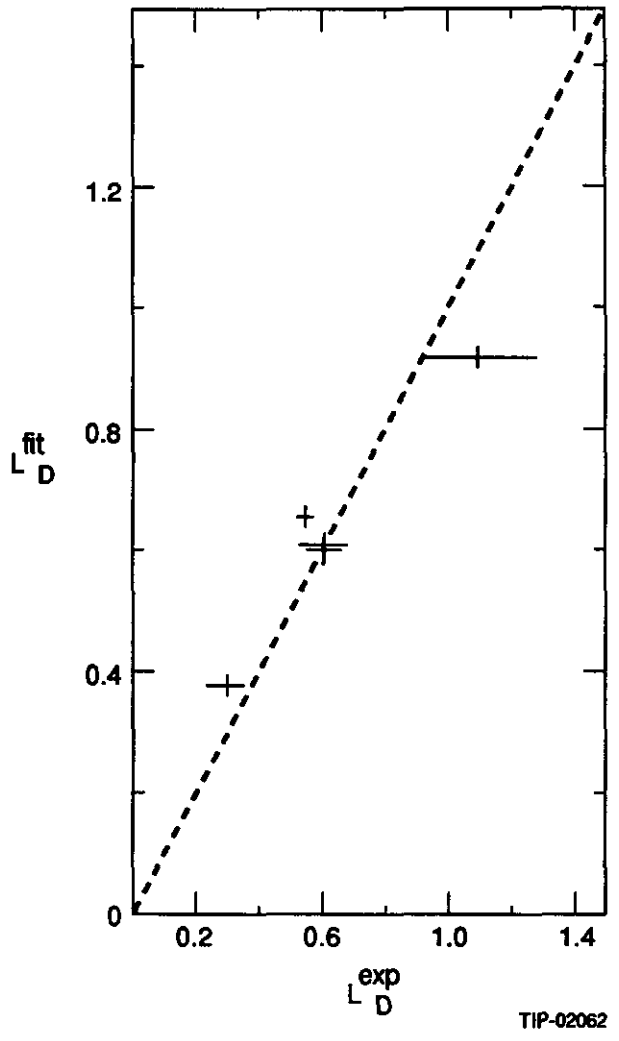

Figure 2. Plot of Fit (4) with $\Lambda = 2.7\,\mathrm{mm/GeV\,A^2}$ Versus Experimental L_D.

3.0 DEFLECTION EFFICIENCY

In the considered model the beam deflection efficiency as a function of angle θ is equal to

$$F(pv/R,\theta) = F_s A_b(pv/R)\exp\left(-\frac{\theta\, E_c(0)}{\lambda\, pv/R\, E_c(pv/R)}\right), \tag{6}$$

where F_s is the probability for capture into the channeling regime for a straight crystal, A_b is the ratio of bent channel acceptance to straight channel acceptance, and λpv is the dechanneling length in a straight crystal ($\lambda \simeq 0.61\,\mathrm{mm/GeV}$ for Si(110) at room temperature[6]). A harmonic approximation for a beam uniformly distributed in angle (Equation (6)) is as follows:

$$F(\rho,\theta) = F_s(1-\rho)^2\,\exp\left(-\frac{\theta/\theta_D}{\rho(1-\rho)^2}\right); \tag{7}$$

TABLE 1. PROPERTIES OF SOME PLANAR CHANNELS.

Channel	R_T (cm)	θ_c (μrad)	L_D (cm)	θ_D
Si(110)	174	5.7	61	0.35
Si(111)L	178	6.2	109	0.61
Si(111)S	287	2.9	9	0.03
Ge(110)	98	7.4	61	0.61
Ge(111)L	106	7.7	89	0.84
Ge(111)S	157	3.8	7	0.05
W(100)	33	11.2	30	0.91
W(110)	23	14.5	55	2.41

NOTE: Properties are calculated for $pv = 1\,\mathrm{TeV}$ and $x_c = d/2 - 2u$. L_D values for Si(110), Si(111)L, Ge(110), W(100), and W(110) are experimental ones taken from Reference 6; others are estimated using Equation (4).

here $\rho = R_T/R$ is a critical radius-to-radius ratio, and $\theta_D = L_D^0/R_T$ is a characteristic angle independent of energy (see Table 1).

We shall illustrate Equation (6), calculating the efficiency of deflecting a proton beam by $\pi/2$ in a tungsten crystal (channel W(110)). Figure 3 shows F/F_s as a function of pv/R, for a crystal at room temperature and cooled to 77 K. Due to uncertainty in L_D and sensitivity to x_c, the error of this result could be from one to two orders.

The efficiency of a $\pi/2$ beam deflection by a germanium crystal was calculated to be 5×10^{-9}, or four orders less than the estimation in Reference 7. That excludes the possibility of germanium as a deflector for this purpose. A tungsten deflector (dislocation-free) could only give a satisfactory efficiency of beam deflection at the right angle. The dependence $L_D(R)$ crucially affects perspectives of proton beam deflection at an angle of order $\pi/2$.[8]

There should also be some dependence on incident beam divergence; L_D for a "pencil" beam could be twice as high[5] as the one used here. However, in the very long crystal required here, channeled beam distribution will relax quickly from the "pencil" case to the "wide" one.

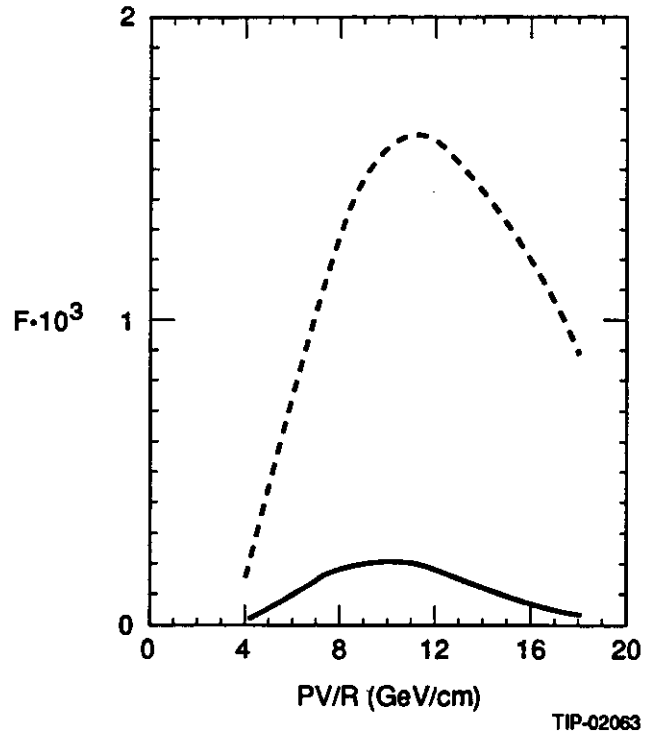

Figure 3. Efficiency of Proton Beam Deflection at $\pi/2$ by Tungsten (110) Channel. Solid line for 300 K, dashed line for 77 K.

For large θ, efficiency is maximal when

$$d[\rho E_c(\rho)]/d\rho = 0, \tag{8}$$

which means $\rho = 1/3$ for the harmonic case, corresponding to

$$F_{\text{harm}}(\theta) \simeq F_s\, 4/9\, \exp(-27/4\ \theta/\theta_D). \tag{9}$$

4.0 DIAMOND-TYPE (111)-CHANNELS

For diamond-type crystals with (111) orientation, the situation is more complicated. Wide (111)L channels and narrow (111)S ones are working simultaneously, giving efficiency in the following way:

$$F_{(111)} = 3/4\, F_{(111)L} + 1/4\, F_{(111)S}, \tag{10}$$

where $F_{(111)L,S}$ are defined by Equations (6) and (7), with corresponding values of $R_T^{L,S}$ and $L_D^{L,S}$ (see Table 1). Usually, a narrow channel has a much smaller L_D and a larger R_T

than a wide channel, and so does not participate in beam deflection. Figure 4 shows the efficiency of beam deflection at 1 mrad by (111) planes.

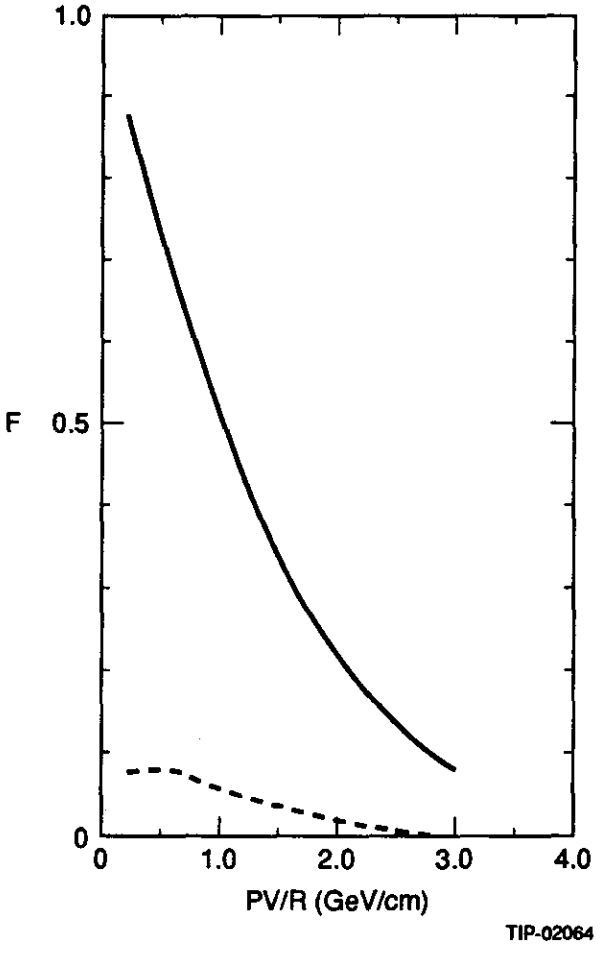

Figure 4. Efficiency (Rel. Units) of Beam Deflection by Si(111) Planes at 1 mrad. (Dashed line is narrow (111)S channel contribution.)

5.0 OPTIMAL CURVATURE

Efficiency (Equations (6) and (7)) could be maximized, solving the equation $dF/d\rho = 0$. That allows for optimal curvature $\rho_{\rm opt}(\theta)$ in the harmonic case

$$\theta/\theta_D = \frac{2\rho_{\rm opt}^2\,(1-\rho_{\rm opt})^2}{1-3\rho_{\rm opt}}. \tag{11}$$

Substituting $\rho_{\rm opt}(\theta)$ into Equation (7), we obtain an upper limit $F_{\rm opt}(\theta)$ for a beam deflection efficiency at angle θ:

$$F(\rho,\theta) < F_{\rm opt}(\theta) = F_s(1-\rho_{\rm opt}(\theta))^2\,\exp\left(-\frac{2\rho_{\rm opt}(\theta)}{1-3\rho_{\rm opt}(\theta)}\right). \tag{12}$$

For any required deflection angle θ, an optimal parameter pv/R could be chosen, so the upper limit of the efficiency does not depend on the beam momentum. Figure 5 shows optimal curvature (a) and corresponding efficiency (b) calculated in the Moliere approximation for different planar channels.

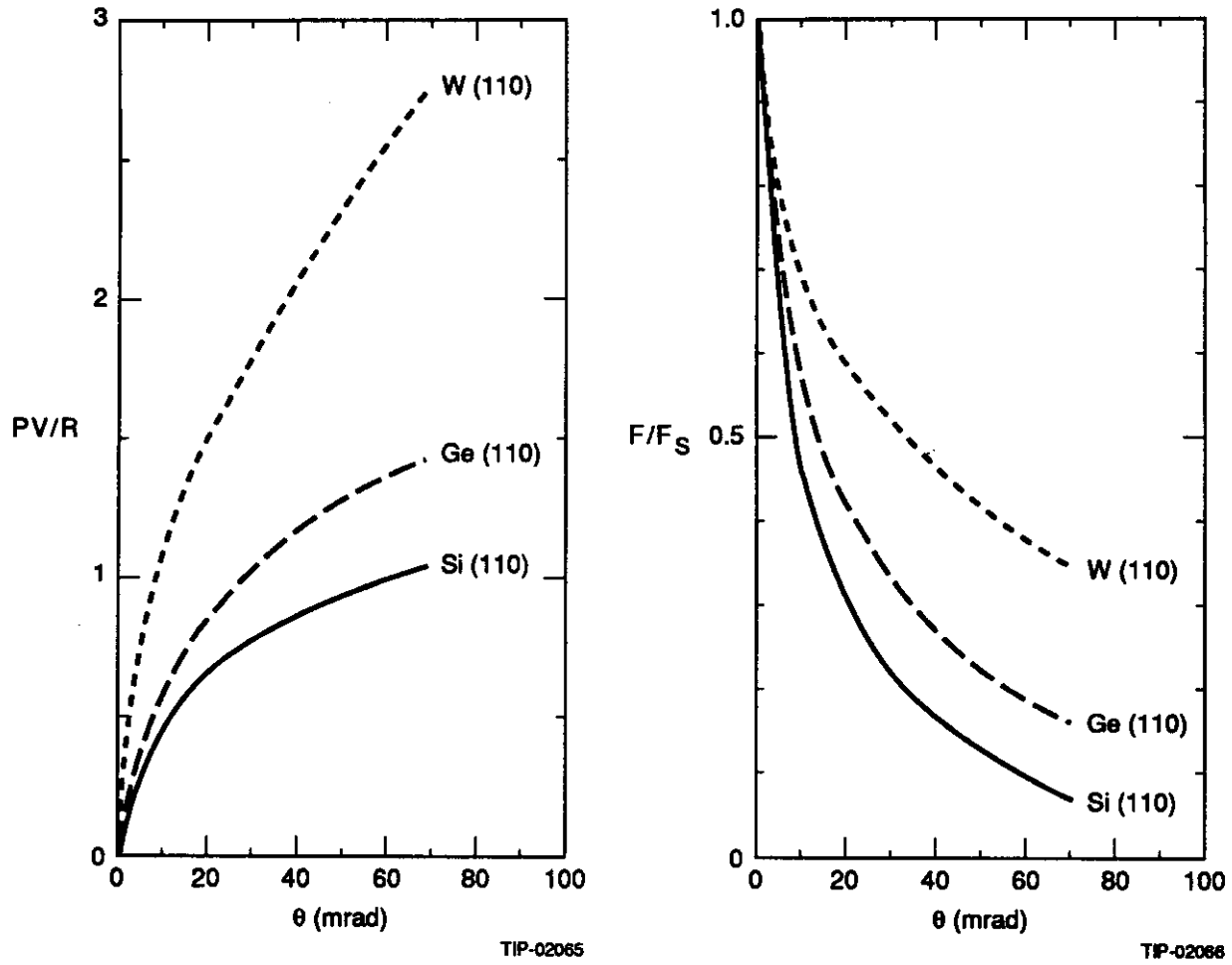

Figure 5. Optimal Curvature pv/R (a) and Corresponding Efficiency F/F_s (b) as Functions of Deflection Angle θ.

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