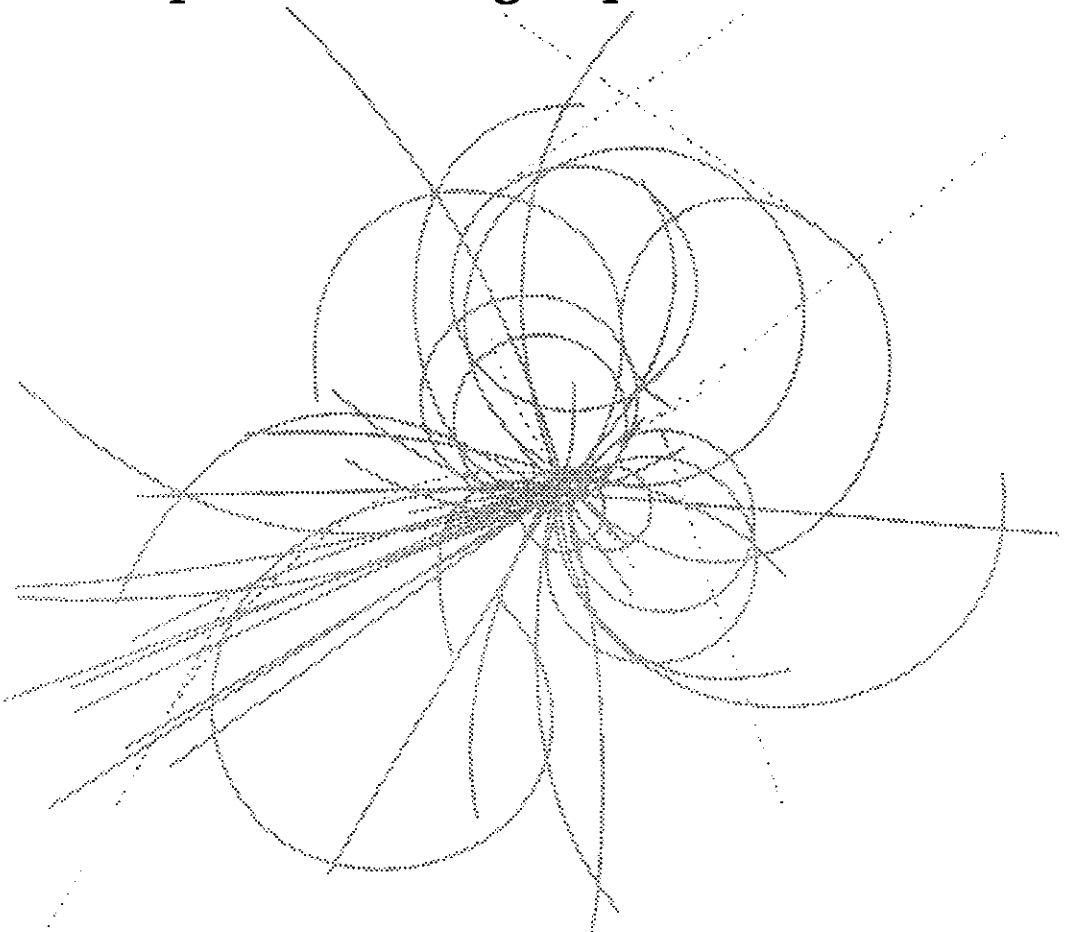
SSCL-N-739

Superconducting Super Collider Laboratory



The Effects of Collider Arc Quadrupole Magnetic Field Errors on the SSC Injection Process

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THE EFFECTS OF COLLIDER ARC QUADRUPOLE MAGNETIC FIELD ERRORS ON THE SSC INJECTION PROCESS

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The purpose of this note is to describe the accelerator effects of field errors in the Collider Arc Quadrupole. Two concerns motivated this work. Allowed multipoles in quadrupoles shift the tune with betatron amplitude directly whereas allowed multipoles in dipoles only shift the tune with betatron amplitude in the presence of momentum errors. Furthermore, the quadrupole is located at the point of maximum beta value in the arc lattice.

In this note the SSC convention for the description of field errors will used. This convention is used in the Quadrupole Magnet Prime Item Development Specification.

The SSC convention for magnetic field errors in quadrupoles is different from the convention for errors in dipoles. Dipole errors are measured by the ratio of the field error at 1 cm to the central field. Quadrupole errors are measured by the ratio of the field error at 1 cm to the quadrupole component of the field at 1 cm. More precisely,

$$iBx + By = 10^{-4} g\ Rr \sum_{m} (i a_m + b_m)\left(-\frac{x+iy}{R_r}\right)^m$$

In this expression g is the quadrupole gradient (T/m), Rr is the reference radius for measurement (0.01 m), and the factor of 10^{-4} takes into account that the coefficients are measured in SSC units. The mean and the rms deviation from the mean of the values of a_m and b_m are sufficient for a description of accelerator performance.

PART I Mean (systematic) values of multipoles:

If the coefficients b_m (n>1) have a non-zero value when averaged over the ensemble of quadrupoles in the collider ring, the betatron tune may depend on momentum deviation δ , and on the x and y betatron amplitudes a_x and a_y . Since a beam consists of particles with a spread in momentum and betatron amplitude, there will be a spread in tune if the average values of b_m are different from zero. Tracking studies which simulate the collider ring made with 5 cm dipoles suggest that if the spread in tune caused by the average value of any multipole is larger than 0.001, that multipole may contribute significantly to the dynamic aperture. The evaluation of tune spread should be made with all combinations of betatron amplitude up to 5 mm and momentum deviation up to 5 x 10^{-4}.

A calculation of the tune shift caused by a field error begins with a calculation of the betatron phase shift caused by the passage of a particle through the error fields. The x-phase shift $\Delta\Psi_x$ is as follows:

$$\Delta\Psi_x = \frac{\cos\Psi_x\ \beta_x^{\frac{1}{2}}\ L\ B_y(x,y)}{\epsilon_x^{\frac{1}{2}}\ B\rho}$$

In this expression L is the length of the quadrupole and all other parameters are evaluated at the location of the quadrupole. The average tune shift is as follows:

$$\Delta\nu_x = \frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{2\pi} \frac{\beta_x^{\frac{1}{2}} L \cos\psi_x B_y(x,y)}{\epsilon_x^{1/2} B_0 \rho} \, d\psi_x \, d\psi_y$$

The coordinates x and y used to evaluate the field error in the above expression are as follows:

$$x = \beta_x^{\frac{1}{2}} \epsilon_x^{\frac{1}{2}} \cos\psi_x + \eta\delta$$

$$y = \beta_y^{\frac{1}{2}} \epsilon_y^{1/2} \cos\psi_y$$

A rough and adequate estimate for the tune shifts due to multipoles is the variation of x tune with x betatron amplitude with y and momentum deviation zero. This is a consequence of the specific design of the collider where momentum dependent tune shifts are small. The error fields of the nth multipole are:

$$B_y(x) = 10^{-4} g R_R^{1-n} b_n x^n$$

The average tune shift for one quadrupole is:

$$\Delta\nu_x = \frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{2\pi} \frac{10^{-4} g R_R^{1-n} b_n \beta_x^{\frac{1}{2}(n+1)} \epsilon_x^{\frac{1}{2}(n-1)} L \cos^{n+1}(\psi_x) \, d\psi_x \, d\psi_y}{B_0 \rho}$$

Evaluation of the average gives the following result:

$$\Delta\nu_x = \frac{10^{-4} g L b_n R_R^{(1-n)} \beta_x^{\frac{1}{2}(n+1)} \epsilon_x^{\frac{1}{2}(n-1)}}{2\pi B\rho} \frac{(n(n-2)\cdots 1)}{((n+1)(n-1)\cdots 2)}$$

One simplification useful. If the phase shift/cell is 90 degrees then:

$$\frac{gL}{B\rho} = \frac{\sqrt{2}}{L_{\frac{1}{2}}}$$

$L_{\frac{1}{2}}$ is the half cell length (90 meters). Tracking results are usually quoted as functions of the x betatron amplitude at the F quadrupole:

$$a_x = \epsilon_x^{\frac{1}{2}} \beta_{xF}^{\frac{1}{2}}$$

In terms of these variables the tune shift caused by the field error in one quadrupole is:

$$\Delta\nu_x = \frac{b_m a_x^{m-1} 10^{-4} \sqrt{2} R_R^{1-m} \beta_x^{\frac{1}{2}(m+1)} \left[m(m-2) \cdots 1 \right]}{2\pi L_{\frac{1}{2}} \beta_{xF}^{\frac{1}{2}(m-1)} \left[(m+1)(m-1) \cdots 2 \right]}$$

Finally in a lattice made up of m cells, the tune shift becomes:

$$\Delta\nu_x = \frac{m k b_m a_x^{m-1} 10^{-4} \sqrt{2} R_R^{1-m} \left[\beta_{xF}^{\frac{1}{2}(m+1)} - \beta_{xD}^{\frac{1}{2}(m+1)} \right] \left[m(m-2) \cdots 1 \right]}{2\pi L_{\frac{1}{2}} \beta_{xF}^{\frac{1}{2}(m-1)} \left[(m+1)(m-1) \cdots 2 \right]}$$

The term caused by the D quad errors is negative because these magnets are powered with the opposite polarity. The above expression can be reduced by again making the assumption that the phase shift/cell is 90 degrees. The final result is:

$$\Delta\nu_x = \frac{m k b_m a_x^{m-1} 10^{-4} R_R^{1-m}}{\pi} \left(\frac{2+\sqrt{2}}{2-\sqrt{2}} \right)^{\frac{1}{2}} \left(1 - \left(\frac{2-\sqrt{2}}{2+\sqrt{2}} \right)^{\frac{1}{2}(m+1)} \right) \frac{\left[m(m-2) \cdots 1 \right]}{\left[(m+1)(m-1) \cdots 2 \right]}$$

This result is valid for odd n. For even n the tune shift is zero.

The number of cells in the collider rings is approximately 400. The following tables give the tune shift for 1 unit of $\langle bn \rangle$ and for the values of $\langle bn \rangle$ taken from the Quadrupole Prime Item Development Specification. The tune shift is evaluated at a betatron amplitude of 5 mm at the QF quadrupoles.

n	$\langle b_m \rangle$	$\Delta\nu_x$
0	1	0
1	1	1.27 x 10-2
2	1	0
3	1	2.79 x 10-3
4	1	0
5	1	5.97 x 10-4
6	1	0
7	1	1.31 x 10-4
8	1	0
9	1	2.95 x 10-5

Table I Tune shift at 5 mm amplitude caused by 1 unit of systematic quadrupole magnet field error $\langle b_m \rangle$.

n	$\langle b_m \rangle$	$\Delta\nu_x$
0	20.0	0
1	---	---
2	0.2	0
3	0.1	2.79 x 10-4
4	0.1	0
5	0.8	4.77 x 10-4
6	0.05	0
7	0.05	6.56 x 10-6
8	0.05	0
9	0.20	5.91 x 10-6

Table II Tune shift at 5 mm amplitude caused by the field error tolerances given in the Quadrupole Magnet Prime Item Development Specification.

The data in Table II show that the Quadrupole PIDS satisfies the condition that the tune shifts due to any multipole $\langle b_m \rangle$ be less than 0.001 at a 5 mm amplitude.

Systematic errors with an even value of n cause tune shifts only in the presence of momentum errors. To first order in momentum deviation δ , these tune shifts are:

$$\Delta\nu_{x,m} = b_m \, m \left[\frac{\eta\delta}{R_v}\right] \frac{\Delta\nu_{x,m-1}}{b_{m-1}}$$

This expression if valid for n even. Since $\frac{\eta\delta}{R_v}$ is of order 0.1 even for a momentum error of 5×10^{-4} , these terms are clearly negligible.

Persistent currents contribute significantly to the terms b1, b5, and b9. M. A. Green (1) has computed these contributions for the example of the 4 cm QC-1 LBL prototype quadrupole. For 2 TeV injection Green finds $\langle b1 \rangle = -4.1$, $\langle b5 \rangle = -.93$, $\langle b9 \rangle = +.04$. Magnetic measurements made on QC-1 are in approximate agreement with Green's calculation.

The persistent current $\langle b1 \rangle$ will depress the tune of the collider rings by -4.1×0.0127 or 0.052 at 2 TeV injection. In magnets made with SSC style conductor and collaring pressures there is a magnetic field drift during the one hour injection period of about 10% of the calculated persistent current value. The mechanism for the drift is not established and probably several processes are taking place. Use of the 10% rule leads to the conclusion that if the current in the quadrupole is held constant, the tune will drift up during the injection period by about 0.005. This is not the whole story, however, since the field in the collider dipole also drifts during the injection period.

The persistent current $\langle b5 \rangle$ and $\langle b9 \rangle$ do not need correction. A 10% drift in these multipoles during injection will not be noticed. The $\langle b5 \rangle$ calculated by Green is slightly larger than the quadrupole PIDS. It still, however, satisfies the tune shift criterion of <0.001 at 5 mm amplitude criterion. It would be desirable to introduce a slight positive geometric $\langle b5 \rangle$ term in the coil design to partially cancel the persistent current $\langle b5 \rangle$ at injection.

If the coefficients a_m have non-zero averages the lowest order effect is to couple x and y motion. Since the collider will operate with horizontal and vertical tunes split by one unit, an average value of a_m is greatly suppressed in its effect. Tracking studies show that the values of $\langle a_m \rangle$ given in the PIDS do not influence beam behavior.

(1) M. A. Green, SC-MAG-297, A Comparison of Calculated and Measured Magnetization Multipoles in the LBL QC-1 One Meter Quadrupole Magnet

PART II Effects of random multipoles:

The variation in the values of multipoles from quadrupole to quadrupole has the same effect as the variation in the values of dipole multipoles, that is to provide the noise that excites particles to ever higher betatron oscillation amplitude and eventual loss. To a good approximation the effects of random variations can be estimated by comparing the integrated field error per quadrupole with the integrated field error per dipole. A direct comparison can be made by multiplying the quadrupole multipoles by gRrLq/BLb where g is the quadrupole gradient, Rr is the reference radius for multipole measurement, Lq is the length of a quadrupole, Lb is the length of a dipole, and B is the central dipole field. For the collider, this factor is 0.11. Another useful comparison is the field error per unit length of magnet. This can be compared by multiplying the quadrupole multipoles by gRr/B. This factor is 0.313.

	Dipole PIDS	Quad PIDS	.313xQuad PIDS	0.11xQuad PIDS
a1	1.25	5.00	1.56	0.55
b1	.50	(10.00)	3.13	1.10
a2	.35	2.00	0.626	0.22
b2	1.15	2.00	0.626	0.22
a3	0.32	1.00	0.313	0.11
b3	0.16	1.00	0.313	0.11
a4	0.05	0.30	0.094	0.033
b4	0.22	0.30	0.094	0.033
a5	0.05	0.30	0.094	0.033
b5	0.017	0.15	0.047	0.0165
a6	0.008	0.05	0.016	0.0055
b6	0.018	0.05	0.016	0.0055
a7	0.01	0.05	0.016	0.0055
b7	0.01	0.05	0.016	0.0055
a8	0.0075	0.05	0.016	0.0055
b8	0.0075	0.05	0.016	0.0055

Table III

This table compares quadrupole errors with dipole errors. The second column is the dipole error specification in dipole units. The fourth column is the quadrupole specification in dipole units. The comparision between the second and fourth column is a comparision of field error per unit length. The fifth column shows the comparison on a per magnet basis.

There are five dipoles and one quadrupole per half cell. The fifth column in table III shows that the integrated field error in the quadrupole is generally smaller than the integrated field error in one dipole magnet. Since the quadrupole random errors add in quadrature with five dipole random errors, the expected effect of the quadrupole random errors is small.

PART IV Numerical simulation of quadrupole effects:

The SSCTRK numerical simulation code has been modified to accept both random and systematic errors. This modification has been checked, in part, by comparing the tune shifts caused by systematic multipoles with the analytic expressions derived in part I.

The quadrupole PIDS was made with the assumption that the quadrupole coil inner diameter was 4.6 cm. Estimated multipole errors for other inner diameter values have been determined by using the following quadrupole multipole error scaling laws.

Random Errors:

$$(\sigma a'_m, \sigma b'_m) = (\sigma a_m, \sigma b_m)\ (Rs/Rs')^{m-\frac{1}{2}}$$

Systematic Errors:

$$\langle a'_m, b'_m \rangle = \langle a_m, b_m \rangle\ (Rs/Rs')^{m-1}$$

Rs is the scale radius (inner coil diameter) of the quadrupole. The above laws do not apply to the variations of the b1 multipole. <b1> and b1 are to first order independent of magnet bore size; they are caused by variations in the quadrupole length.

Numerical simulations of collider injection have been made including multipoles to 8th order (n=7). No sorting of magnets has been made and no systematic errors above sextupole have been corrected. A chromaticity error of 5 units has been made. The job card for the 4.6 cm quadrupole case is given in Appendix A.

Figure 1, 2, 3, and 4 show 100,000 turn simulation of the collider injection with 3, 4, 4.6, and 5 cm quadrupoles and 5 cm dipoles. These figures show that while the 3 cm quadrupole clearly dominates the aperture, the other choices of quadrupole bore do not have a statistically significant influence.

Another way to look at the influence of quadrupole errors is to compare a collider with dipole errors and no quadrupole errors with a collider with quadrupole errors and no dipole errors. Figure 5 shows a simulation made with only dipole errors while Figures 6, and 7 show simulations made with no dipole errors and 3, and 4 cm quadrupole errors.

PART V Conclusions:

The inner diameter of the coil in the Collider Arc Quadrupole can be made as small as 4 cm without significantly reducing the dynamic aperture determined by the field errors in 5 cm dipoles. If the injection energy is at least 2 TeV, the only persistent current effect that requires correction is an offset and small drift in the tune of the collider during the injection period.

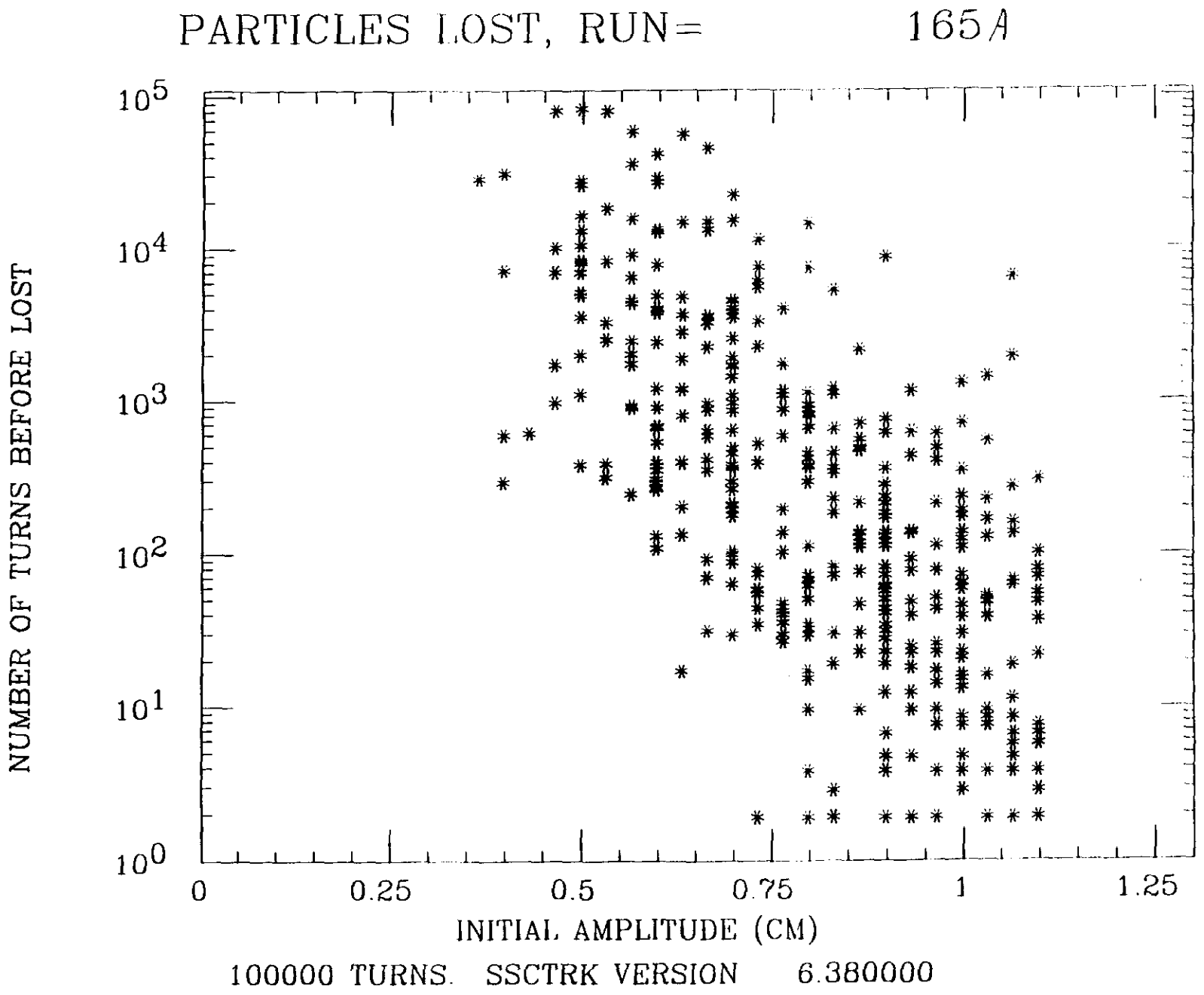

Figure 1 Numerical simulation of SSC injection with 5 cm dipole errors and 3 cm quadrupole errors. Quadrupole errors limit the aperture.

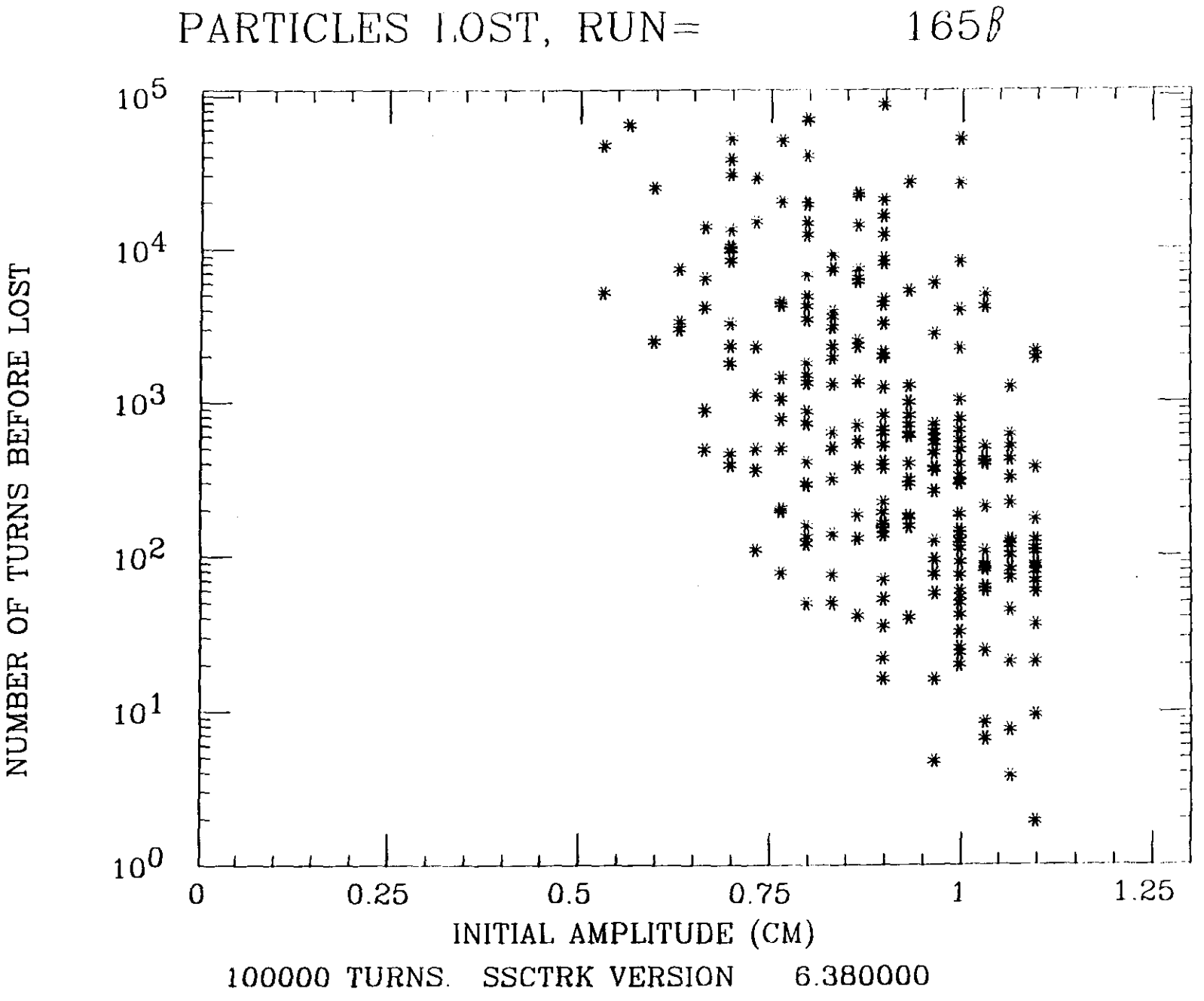

Figure 2 Numerical simulation of SSC injection with 5 cm dipole errors and 4 cm quadrupole errors.

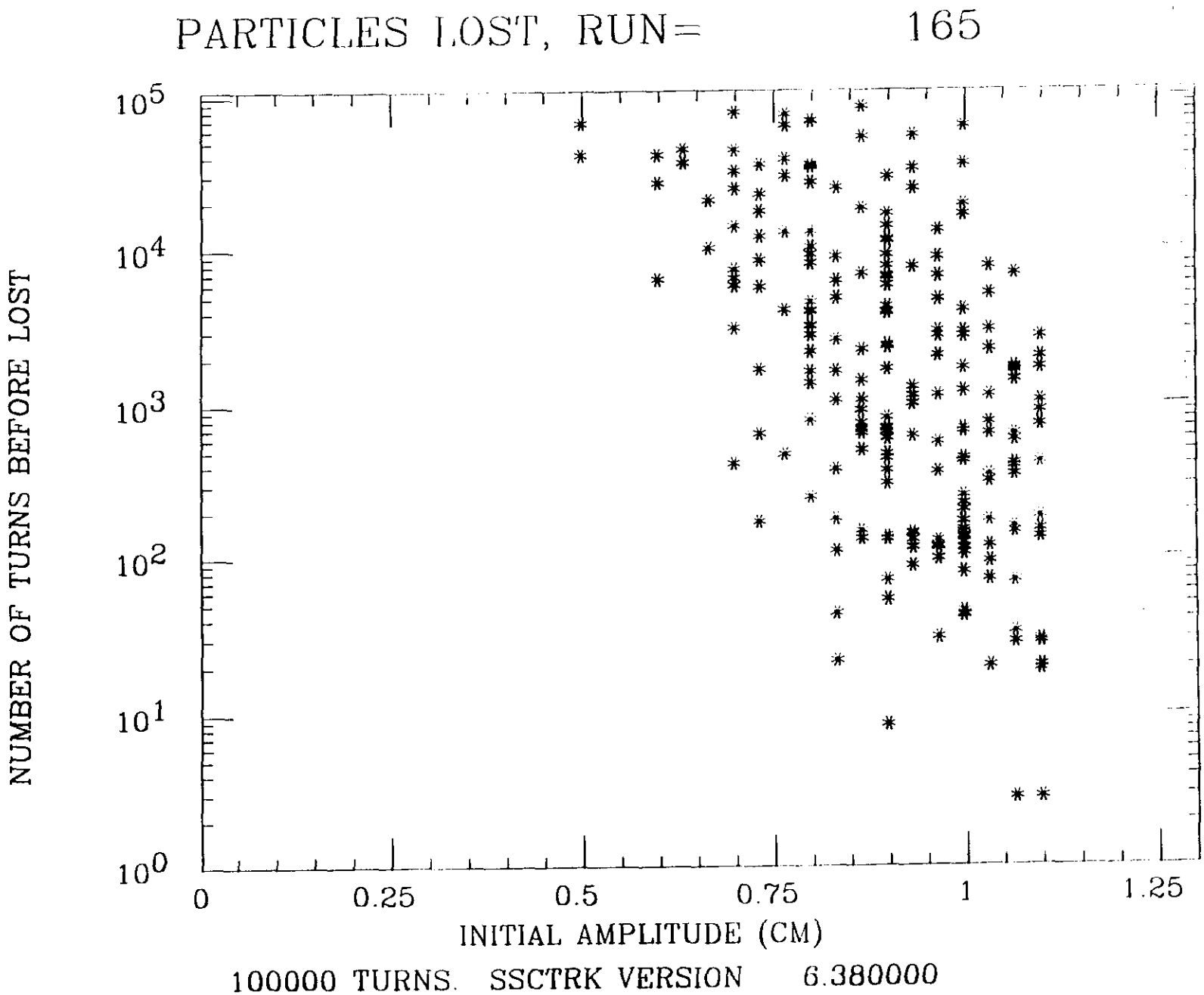

Figure 3 Numerical simulation of SSC injection with 5 cm dipole errors and 4.6 cm quadrupole errors. This error set is taken from the current Dipole and Quadrupole PIDS.

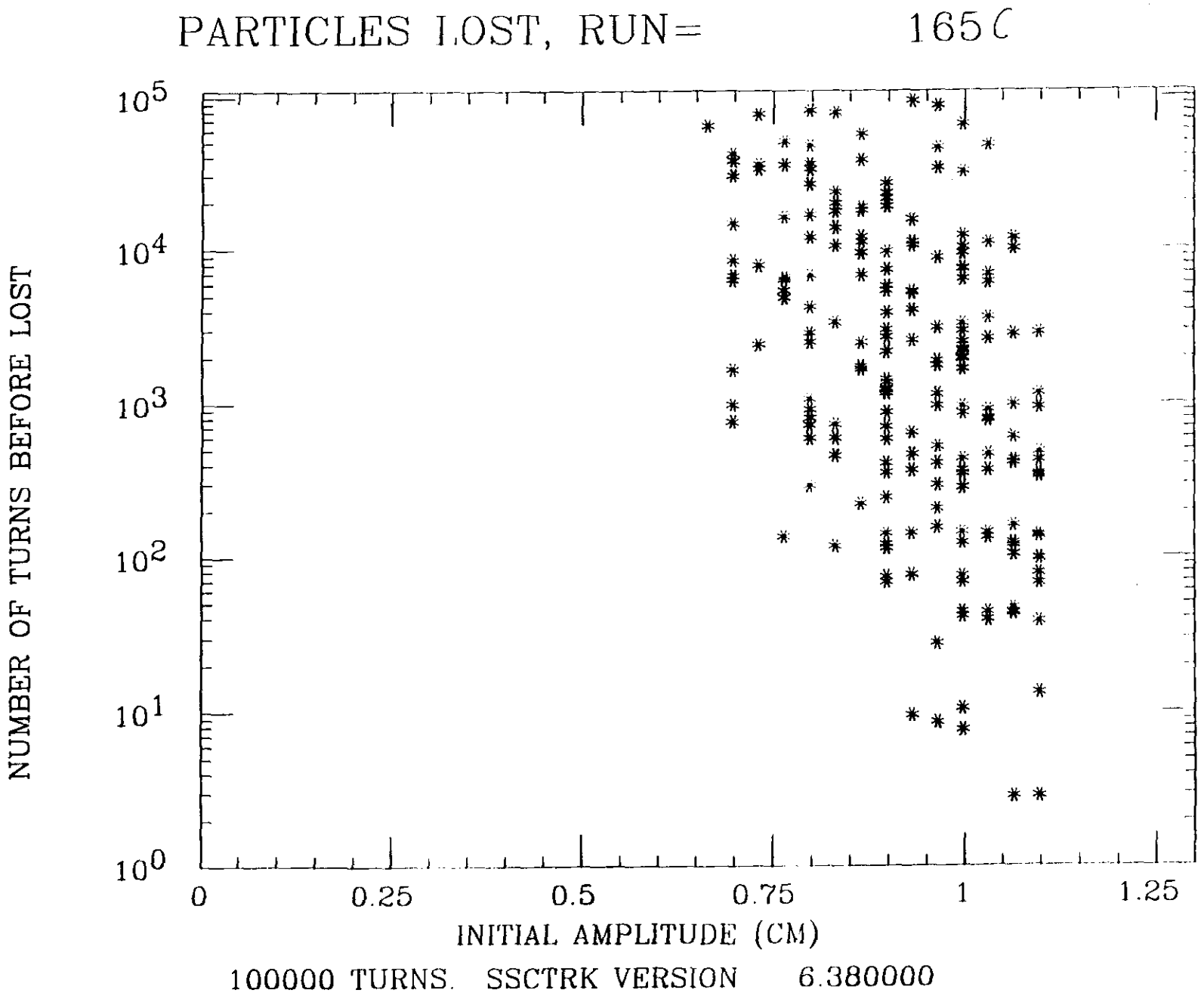

Figure 4 Numerical simulation of SSC injection with 5 cm dipole errors and 5 cm quadrupole errors.

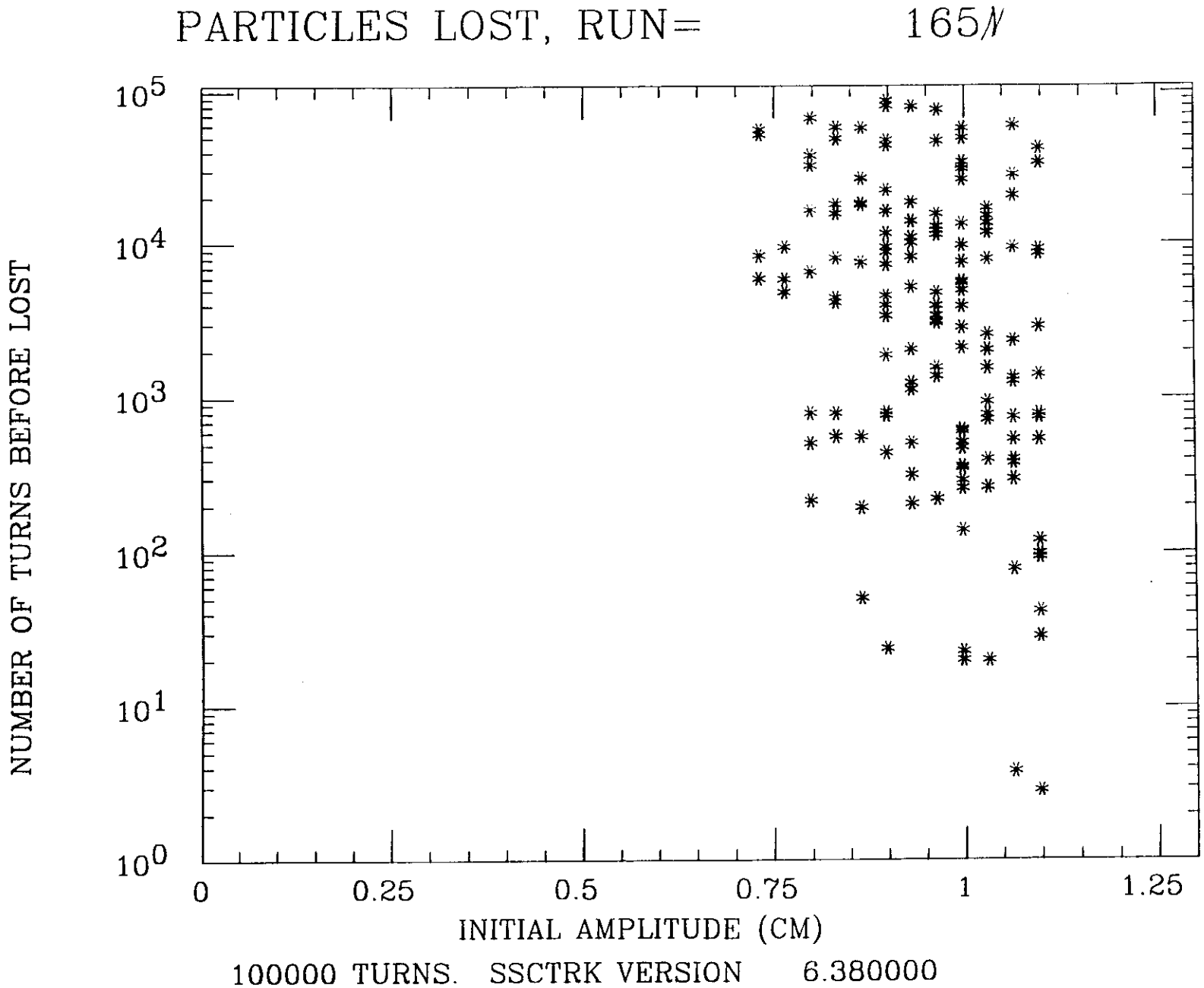

Figure 5 Numerical simulation of SSC injection with 5 cm dipole errors and no quadrupole errors.

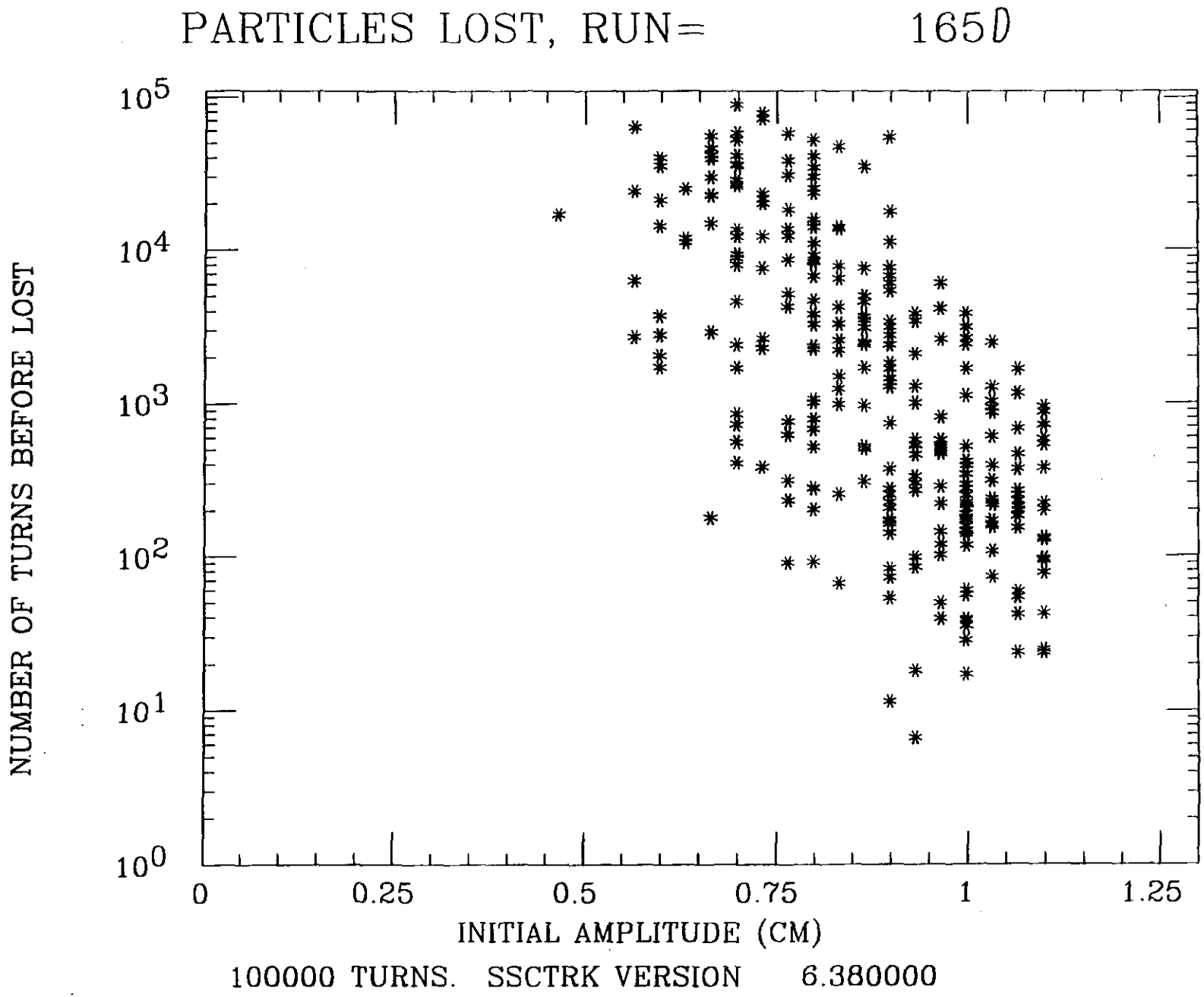

Figure 6 Numerical simulation of SSC injection with no dipole errors and 3 cm quadrupole errors.

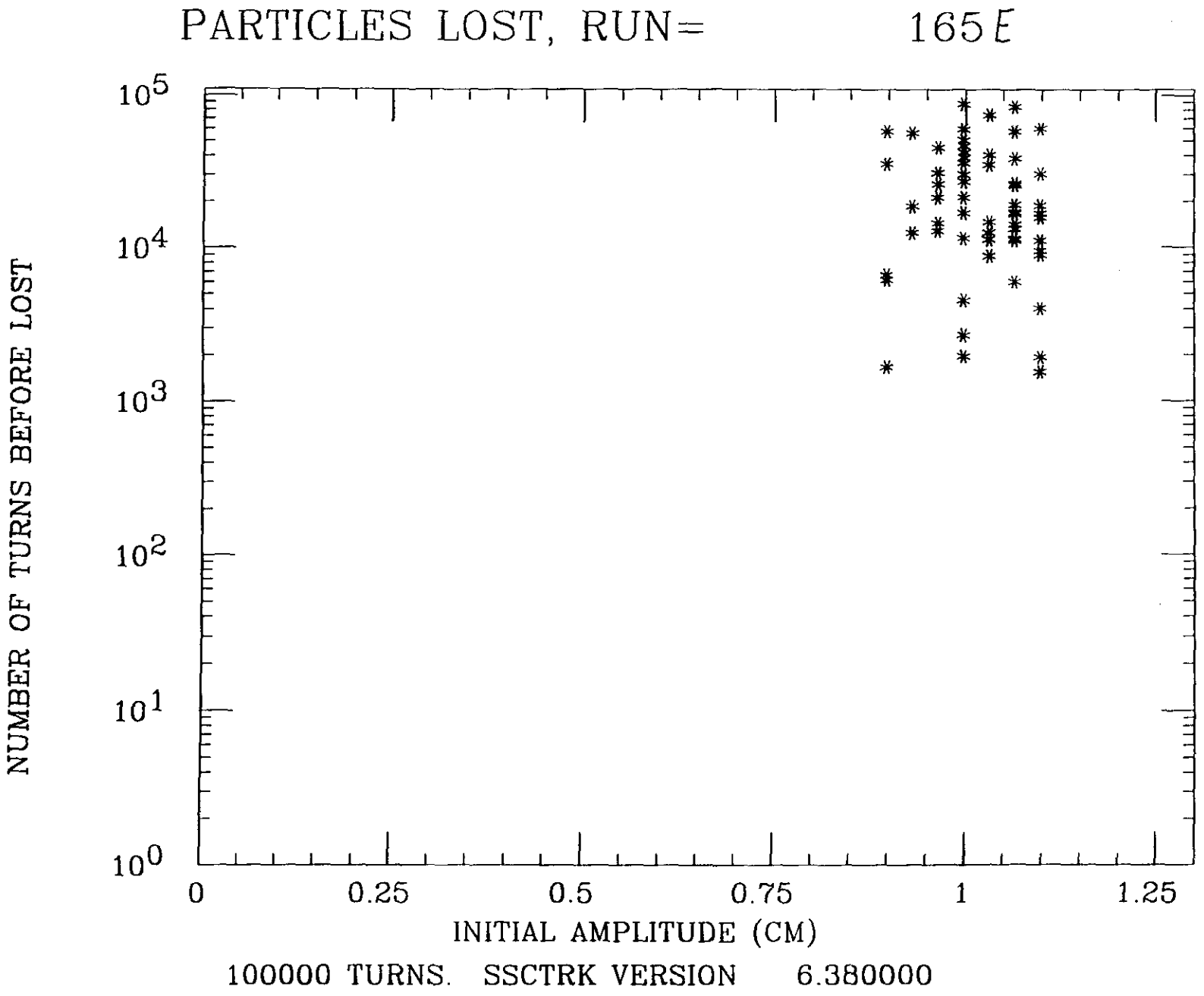

Figure 7 Numerical simulation of SSC injection with no dipole errors and 4 cm quadrupole errors.

Appendix A

Job card used for 4.6 cm quadrupole simulation.

```
&inputs
     resume=0,
     run   =165,
     turns =100000,
     every =5000,

     r0    =11.3d+5,
     mags  =4000,
     apscal=.false.,
     bore  =4.d+0,
     fidbor=4.d+0,
     e0    =2.d+12,
     mpoles=7,
     bends =5.0d+0,

     dx0   =(1.0d-1,1.0d-1),
     aplim =2.0d+0,
     oksmer=-1.,
     inx0  =( 10.d-1, 10.d-1),
     maxdx0=(1.0d-1,1.0d-1),
     th0   =(0.,0.),

     mudeg=90.d+0,
     dnux =1.425d+0,
     dnuy =0.410d+0,

     seed0 =-1,
     sedcas=16,
     xerrab=10.d-2,
     xerr2 =10.d-2,
     drift =.false.,
     derrab=0.d+0,
     derr2 =0.d+0,

     asys= 0.04d+0, .032d+0,.026d+0,.010d+0,.005d+0,.005d+0,.005d+0,
     bsys= 0.04d+0,-2.00d+0,.026d+0,.080d+0,.005d+0,-.02d+0,.005d+0,
     asig= 1.25d+0, .35d+0,.32d+0,0.05d+0,.050d+0,.008d+0,.01d+0,
     bsig= 0.50d+0,1.15d+0,.16d+0,0.22d+0,.017d+0,.018d+0,.01d+0,

     qasys= 2.00d+0, .20d+0,.10d+0,.10d+0, .10d+0,.050d+0,.05d+0,
     qbsys= 0.00d+0, .20d+0,.10d+0,.10d+0,-.80d+0,.050d+0,.05d+0,
     qasig= 5.00d+0, 2.0d+0,1.0d+0,.30d+0, .30d+0,.050d+0,.05d+0,
     qbsig= 10.0d+0, 2.0d+0,1.0d+0,.30d+0, .15d+0,.050d+0,.05d+0,

     aset=  0.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0,
     bset=  0.d+0, 0.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0,
     alset= 0.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0,
     blset= 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0, 3.d+0,

     blks=5,
     blksig=0.d+0,
     setlev=2.1d+0,

     ngridx=4,
     ngridp=1,
     maxdp0=5.0d-4,
     z0    =00.0d+0,

     ramp0 =0.0d+0,
     rfv0  =2.0d+7,
     rffreq=3.75d+8,
     fdbtim=1.0d+4,
     m720  =0.0d-2,
     m10   =0.0d-2,

     bumpel= 0, 0,
```

```
     bumpdx=(0.d+0,0.d+0),
     incsf =0.375d-1,
     incsd =0.375d-1,

     enoise=0.d-2,
     modf  =0.d+0,
     ampf  =0.d+0,

     chekin=3,

     select=.false.,

     ncas=64,
     liscas=1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,
            22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,
            40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,
            58,59,60,61,62,63,64,

     alldon=.false.,
     nexjob=.false.,

&end
&inputs
  alldon=.true.,
  nexjob=.false.,
&end
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