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COMMENT ABOUT CRYOGENIC STABILIZATION AND THE BUS BAR CONDUCTORS

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ABSTRACT

A slightly reinterpretation of the Cryogenic Stability Criteria brings about a close expression to calculate the copper to superconducting ratio needed to satisfy these criteria. This ratio depends on the fraction of perimeter in contact with the Helium, the power generated by the normal zone and the stationary temperature of the conductor. This approach is applied to the proposed bus bar conductor for the SSC.

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1. Introduction

The superconductor bus bar cable can make the connection of every magnets along the chain of the collider ring accelerator.[1] This cable has a very long extension and it suffers of several perturbations during the operation of the machine. When a normal zone appears in the conductor the heat may be transferred to and removed by the Helium (gas or/and liquid depending on its thermodynamic state) flowing on its surface. This mechanism is the only means for the cable to recover its superconductor state, consequently, it can be protected from large disturbances using a proper copper to superconducting (s.c.) ratio with passages where the Helium can flow on its surface, or, turning the power supply off allowing the current to decay fast enough through a dump resistance. This last method requires the monitoring and detection of a normal zone incursion of bus bar cable and to turn off the power supply any time there is a large disturbance on this cable, making the operation of the accelerator less reliable. On the other hand, because of the very small space available inside the magnet is difficult to make passages inside or around the bus bar conductor for the Helium flow, making difficult to assure a cryogenic stabilization of this conductor and requiring a good estimation of the copper to s.c. ration which takes account the fraction of perimeter in contact with the Helium.

Once a normal zone appears in a superconducting bus bar cable, we want that this normal zone shrink itself under the action of the heat transferred to the Helium. If the normal zone is large, the recovery of its superconducting state will be possible if the copper to s.c. ratio, λ, is big enough and there is Helium flowing on the surface of the conductor which takes away the heat generated. At this time, there are two cryogenic criteria to choose this ratio; the first one was formulated by Stekly and Zar[2] and Stekly, Thome and Strauss;[3] the second one was formulated by Maddock, James and Morris[4] which is called "The Equal-Area Theorem."

A very brief discussion of both criteria will be given and with a slight reinterpretation of them, the copper to s.c. ratio will be given as a function of the fraction of the conductor perimeter in contact with the Helium.

2. Review of the Cryogenic Criteria

Suppose that a normal zone appears in a superconducting cable because of a large disturbance on it, then, most of the current will flow into the matrix copper because of the very high resistivity value that the NbTi has in its normal state. Let us calculate the relative current density flowing in the NbTi. Because the voltage across the normal zone will be the same for the copper as well as for the NbTi, then the following relation is satisfied

$$\frac{L\rho_{cu}I_1}{a_{cu}} = \frac{L\rho_{sc}I_2}{a_{sc}} , \tag{1}$$

where I_1 and I_2 are the currents flowing in the copper and the NbTi elements; L is the length of the normal zone; ρ_{cu} and ρ_{sc} are the resistivities of these elements; a_{cu} and a_{sc} are their respective areas. The density of current flowing in the NbTi is given by

$$J_2 = \frac{\rho_{cu}J_1}{\rho_{sc}} . \tag{2}$$

The total heat generated per unit time will be that one generated by the copper and NbTi elements, using the relation (2), it can be expressed as

$$\rho_{cu}J_1^2 + \rho_{sc}J_2^2 = \rho_{cu}J_1^2\left[1 + \frac{\rho_{cu}}{\rho_{sc}}\right] , \tag{3}$$

but the ratio ρ_{cu}/ρ_{sc} is of the order 10^{-4}, so, we can neglect the contribution of the heat generated by the NbTi. The equation that governs the temperature in the conductor can be written as

$$(\delta c)\frac{\partial\theta}{\partial t} = \nabla\cdot\left(k(\theta)\nabla\theta\right) + \rho J^2 - \frac{(\theta-\theta_o)hP}{A} , \tag{4}$$

where ρJ^2 is the rate of heat density generated by the copper matrix; (δc) is the average overall the conductor components of the density, δ, times the specific

heat, c; $k(\theta)$ is the thermal conductivity; P is the perimeter of the conductor; h is the heat transfer coefficient; θ_o is the batch temperature (assumed constant); ∇ is the gradient and $\nabla\cdot$ is the divergent of the space; and A is the total cross section area of the cable, given by

$$A = a_{cu} + a_{sc} \,. \tag{5}$$

First Criterion

This one can be state grossly in the following way, assume the temperature does not depend on the coordinates (the first term of the right hand side of (4) is zero) and assume that the variation of the temperature with respect the time is negative (shrinkage condition) whenever this one is less or equal than the critical temperature of the superconductor, θ_c, then the equation (4) transform in the following relation

$$A\frac{\rho J^2}{(\theta_c - \theta_o)hP} < 1 \,, \tag{6}$$

the copper to s.c. ration, λ, can be obtained from this relation.

Second Criterion

In the criterion (6) we ignored the complicated behavior that function "$(\theta - \theta_o)h$" has with respect the temperature as it is shown in the Figure 1. Let us now take account this behavior and consider we have an stationary state (the temperature of the conductor does not change with time), then defining the vector function $\vec{S}$ as

$$\vec{S}(\theta) = k(\theta)\nabla\theta \tag{7}$$

and using the above considerations, the expression (4) is written as

$$\frac{1}{2k(\theta)}\frac{dS^2}{d\theta^2} = \frac{(\theta - \theta_o)hP}{A} - \rho J^2 \,, \tag{8}$$

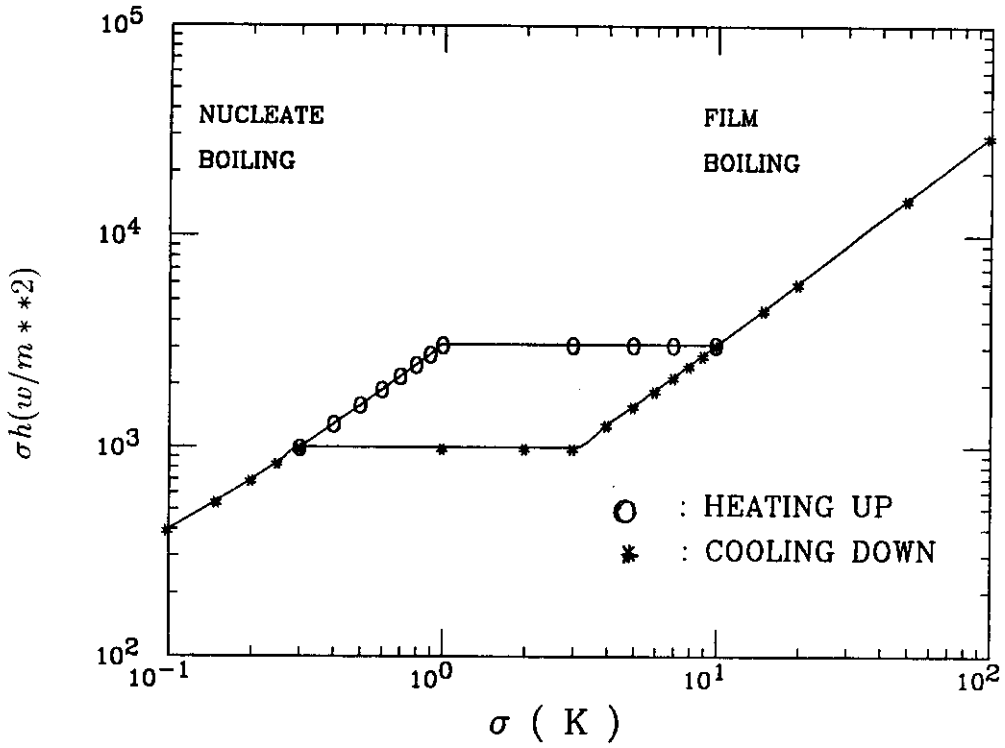

Figure 1. Heat transfer function between conductor and helium.

where S is the magnitude of the vector (7). Multiplying (8) by $k(\theta)$ and integrating with respect the temperature, we have

$$\frac{1}{2}[S^2(\theta_1) - S^2(\theta_o)] = \int_{\theta_o}^{\theta_1} [(\theta - \theta_o)hP/A - \rho J^2]k(\theta)d\theta \ . \tag{9}$$

Assuming now that the flow of heat is constant along the conductor ($S(\theta_1) = S(\theta_o)$), we get

$$\int_{\theta_o}^{\theta_1} [(\theta - \theta_o)hP/A - \rho J^2]k(\theta)d\theta = 0 \tag{10}$$

furthermore, the function $k(\theta)$ is a positive function of the temperature ,so the relation (10) is equivalent to the expression

$$\int_{\theta_o}^{\theta_1} [(\theta - \theta_o)hP/A - \rho J^2]d\theta = 0 \tag{11}$$

which is called "the equal-area theorem" because it expresses explicitly the fact that the area generated by the heating process is the same as the area generated by the cooling process. This, in turn, implies that the conductor will have a stationary condition where its temperature will not depend on the location on the conductor. In practice, however, the thermal conductivity spread the heat of the normal zone along the conductor returning into a superconducting state.

In what follows it will be assumed that the Pool–boiling[5] heat transfer function, plotted in the Figure 1, will be valid during the time that the Helium is flowing on the surface of the conductor.

3. Calculation of λ using a modified First Criterion

According to the equation (4) if the temperature in the conductor does not depend on the position, the evolution of the temperature with respect to time is given by

$$(\delta c)\frac{d\theta}{dt} = \rho(\theta)J^2 - (\theta - \theta_o)Ph(\theta - \theta_o)/A \ , \tag{12}$$

where we have written the dependence of the resistivity and heat transfer with respect the temperature. Figure 2 shows the qualitative values of the joule heating term ρJ^2 with respect the temperature. From this figure we see that above the temperature θ_{max} ($\sigma_{max} = \theta_{max} - \theta_o$) the joule heating remains constant, but for higher values than this one it starts to grow very fast and the consideration about having a constant batch temperature θ_o (because of the Helium flow) may not be valid. For this reason we will restrict ourselves to consider the case where the temperature of the conductor is not higher than θ_{max} (it is about 20K for copper with a residual resistivity ratio, R R R, less than 300).

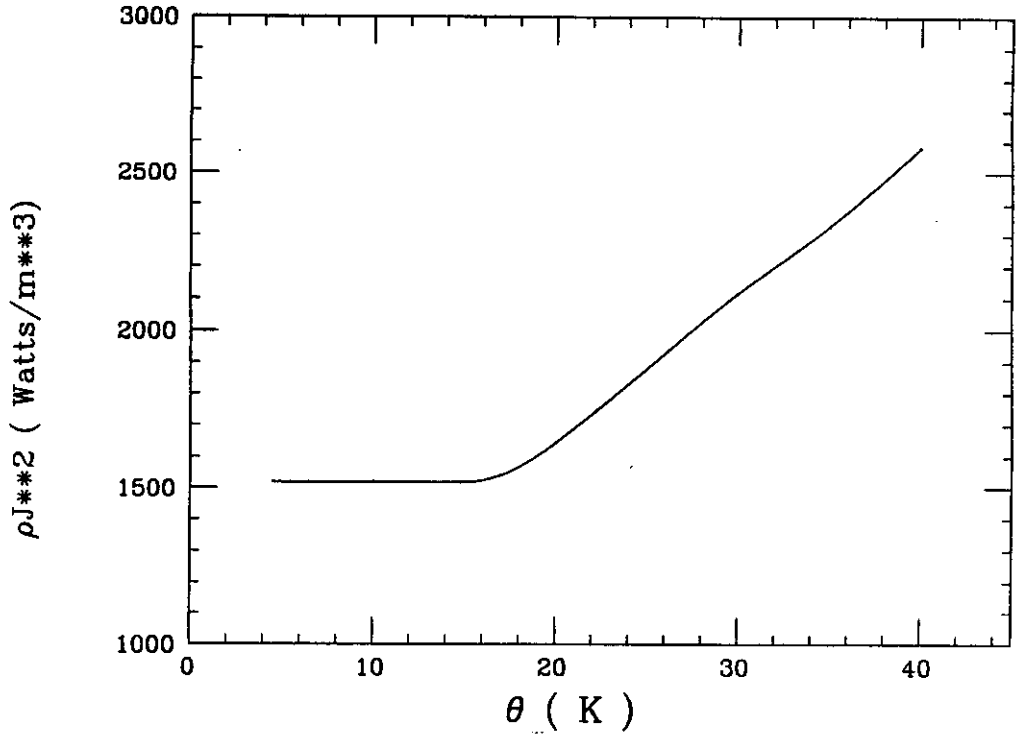

Figure 2. Temperature dependence of the joule heating.

Let θ be the temperature reached by the conductor after a large disturbance. This temperature is between θ_g and θ_{max}, and we want that this temperature decreases as the time evolves, that is, we want the relation (12) be negative for any temperature in that interval. This condition is expressed through the following relation

$$\rho J^2 - \sigma h(\sigma) P/A \leq 0 \,, \tag{13}$$

where σ is given by

$$\sigma = \theta - \theta_o \tag{14}$$

and the relation between batch temperature, θ_o, generating temperature, θ_g, and critical temperature, θ_c, is shown in Figure 3.

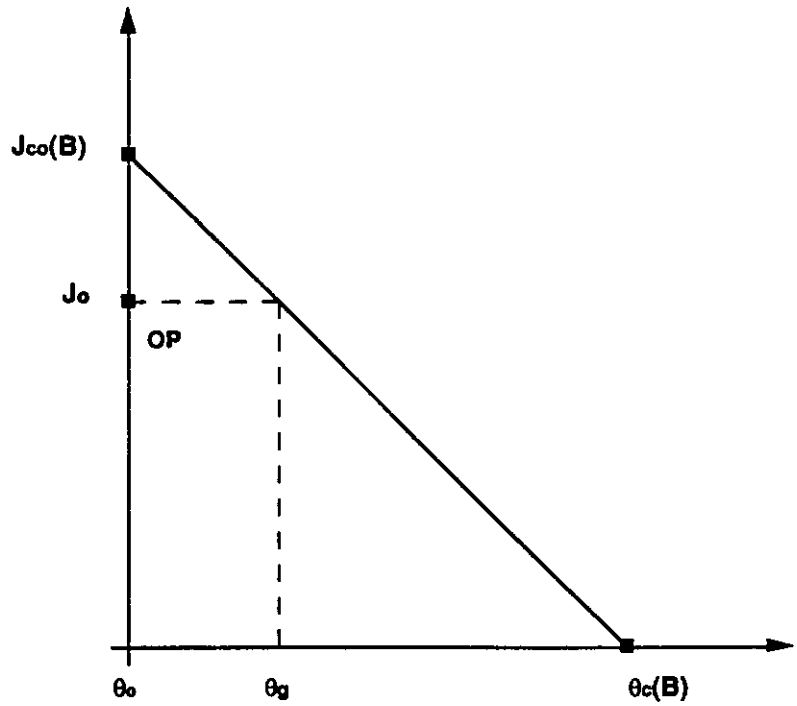

Figure 3. Critical current density as a function of temperature showing the temperature at which the ohmic heat generation starts, θ_g.

Assuming we have a cylinder conductor, its perimeter can be expressed in terms of its cross section area as

$$P = 2\xi\sqrt{\pi A}\ , \tag{15}$$

where ξ is the fraction of the perimeter in contact with the Helium. If the copper area is given in terms of the copper to s.c. ratio, λ, as

$$a_{cu} = \lambda a_{sc} \tag{16}$$

we can use these relations in (13) to obtain the following expression

$$\lambda^4 - q^2\lambda - q^2 \geq 0\ , \tag{17}$$

where q is defined by

$$q = \frac{\rho I^2}{2\xi a_{sc}^{3/2}\sqrt{\pi}\sigma h(\sigma)} \tag{18}$$

being I the current flowing in the conductor. The left hand side of the relation (17) represents a polynomial of fourth degree with respect to λ, it has two complex roots, one negative root and its positive root which has the physical meaning, is given by

$$\lambda_1(\sigma,\xi) = \frac{\sqrt{\mathrm{R}^{2/3} - 4\mathrm{q}^2/3}}{2\mathrm{R}^{1/6}}\left[1 + \sqrt{1 + \frac{2q^2\mathrm{R}^{1/2}}{(R^{2/3} - 4q^2/3)^{3/2}}}\right], \tag{19}$$

where R is defined as

$$R = \frac{q^4}{2}\left[1 + \sqrt{1 + \frac{256}{27q^2}}\right]. \tag{20}$$

The relation (17) tell us that the allowed values for λ are any higher or equal than λ_1 as it is shown graphically in the Figure 4. The behavior of λ_1 with respect to σ is shown in the Figure 5 for the inner cable of the SSC dipole magnet ($a_{sc} = 5.1383$ mm^2, $I = 6500$ A, $\rho = 2 \times 10^{-10}\Omega$m) and for several ξ values. As we can see from these plots , the naive application of the criterion (6) for $\xi = 0.25$ and $\sigma_c = 2.8$, give us the value of $\lambda_1 = 66$ which is unneeded too high. However, taken the more flexible condition (17), we are able to allow the conductor to have higher stationary temperatures reducing, consequently, the amount of copper we need to put into it. A safe selection is any value such that $\sigma \leq 12$K. For the values $\xi = 0.25$ and $\sigma = 9.2$K, we have $\lambda_1 = 15$.

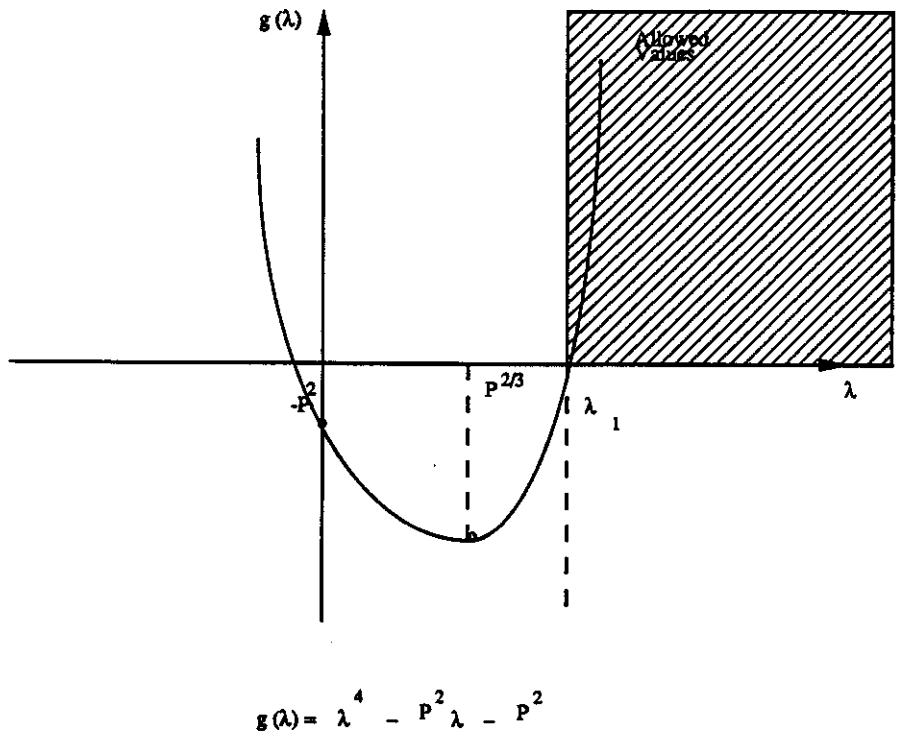

Figure 4. Allowed λ values.

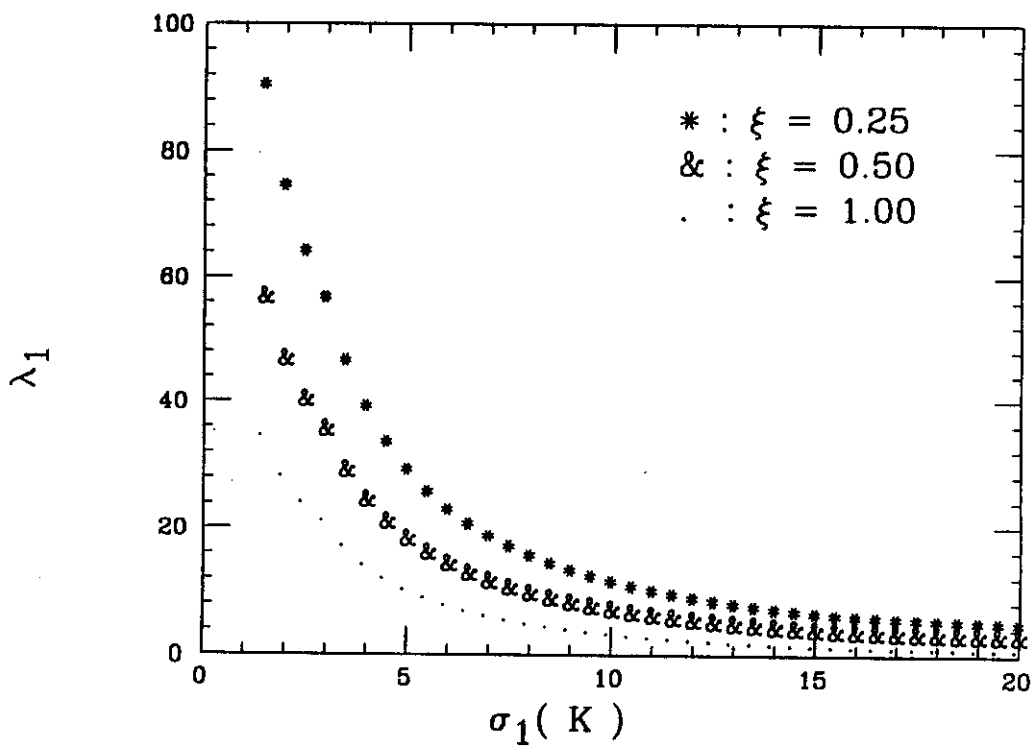

Figure 5. λ_1 values as a function of the fraction of perimeter in contact with the helium, ξ.

4. Calculation of λ using a Modified Second Criterion

Using the expressions (5), (15) and (16) in (11), the equal-area theorem is expressed by the following relation

$$\int_{\theta_g}^{\theta_{max}} \left[\frac{\rho I^2}{a_{sc}^2 \lambda^2} - \frac{2\sqrt{\pi}\xi\sigma h(\sigma)}{\sqrt{a_{sc}(1+\lambda)}} \right] d\theta = 0 \ , \tag{21}$$

but because the first term inside the integral is constant, this relation can be transformed in the following expression

$$\lambda^4 - q_*^2 \lambda - q_*^2 = 0 \ , \tag{22}$$

where q_* is defined by

$$q_* = \frac{\rho I^2}{2a_{sc}^{3/2}\sqrt{\pi}\xi \left[\frac{1}{\sigma_{max}-\sigma_g} \int_{\sigma_g}^{\sigma_{max}} \sigma h(\sigma) d\sigma \right]} \ , \tag{23}$$

σ_g and σ_{max} and are given by

$$\sigma_g = \theta_g - \theta_o \ , \tag{24a}$$

and

$$\sigma_{max} = \theta_{max} - \theta_o \ . \tag{24b}$$

The solution of (22) has the same form as (19) has, that is

$$\lambda_*(\xi) = \frac{\sqrt{R_*^{2/3} - 4q_*^2/3}}{2R_*^{1/6}} \left[1 + \sqrt{1 + \frac{2q_*^2 R_*^{1/2}}{(R_*^{3/2} - 4q_*^2/3)^{3/2}}} \right] \tag{25}$$

with the substitution of q by q_* wherever it appears in (20).

Let us apply this expression to the inner bus bar conductor for the SSC using the following values $\xi = 0.25$, $\sigma_g = 1.5$K and $\sigma_{max} = 13$K in the relations (23),(24) and (25), the copper to s.c. ratio obtained is $\lambda_* = 14$. Figure 6 shows the variation of λ_* with respect fraction of perimeter in contact with the Helium, ξ, for this particular example.

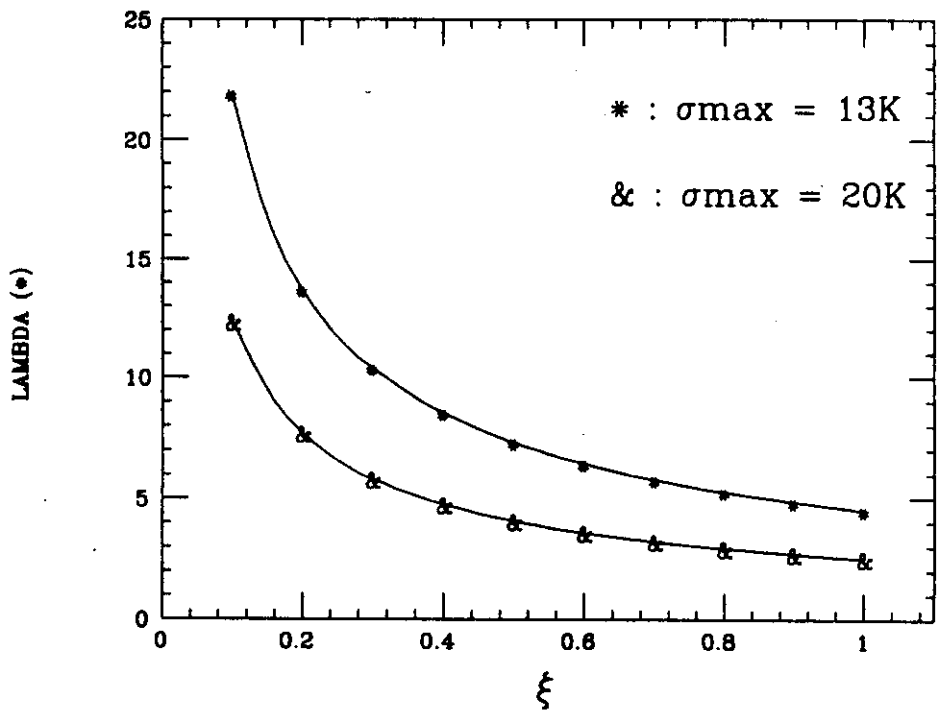

Figure 6. λ_* values as a function of the fraction of perimeter in contact with the helium, ξ.

5. Conclusions

Because of the reduced space available in the magnets, to use the cryogenic stability criteria to calculated the copper to s.c. ratio needed for the bus bar conductor in accelerator rings, is required the consideration of the fraction of perimeter in contact with the Helium. This restriction was considered using the first and second criteria, the assumption (15) and the slight modification (13) bringing about the expressions (19) and (25). As we can see from the Figures 5 and 6, the copper to s.c. ratio increase very rapidly as the fraction of the perimeter in contact with the Helium decrease, in fact, this rapid increase correspond to

the increase of the "q" value, relation (18) or (23) (it is not difficult to see that $\lambda_1 \simeq q^2$ when $q \gg 1$), thus, given the operating current and the fraction of perimeter in contact with the Helium, we may choose the copper with the highest residual resistivity ratio (RRR) as possible to reduce the λ_1 value. Although a cylindrical shape has been considered here, these results can be applied to any closed connected cross section shape with the same area and perimeter as a cylinder has (equivalent shape).

A poor heat transfer process is equivalent to a small fraction of perimeter in contact with the Helium, then, especial attention must be taken to the form of the heat transfer function as shown in the Figure 1. If the fraction of the perimeter in contact with the Helium is close to zero, the above considerations can not be applied, consequently, the bus bar conductor has to be protected against normal zone growing (quench), one way to do this is selecting the copper to s.c. ratio in such a way that, once a normal zone is detected and the power supply is turned off, the maximum temperature of the normal zone is not allowed to exceed more than a selected value[6] depending on the characteristic current–time–decay of the circuit.

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