SSCL-N-754

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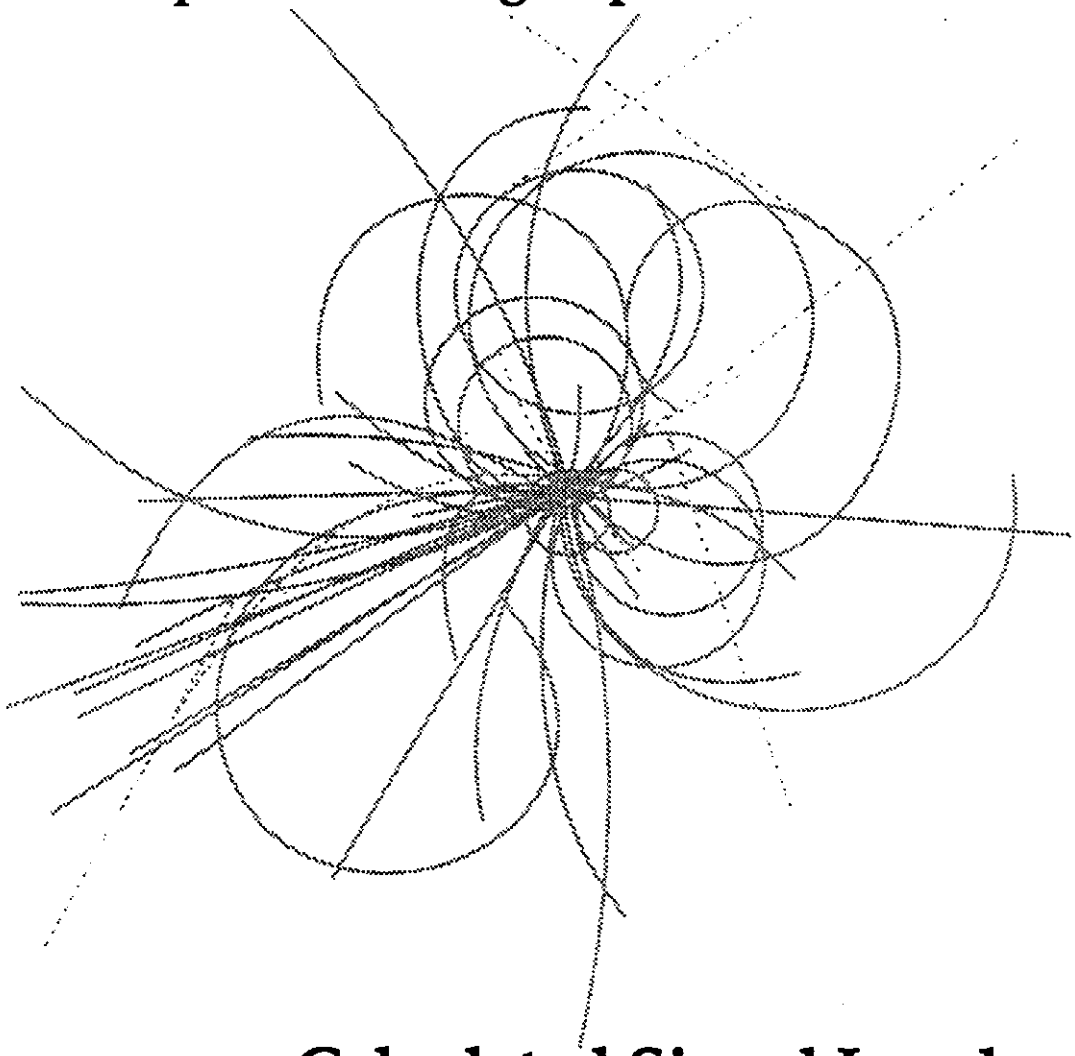

Calculated Signal Level From LEB BPM

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G. Aiello

Accelerator Division
Superconducting Super Collider Laboratory†
2550 Beckleymeade Avenue
Dallas, Texas 75237

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1.0 Introduction

The output signal from the LEB Beam Position Monitor system is calculated in this note in order to evaluate the dynamic range and the resolution required for the system.

The beam is not bunched before the first 100 turns. For this reason this analysis is restricted to a time which goes from 0.2 ms after injection up to extraction. During this acceleration time the beam sigma changes from 3.4 ns down to 0.7 ns while the RF is ramped from 47.5 MHz to 60 MHz [1].

The signal from the stripline has been analyzed both in the time domain and frequency domain and both the signal response and the RF component at different current and different time after injection have been calculated. The results from the two methods are compared in order to check the calculations.

The program Pro-Matlab [2] running on a Sun Workstation has been used to get the numerical results.

2.0 Time Domain Analysis

The beam current of a single bunch is given by:

$$I_b(t) = eN\, f(t) \qquad \text{(EQ 1)}$$

with $e = 1.6 \times 10^{-19}$ C; N = number of particles per bunch and f(t) is the normalized distribution function. If the beam is gaussian it is:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} \qquad \text{(EQ 2)}$$

with σ = bunch length [s].

When the current at (EQ1) reaches the upstream end of the stripline, it induces two signals: one is seen at the output, the other one propagates downstream with velocity c. The amplitude of this signal is given by:

$$V_0(t) = \frac{1}{2} g Z_L I_b(t) \qquad \text{(EQ 3)}$$

with L = cable length [m], $A(f_0)$= attenuation [nepers/m] = 1/8.687 [dB/m]. The parameter β depends on the cable.

The slope of the loss versus frequency dependance in the log-log plot is not always 0.5: for this reason it is convenient to choose a frequency comparable to the impulse risetime [4]. Different approaches can be used [5] which consider the deviation from the ideal coaxial cables [6].

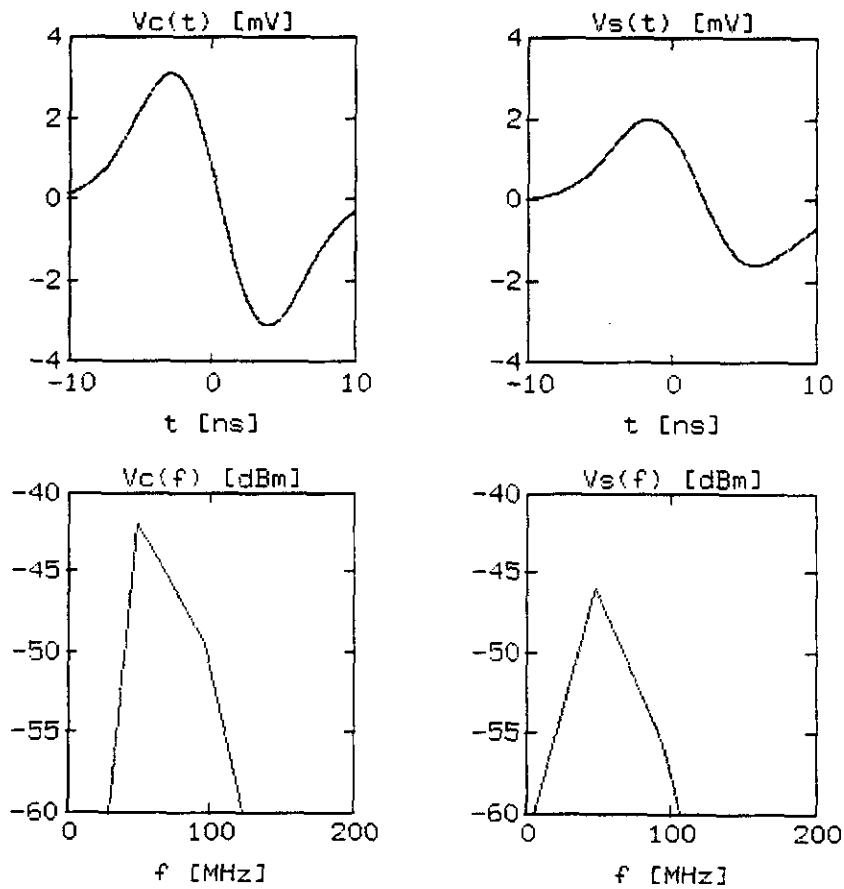
The signal downstream the cable is given by the convolution of the input signal with the impulsive response of the cable:

$$V_s(t) = V_c(t) \otimes h(t) \qquad \text{(EQ 7)}$$

The signals calculated for different bunch lengths are shown in figure 2 and 3, while the parameters used in the calculation are shown in table 1. A 100 meter long RG-213/U has been considered.

In order to calculate the RF amplitude upstream and downstream of the cable, the signal spectrum has been calculated using the FFT algorithm [7]. The results of the calculation for different bunch length and different modes of operations are shown in table 2.

FIGURE 2. Time domain signal and spectrum. Bunch length = 3.4 ns; Injection time = 0.2 ns. a) signal at the stripline; b) signal after the cable.



3.0 Frequency Domain Analysis

The same analysis can be done in the frequency domain [8]. In order to calculate the response downstream the cable in the frequency domain it is necessary to make a Fourier decomposition of the input waveform, then to calculate the gain and the phase change for each of the frequency components contained in the input waveform, and finally to reassemble them to obtain the time domain response [9].

The gaussian distribution function at (EQ2) can be analitically written in Fourier series:

$$f(t) = \frac{1}{T} + \sum_{1}^{\infty} I_k \cos(k\omega_o t) \qquad \text{(EQ 8)}$$

with T = bunch spacing; $\omega_0 = 2\pi/T$; and the Fourier series coefficient being:

$$I_k = \frac{2}{T} e^{-\frac{k^2 \times \omega_o^2 \times \sigma^2}{2}} \qquad \text{(EQ 9)}$$

The response from the stripline has been calculated from (eq1), (eq2), (eq3) and (eq4) substituting (EQ2) with (EQ8).

The signal downstream the cable has been calculated considering the attenuation and the phase shift for each of the frequency components. Being $\gamma_k = \alpha_k + j\beta_k$ the attenuation per unit length at a given frequency and L the cable length, the attenuation coefficient is:

$$A_k = e^{-\alpha_k L} \qquad \text{(EQ 10)}$$

with α_k in Nepers/m, while the cosinus argument becomes:

$$k\omega_o t - \beta_k L \qquad \text{(EQ 11)}$$

with $\beta_k = \alpha_k + k\omega_0 T_l$, T_l the delay per unit length. The first part is responsible for the phase shift, the second one for the cable delay. The signal downstream the cable becomes:

$$V_s(t) = \frac{1}{2} eNgZ_L \times \sum_{k=1}^{\infty} A_k I_k \left[\cos(k\omega_0 t - \beta_k L) - \cos((k\omega_0)(t - \frac{2l}{c}) - \beta_k L) \right. \qquad \text{(EQ 12)}$$

Since we are not interested to the delay between Vc(t) and Vs(t) in this analysis, we suppose $\beta_k = \alpha_k$.

The amplitude attenuation coefficient α_k has been interpolated from the loss versus frequency diagram [10]. It has been found that the loss is not proportional to the square root of the frequency, as assumed at section 2, the slope of the log-log plot being 0.64 instead of 0.5.

- collider test operation: $N = 2x10^8$.

The bunch length change during the acceleration ramp induces a change in the response signal from the stripline. The results are shown in table 2.

TABLE 2. Calculation results.

				Time analysis		Frequency analysis	
Inj. Time	**RF**	**Bunch length**	**N p⁺**	**Vc (t)**	**Vs (t)**	**Vc (t)**	**Vs (t)**
[ms]	**[MHz]**	**σ [ns]**		**[dBm]**	**[dBm]**	**[dBm]**	**[dBm]**
0.2	47.5	3.4	$2x10^8$	-41.91	-46.02	-41.93	-46.17
			10^{10}	-7.93	-12.04	-7.95	-12.19
			$5x10^{10}$	+6.05	+1.94	+6.03	+1.79
49	60	0.7	$2x10^8$	-33.73	-38.40	-33.72	-38.64
			10^{10}	+0.25	-4.42	+0.26	-4.66
			$5x10^{10}$	+14.23	+9.56	+14.24	+9.32

The discrepancy between the results from the two methods is due to the fact that the real slope of the cable attenuation function is different from the slope used in the time domain analysis.

5.0 Conclusions

Both the time domain and frequency domain calculations have been done to check the results. The comparison between the methods shows a good agreement. The time domain method presents some problems because of the different slope between the cable loss versus frequency log-log plot and the theoretical one. On the other hand the frequency domain analysis presents a frequency-windowing ripple to the output waveform. As a consequence the time domain analysis is recommended when the real shape of the signal is needed, while the frequency domain one when the value of few harmonics has to be calculated.

Different operational modes have been considered and the RF frequency component amplitude of the signal upstream and downstream the cable from 0.2 ms to 49 ms from the injection time has been calculated. The conclusions from the calculations are:

- Cable attenuation: $\Delta V = 4.5$ dB;
- Different modes of operation produce: $\Delta V = 48$ dB;
- The acceleration ramp produce: $\Delta V = 7.5$ dB;
- Maximum output voltage: $V = +9.3$ dBm;
- Minimum output voltage: $V = -46.2$ dBm.
- Total voltage range: $\Delta V = 55.5$ dB.