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Transients of electric current in the safety leads.

Experimental results show that during a quench in a superconducting magnet the rate of change of the electic current in the safety leads is in the range of 35,000. to 65,000. amperes/second. Some published empirical values are as high as 220,000.amperes/second.

According to the Kelvin's Skin Effect Theory, a fast changing current in a conductor will have the tendency to flow near the surface of the conductor thus being restricted to a smaller part of the total sectional area and producing the effect of increasing the resistance (see for example ref1).

The results of the skin effect may lead to the fast increase in the skin temperature, causing degradation of the mechanical properties of the conductor. The Ampere forces in the conductor in addition to the thermal stresses and thermal degradation of the properties may have an effect of pealing the skin layer thus decreasing the cross section and burning the leads.

Following the Maxwell equations, the differential equation for the current density in a conductor is as follows

$$\nabla^2 i = \mu\sigma\frac{\partial i}{\partial t} \qquad (1)$$

where i is the current density,
σ is the conductivity,
and μ is the permeability

Equation (1) may be writen as follows

$$\frac{\partial^2 i}{\partial r^2} + \frac{1}{r}\frac{\partial i}{\partial r} = \mu\sigma\frac{\partial i}{\partial t} \qquad (2)$$

Separation of variables in the form

$$i(r,t) = R(r)T(t) \qquad (3)$$

will result

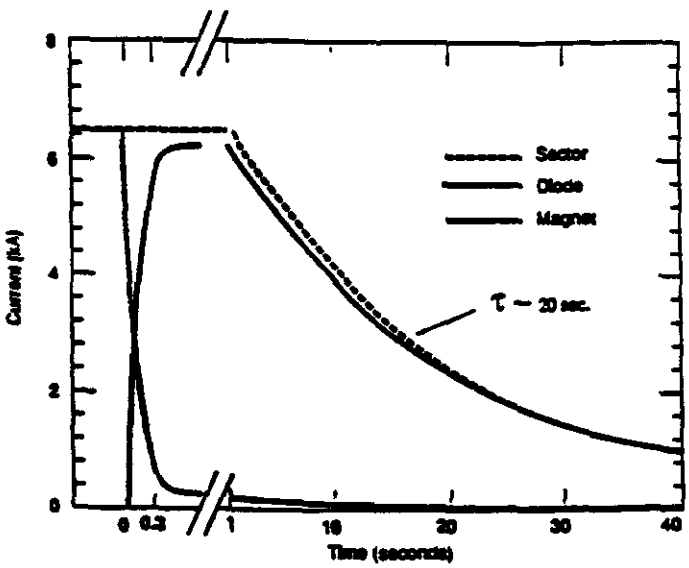
$$\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} + C\mu\sigma R(r) = 0 \qquad (4)$$

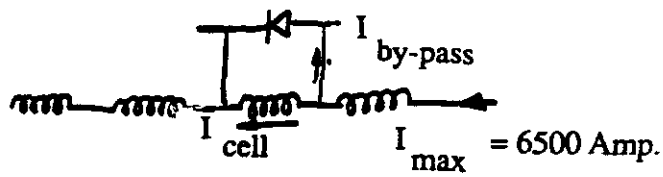
$$\frac{dT(t)}{dt} + CT(t) = 0 \qquad (5)$$

where C= constant

Initial and boundary conditions:

The main requirement from the power system during a quench in one magnet (or more correct - one cell), is to maintain the current in the rest of the magnets unchanged until the quench protection system is operating. This is obtained by switching the current from the quenched magnet to the by-pass (see fig 1). This process may last up to 0.2 seconds. The time constant of the current commutation is L/R where L is the magnet inductance and R is the quenched magnet resistance (for practical purposes this value may be assumed constant and may be derived from empirical data).





$$I_{by\text{-}pass}(t) + I_{cell}(t) = I_{max} = \text{const}$$

Figure 1 : Current commutation.

The total current in the power leads and in the by-pass barrs is

$$I(t) = I_{max}(1 - e^{-t/\tau}) \qquad (6)$$

For normal operation Imax = 6,500 ampere.

Integrating equation (5) we obtain

$$T(t) = A(1 - e^{-Ct}) \qquad (7)$$

Where A is constant

From (6) we get

$$C = 1/\tau = R/L \qquad (8)$$

Equation (4) is a Bessel equation of zero order. Two boundary conditions are necessary to solve this equation . These will be the symmetry condition:

$$\frac{dR(r)}{dr} = 0, \; r = 0 \qquad (9)$$

and the given current integral.

$$\iint_a i \, da = I(t) \qquad (10)$$

where a is the cross section area.

The solution of equation (4) is a Bessel function of zero order

$$R(r) = A_1 J_0(fr) + A_2 Y_0(fr) \qquad (11)$$

A finite value of (i) at r=0 results A2 = 0. This fulfills also the symmetry condition.

The final form of the equation is as follows:

$$i(r,t) = A_1(1 - e^{-t/\tau}) J_0(\sqrt{(\mu\sigma/\tau)}r) \qquad (12)$$

Integration of equation (12) over the cross section of the conductor and using equation (6) gives:

$$I_{max} = A_1 \int_{r_1} \int_a J_0(\sqrt{(\mu\sigma/\tau)}r)\, da$$

$$I_{max} = 2\pi A_1 \int_0 J_0(\sqrt{(\mu\sigma/\tau)}r) r\, dr = 2\pi A_1 r_1 J_1(\sqrt{(\mu\sigma/\tau)}r_1) \quad (13)$$

Where r1 is the radius of the conductor.

The final solution for the equation is:

$$i(r,t) = \frac{I_{max}}{2\pi r_1 J_1(\sqrt{(\mu\sigma/\tau)}r_1)}(1 - e^{-t/\tau}) J_0(\sqrt{(\mu\sigma/\tau)}r) \quad (14)$$

The local heating power due to the electric current is:

$$W = I^2/\sigma \quad (15)$$

For the first approximation σ may assumed constant.

To obtain a more accurate solution including the σ and R as functions of temperature, the former results should be introduced in the thermal equations and solve them iteratively. (A more accurate procedure is to solve equation (2) and the thermal equations simultaneously.)

References

1. Simon Ramo, J. R. Whinnery, T. Van Duzer :
Fields and Waves in Communication Electronics.
John Wiley , N.Y.