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Quasi-local Lumped Correction of IR Quad Nonlinearities

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At the time of the 1986 CDR, it was known that the linear aperture could be greatly restricted by nonlinear fields in the IR quads, [1] because of their relatively large strengths, and because of the very large values of β at the quads, which magnifies the nonlinear effects. Also, it is not possible to rely on statistical cancellation of random multipole components, even in non-allowed multipoles, because of the small number of IR quads. It was found that it would be necessary to correct b_2, b_3, b_4, b_5, a_2, a_3, a_4, a_5, and the existence of trim-coils within the quads for local cancellation of all these multipoles was postulated. Correction by at least an order of magnitude is desired.

Since the CDR it has been found that using small discrete correctors distributed properly between the dipoles in the arcs can be sufficient for cancellation of dipole-induced nonlinearities, [2] and such correction is currently planned for the SSC (and LHC). [3] An extension of that method for the IRs, using small discrete elements distributed between IR quad elements, is also possible and can replace the need for some or all of these internal trim coil elements. In this note, this approach is discussed and a sample calculation is presented.

For an initial example, we consider a sample SSC IR triplet; parameters are displayed in Table 1 and betatron functions are displayed in Figure 1. The IR center has $\beta^* = 0.5$ m and a magnet-free drift of 20 m leads into the triplet, where betatron functions vary up to maxima of ~8000 m in both transverse dimensions. The total quad length is 51 m, and the mean focusing strength is 215 T/m (for 20 TeV protons).

We consider a 12-pole systematic multipole error (b_5) of unit strength (10^{-4} of the focusing field at 1 cm). b_5 is chosen since it is a multipole which is allowed in quadrupole symmetry and therefore can have a large systematic value, it directly causes amplitude-dependent tune shifts, and it is the highest order term for which correction is needed (and therefore the most difficult to correct).

To set up a correction configuration, we split each of the F, D, F triplet units into two half-length magnets and place short corrector elements in the center gaps between those magnets, as well as at the ends of the F, D, F units (see Fig. 2). The correctors are powered to cancel the total nonlinear field quasi-locally in the quad units following Simpson's Rule $S_{5F,D} = -(1/6, 4/6, 1/6)\, B' b_5 L_{F,D}$.

There are four first-order amplitude-dependent tune-shift terms driven by b_5. There are also five momentum-dependent terms; however these all contain powers of the dispersion function which is near zero in the IR quads and are initially ignored. The amplitude-dependent tune shifts are:

$$\Delta\nu_x = \frac{1}{2\pi B\rho}\left[\frac{5}{4} I_x^2 \int B' b_5 \beta_x^3 \, ds\right.$$

$$\left.\frac{-15}{2} I_x I_y \int B' b_5 \beta_x^2 \beta_y \, ds \;\; \frac{+15}{4} I_y^2 \int B' b_5 \beta_x \beta_y^2 \, ds\right]$$

$$\Delta\nu_y = \frac{1}{2\pi B\rho}\left[\frac{-15}{4} I_x^2 \int B' b_5 \beta_x^2 \beta_y \, ds\right.$$

$$\left.\frac{15}{2} I_x I_y \int B' b_5 \beta_x \beta_y^2 \, ds \;\; \frac{-5}{4} I_y^2 \int B' b_5 \beta_y^3 \, ds\right]$$

where I_x and I_y are the horizontal and vertical actions, which are related to the amplitudes A_x, A_y, by $A_{x,y} = \sqrt{2\beta_{x,y} I_{x,y}}$, β_x, β_y are the betatron functions, $B'b_5$ is the multipole strength and the integrations are over the quads and correctors. These are dependent on the integrals over the quads of the four factors: β_x^3, $\beta_x^2\beta_y$, $\beta_x\beta_y^2$, β_y^3 multiplied by the nonlinearity strengths.

The correctors contribute to the tune shifts as sums of strengths weighted by the same betatron function factors; they cancel the IR quad contributions to the same accuracy as the integrals over quad units are accurately approximated by Simpson's Rule.

Table 2 shows comparisons of the F, D, F quad integrals and correction factors. Most terms are corrected by factors of 100. One integral (the integral of β_x^3 over the D quad) misses by 25%, however that D quad contribution is a very small part (< 5%)of the total corresponding tune shift term. The next worst term (the integral of β_y^3 over the D quad) misses by only 8%. The largest tune shift terms are due to coupled motion integrals ($\beta_x^2\beta_y$, $\beta_x\beta_y^2$); these largest terms are all reduced by two orders of magnitude. Overall, correction by ~two orders of magnitude is obtained, much better than necessary. Correction of b_5 was demonstrated; correction of lower multipoles should be better.

The momentum-dependent tune-shifts would also be corrected by similar factors: these terms would be initially at least an order of magnitude smaller because of the small dispersion. Because beam crossings are at an angle, the beams pass through the quads with off-center displacements. This displacement implies significant feed-down multipoles. Tune shifts from the feed-down multipoles will also be compensated by ~ two orders of magnitude.

Resonance driving terms are also represented by integrals of betatron factors times phase factors over the quads. They are also reduced by orders of magnitude by the present method . (Note, the phase factor parts of the integrals vary little over the IR triplet because of the large betatron functions.)

In this example, we have compensated the effects of a systematic multipole component. The method should be modified for compensation of "random" multipole content, particularly when the random variation is between quads in the same F or D focusing units. A "trapezoidal" rule should be used to obtain corrector strengths. Correction of the random content would not be

quite as good; however random content effects are already reduced somewhat by the random variation.

Since the correction is better than necessary, a sparser correction algorithm could also be considered. In particular, correction of the first F quad (nearest the IR) can be reduced, since the beam is not sufficiently magnified in that first quad to cause large tune shifts. Also, variation of correctors from integration rule values to cancel dominant nonlinear terms has not been considered. Non-local correction using elements outside the triplet has also not been included. These and other variations should be explored before final specification of a correction system.

In summary, we have demonstrated that lumped quasi-local compensation of IR nonlinearities is possible. A specific high-order example (correction of systematic b_5) has been evaluated and obtains correction by two orders of magnitude.

Table I - Elements of Triplet

Element	Length	$B'/B\rho$
Drift	20 m	0
F-Quad	15 m	0.0032
D-Quad	24 m	-0.003075
F-Quad	12 m	+0.003237

Table II - Tune Shift Terms in IR Triplet

Element, Term	$\Delta\nu_{x,y}$ ($b_5 = 10^{-4}$, $\varepsilon_y = \varepsilon_x = 10^{-8}$)	$\Delta\nu$ (corrected)
F_1, β_x^3	0.000019	-0.0000001
$F_1, \beta_x^2\beta_y$	0.0017	0.000013
$F_1, \beta_x\beta_y^2$	0.00216	-0.000043
F_1, β_y^3	0.00011	-0.000014
D, β_x^3	0.000248	-0.000059
$D, \beta_x^2\beta_y$	0.0208	-0.00026
$D, \beta_x\beta_y^2$	0.0572	0.00086
D, β_y^3	0.00505	-0.00042
F_2, β_x^3	0.00220	0.0000001
$F_2, \beta_x\beta_y$	0.0496	-0.000043
$F_2, \beta_x\beta_y^2$	0.0334	0.000038
F_2, β_y^3	0.000676	-0.0000094

ε_y and ε_x are the emittance in M-R and re related to the amplitudes and actions by $A_{x,y} \geq \sqrt{\beta_{x,y}\varepsilon_{x,y}}$ and $\varepsilon_{x,y} = 2\, I_{x,y}$, respectively. 10^{-8} m-R corresponds to A = 2 mm at β = 400 m, a typical maximum value in the arcs. In the uncorrected $\Delta\nu$ calculations, signs have been ignored. A negative sign in the corrected $\Delta\nu$ numbers indicates the particular term is overcorrected by the 3-point correctors.

References

1) SSC Conceptual Design, SSC-SR-2020 March 1986 (p.159).

2) D. Neuffer - Nucl. Inst. and Meth. A274, 400 (1989).
D. Neuffer and E. Forest, Physics Letters A135, 197 (989).

3) G. Brianti, International Conference on High Energy Accelerators, Tsukuba, Japan, August 1989.

Figures

1) Lattice functions β_x, β_y in the IR triplet.

2) Triplet layout with corrector locations (marked by C's).

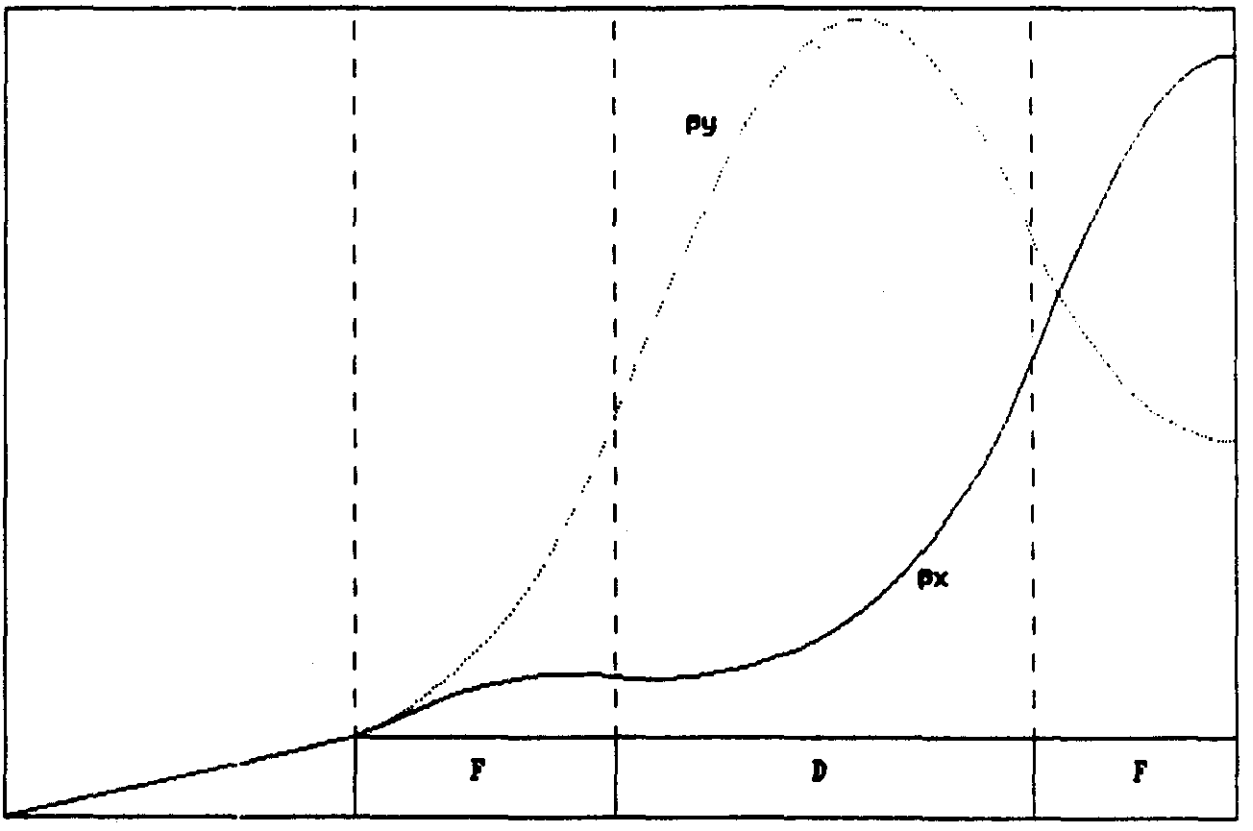

Figure 1

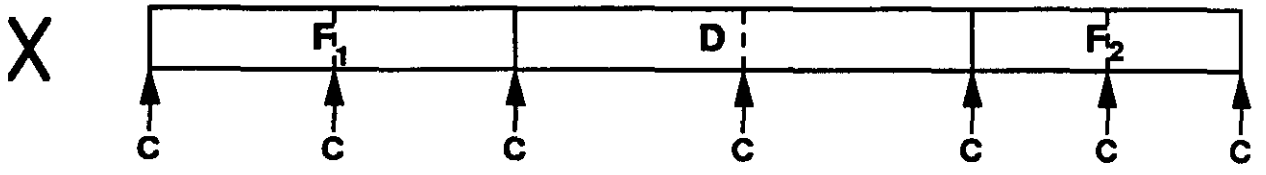

Figure 2