SSC-N-702
June, 1990

Summary Report of the Mini-workshop on Beam-beam Simulations

Introduction

In view of the recent attention given to the subject of beam-beam simulation, an informal mini-workshop was held at the SSCL during 5/21–23, 1990. The purpose of this mini-workshop was to (1) compare the techniques and results of the various beam-beam simulation efforts and (2) evaluate what to do next. There were three groups in attendance from SSCL, the University of Texas at Austin, and Cornell University. No attempt was made to include all the important beam-beam simulation efforts elsewhere. Highlights of the discussions are summarized in this report.

Participants

SSC group :

Alex Chao (chairman)
Miguel Furman
Narayan K. Mahale

Cornell group :

Robert Siemann
Srinivas Krishnagopal

UT Austin group :

Toshiki Tajima
Jim Koga (secretary)

agenda 5/21-5/23/90

May 21, 1990

- Furman
- Mahale

May 22, 1990

- Siemann
- Koga
- Tajima

May 23, 1990

- Krishnagopal
- Mahale/Furman
- Tajima

Topics

- SSC effort
  - Coherent Dipole Beam-beam Simulations
- UT Austin effort
  - Beam-beam Interaction Effects on Particle Dynamics
- Cornell effort
  - DCI Simulations
  - Phase Averaging in the Beam-beam Interaction

SSC effort

Coherent Dipole Beam-beam simulations have been performed to determine the stability boundary of the beam-beam strength parameter ξ versus the total tune ν of a collider ring. Details of the simulation technique can be found in report SSC-62 [1]. The simulations can take into account both head-on and long range collisions. In the current study only head-on collisions are considered. This is a tool checking stage. Some further concerns for the SSC are long range collisions, different number of particles in each bunch, $x-y$ coupling, random changes in the tunes, and the effects of gaps in the beam (i.e. abort, systematic, and random). Once this tool is checked, other effects will be included for SSC runs.

The configuration consists of 4 equally distributed IR's (interaction regions) with 2 bunches per beam. The total tune, ν, is scanned to determine the $\xi-\nu$ stability boundary where $\nu=\nu_{12}+\nu_{23}+\nu_{34}+\nu_{41}$ and ν_{12} refers to the arc tune between interaction regions 1 and 2, etc... Two arc tunes are fixed: ν_{23} and ν_{41}. ν_{12} and ν_{34} are set by $\nu_{12}=\nu_{34}=(\nu-\nu_{12}-\nu_{41})/2$.

For unequal phase advances, $\nu_{23}=2.75$ and $\nu_{41}=2.25$, a novel boat sail shape is apparent (figure 1). Each point in the figure represents a single tracking simulation. Particles in the unstable regions are lost within 100 rotations. For values of the tune between $\nu\approx 8.55-8.75$ a stable region is bounded in ξ between two unstable regions. This result is independent of the particle initialization in the simulation and whether $p-p$ or $p-\bar{p}$ collisions are used. To check the tracking results eigenvalues of the transfer matrix were calculated and the largest eigenvalues of the matrix were determined. Figure 2 shows the results of scanning the tune-shift parameter ξ for value of the total tune of 8.65. The results show a stable region between $\xi=0.050$ and 0.080 bounded by two unstable regions. This is confirmed within numerical accuracy by the results of the simulations at the same value of the tune.

When $\nu_{23}=2.25$ and $\nu_{41}=2.0$, an island region is observed between $\nu\approx 6.6-7.0$ (figure 3). Again a stable region is bounded in ξ between two unstable regions. When $\nu_{23}=2.5$ and $\nu_{41}=2.0$, standard triangular stability curves are obtained. No stable regions are bounded in ξ by two unstable regions. When all four tunes are equal, the half integer resonance disappears and a single triangular stability curve is obtained.

These results differ from matrix calculations of Keil [2]. Keil examined the maximum eigenvalue for a single value of the beam strength parameter $\xi=0.03$ and scanned in the total tune. Keil claims that at $\nu=0.5$ the stability boundary is vertical for the particular case shown in figure 12 from reference [1]. This apparent discrepancy will need to be resolved.

UT Austin effort

The objective of this study is to determine beam-beam interaction effects on particle dynamics using a collective plasma model at the interaction point. A 1 dimensional model is employed at the interaction point so only transverse oscillations due to the counterstreaming beams are studied. The rest of the machine is treated by simple harmonic transport (betatron oscillations). By employing a fully self-consistent model at the interaction point it is hoped that an assessment of the relative importance of collsions as a whole and individual "soft" collisions (collective effects) can be determined. One of the fastest growing collective effects which can occur in a plasma is the filamentation instability. There are two factors which determine its effect on collective motion of counterstreaming beams. They are the timescale of the interaction and the transverse size of the beam. The timescale of the interaction τ_{int} is determined by the length of the beam bunches L_b where $\tau_{int} = L_b/2c$. The maximum growth rate of the filamentation instability for large beams is $\Gamma_{max} = \omega_b/2$ for $p - \bar{p}$ collisions and $\sqrt{2}\omega_b$ for $p - p$ collisions where $\omega_b = \sqrt{4\pi e^2 n_b/\gamma m}$ is the beam plasma frequency, n_b is the beam density, γ is the relativistic factor, and m is the proton mass [3]. The factor $\Gamma_{max}\tau_{int}$ determines the fraction of the growth rate time the beams interact. This number for the SSC is 0.05. So the beams interact for a small fraction of the growth time of the instability. Therefore, the only way the filamentation instability may be of some importance to the beam dynamics is by repeated interactions over many turns. The transverse size of the beam is another factor limiting the effects of the filamentation instability. The typical scale of the filamentation instability is the collisionless skin depth $\lambda_c = c/\omega_b$. For the SSC the ratio of the beam width w_b to λ_c is $\approx 2 \times 10^{-5}$. It has been found that the filamentation instability is suppressed when this ratio is small [4]. The simulations have shown that this does occur. Analytic calculations are being carried out in the current 1 dimensional collider geometry.

Long runs of 1000 turns have been performed. In order to keep the time between rotations reasonable ($\approx$ 1000 time steps) a beam width larger than the SSC was used. This is due to constraints of following light waves in the electromagnetic code. The ratio w_b/λ_c is still small at $\approx 10^{-3}$. Measurements of the tuneshift for both beams for small amplitude particles are shown in figure 4. The tune-shift oscillates about an average of 1.87×10^{-3}

and 1.93×10^{-3} for beams 1 and 2 respectively. The predicted tune-shift for a one dimensional Gaussian beam using SSC parameters is 2.1×10^{-3}. Power spectra of these oscillations show both high and low frequency components (figure 5). The large peaks correspond to oscillations which occur every ≈ 6 rotations. Larger peaks which are not plotted occur at the highest frequencies and can be attributed to noise in the simulations. Poincare sections in $x - x'$ space have been generated for particles in various initial positions in phase space. A preliminary investigation has shown what seem to be resonance islands of 38th order.

Some comparison of short runs with various different dynamics has been done. These simulations incorporated vacuum instead of periodic boundary conditions obtaining closer agreement with theory. The aspects of the 1 dimensional fully self-consistent model were eliminated in stages (figure 6 for beam 1). The final stage involved non-moving particles for one beam and a number of non-interacting particles for the other beam (labeled minimum code in the figure). This case is similar to the "weak-strong" approximation used in tracking codes. The main difference between the self-consistent model and the "weak-strong" model at least for the small number rotations is the oscillations in the tune-shift. A comparison will be done for a large number of rotations where collective effects may begin to play some role.

The preliminary conclusions are that (1) collective effect accumulation is weak at least over short runs, (2) the filamentation instability is suppressed by the finite extent of the beam, (3) average values of the tune-shift nearly agree with theory after substantial calibration efforts, and (4) some oscillations are seen in the tune-shift and emittance.

Some of the shortcomings of this fully self-consistent method have become clear:

- too costly/ small number of particles
- a large number of time steps are neccesary for one rotation. For realistic SSC parameters the simulation time step size Δt is about $3 \times 10^{-5} \tau_{int}$.
- only 1 dimensional collective effects are included
- need to look at $x - y$ coupling (betatron resonance) and $x - y - z$ coupling (betatron-synchrotron resonance)
- need increased resolution and much longer runs.

Most of these items can be eliminated by incorporating a new magnetoinductive model where the displacement current is neglected (Darwin approximation)[5]. As a result Δt becomes comparable to τ_{int} and a factor of $10^4 - 10^5$ increase in speed is possible. This will allow:

- extension to $x - y$ and $x - y - z$ dimensions.
- investigation of betatron resonance and a 4D map comparison
- investigation of synchrontron-betatron resonance

Another method for increased smoothing, accuracy, and reduced cost is the δf algorithm technique. In this method deviations from a steady Gaussian δf are used in a smoothing process [5]. This method could be applied to other simulations such as the Cornell "strong-strong" beam simulations [6]. Future developments of the beam-beam interaction research include:

- comparison with "weak-strong" simulation
- calibration with the theory and present results for the magnetoinductive model
- reduction to one time step for particle push for a given set of particles in the other bunch and a few time steps to include the bunch length
- calibration with other existing machines
- applications to the SSC, or other machines
- diffusion model development (D vs. radius, etc...)

Cornell effort

DCI simulations

The goal of these simulations was to determine reasons for DCI results which show saturation in the tune-shift versus current in a round beam configuration (figure 7a). Round Beam B-factories with $Q \geq .75, .25$ achieve $\xi > 0.1$ in simulations that included transverse motion. Based on this one would have expected similar performance from DCI which had a close resemblence to such round beam colliders. The limit for DCI was measured to be $\xi_{limit} \approx 0.018$. Simulations have been performed for two machines with parameters close to those of DCI. ONE is a one interaction region machine with half the DCI circumference. It was primarily used to study resonance structure and radiation damping effects. TWO is a two interaction region machine with the same circumference as DCI. It was used to study the effects of betatron phase advance errors. The simulations did not include dispersion or longitudinal motion. These approximations are justified by :

- the dispersion contributions to beam sizes are small when running on the coupling resonance.
- $\beta^* \gg \sigma_L$
- variation of Q_s is not important [7].

The simulations treat the radiation and coupling as uniformly distributed along the arc (smooth approximation). A result is that there is coupling only when the integer parts of the vertical and horizontal tunes are equal. This is not the case for real machines. Transport between the IP's was done with a 4×4 linear matrix. Simulations of TWO allowed different phase advances between IP's.

There are four possibilities which explain the low ξ_{limit} observed in DCI :

- operating point
- radiation energy loss $\Delta E/E$ between crossings
- ϵ making
- optical errors

Radiation damping $\delta = \Delta E/E$ was examined using ONE with $Q_v = .8625$ and $Q_h = 1.8625$ just below $\frac{7}{8}$ resonance. Only even order resonances are important. The resonance strengths should decrease as the resonance order increases. With fixed current and radiation damping 6th, 8th, and 10th order resonances were observed in decreasing strength. The tunes were scanned over the 8th order resonance for different values of radiation damping δ. There is a small dependence on δ for $\delta < 2 \times 10^{-5}$ and an abrupt change in behavior at $\delta = 5 \times 10^{-5}$. Away from resonance the behavior is the same, but there are differences near resonaces. Radiation damping has some effect, but ξ_{limit} is maintained well above $\xi = 0.018$ where saturation was observed in experiments. Radiation damping alone is not sufficient to explain results.

ϵ making is given by properties of the synchrotron radiation. In general $\epsilon_h \gg \epsilon_v$ so emittance in the vertical direction was simulated by 1. random and 2. coupling methods. This effect was not important.

Optical errors were examined using the TWO machine parameters. A horizontal phase advance error, ΔQ_h was put into one of the arcs. The other arc had the opposite sign error to maintain the tune Q at the nominal value. One expects beam-beam resonances for $2pQ = m$ where p is the order of the resonance. If m is even, then the reduced Hamiltonian, H_{red} is independent of ΔQ. If m is odd, $H_{red} \propto p\Delta Q$. $Q_h = 3.725$ is near $3\frac{3}{4}$. The 4th order resonance is not excited if $\Delta Q = 0$. Q_h is also near $2\frac{14}{8}$. The 8th order resonance is excited independent of ΔQ. All other factors in limiting ξ influence the strength of the 8th except optical errors. The simulations were done in two dimensions so Q_h, ΔQ_h, Q_v, and ΔQ_v were set. ΔQ_h was varied and $\Delta Q_v = 0$. When ΔQ_h is introduced, this substantially affects ξ level. When $\Delta Q_h \approx 0.004 - 0.008$, results start agreeing with experiment (figure 7b).

There were no direct measurements of ΔQ in DCI. Also the errors in the quadrupole magnets were never measured. The errors were measured for similar quadrupoles used in a machine called super ACO. The errors measured were $\sigma_{Q_v} = 0.002$ and $\sigma_{Q_h} = 0.0005$ with $< \sigma_{Q_v}\sigma_{Q_h} >= -0.7$. A "2 sigma machine" was simulated with $\Delta Q_v = 0.004$ and $\Delta Q_h = -0.001$. The measured tune-shift limit ξ_{limit} was 0.0195 which is close to the tune-shift limit observed in DCI.

These simulations have shown that reasonable phase advance errors can explain the DCI performance.

A puzzle from DCI performance which has not been resolved is a measurement of the limiting tune-shift versus tune where $Q_h = Q_v$. The experimental and simulation results seem to agree except for when the tune is slightly above 0.8 where the simulation indicates a higher ξ_{limit} than that observed. It needs to be resolved whether the tunes in the data were measured tunes or from the quadrupole magnet settings. If the latter case is true, then the extra point can be attributed to a shift of all points in the tune to higher values.

Phase Averaging in the Beam-beam interaction

In the usual approximations, the longitudinal extent of the beam-beam interaction is ignored and is treated as a delta-function. The consequence of ignoring the Gaussian longitudinal distribution is calculated in a Hamiltonian model [8]. It manifests itself as a Gaussian form-factor in the expression for resonance strengths. Physically this arises from the fact that the driving force is applied not at a single betatron phase, but is spread over a wide range of (the test particles's) betatron phase. Hence, this phenomenon is referred to as phase-averaging.

The Hamiltonian analysis can be extended to derive energy-transparency conditions for asymmetric colliders [9]. If one requires that both beams see the same set of resonances and strengths, then the constraints are stringent. In particular, one energy-transparency condition requires to have proportionally more particles in the low-energy beam.

Simulations that incorporate phase-averaging via a thick-lens treatment agree well with the theoretical model. Simulations that in addition allow for a Gaussian distribution of test particles predict an optimal bunch length, $\sigma_l \approx \beta^*$, for round beam in a single-ring e^+e^- collider.

The latter simulations were also applied to compare flat and round profiles. For flat beams the tune-shift parameter decreases monotonically with increasing σ_l/β^*. The maximum tune-shifts achieved in paramter scans are ≈ 0.05. For round beams the tune-shift limit is 2-3 times larger.

The emphasis in all the work so far is on the nonlinear dynamics within the framework of single isolated resonances.

References

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[2] E. Keil, private comm.

[3] H. S. Uhm and C. S. Liu, Phys. Rev. Lett., **43**, 914, 1979.

[4] T. Tajima, Phys. Flds., **22**, 1157, 1979.

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[7] R. Siemann, private comm. with DCI group.

[8] S. Krishnagopal and R. Siemann, Phys. Rev. D,**41**, 2312, 1990.

[9] S. Krishnagopal and R. Siemann, Phys. Rev. D,**41**, 1741, 1990.

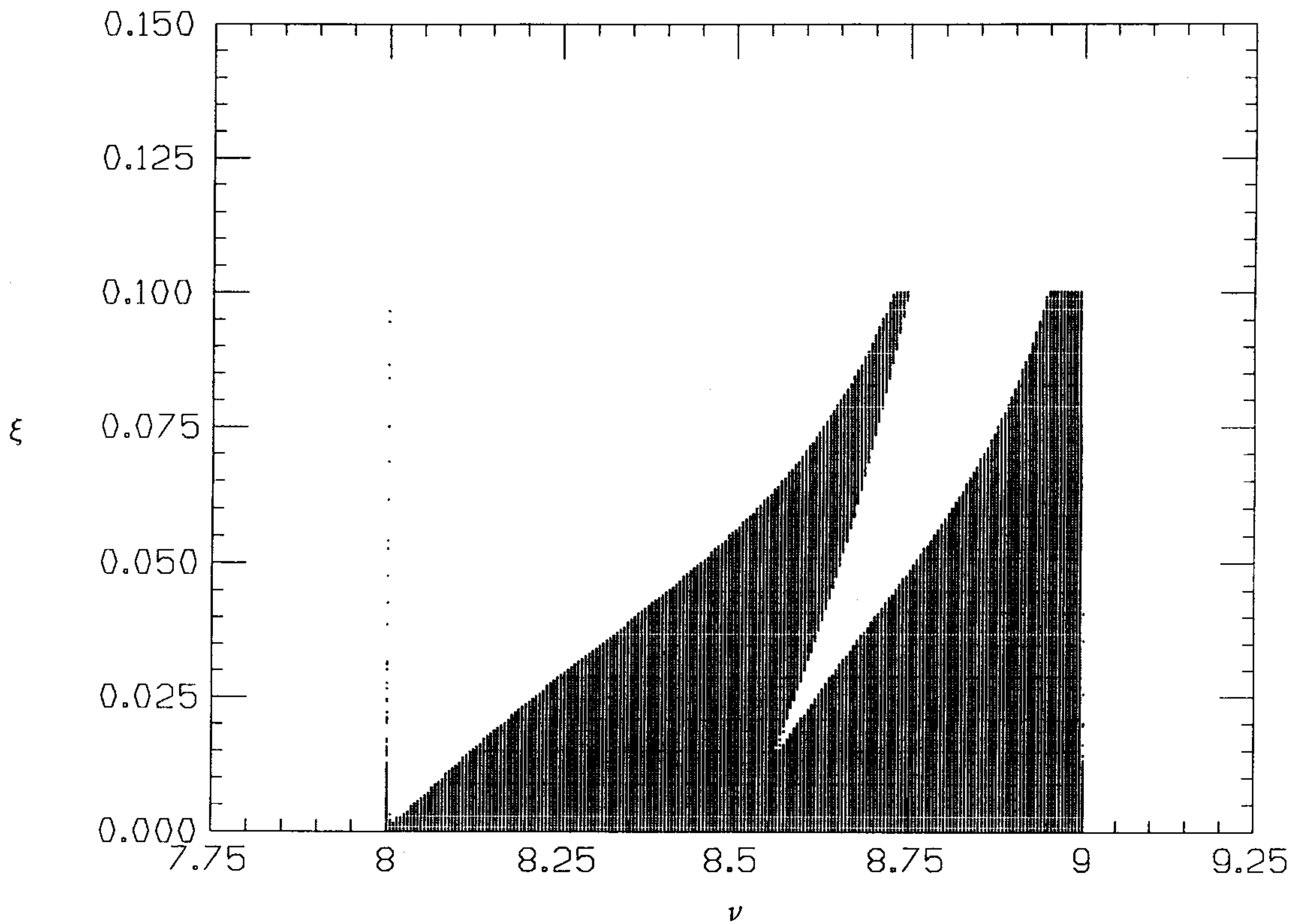
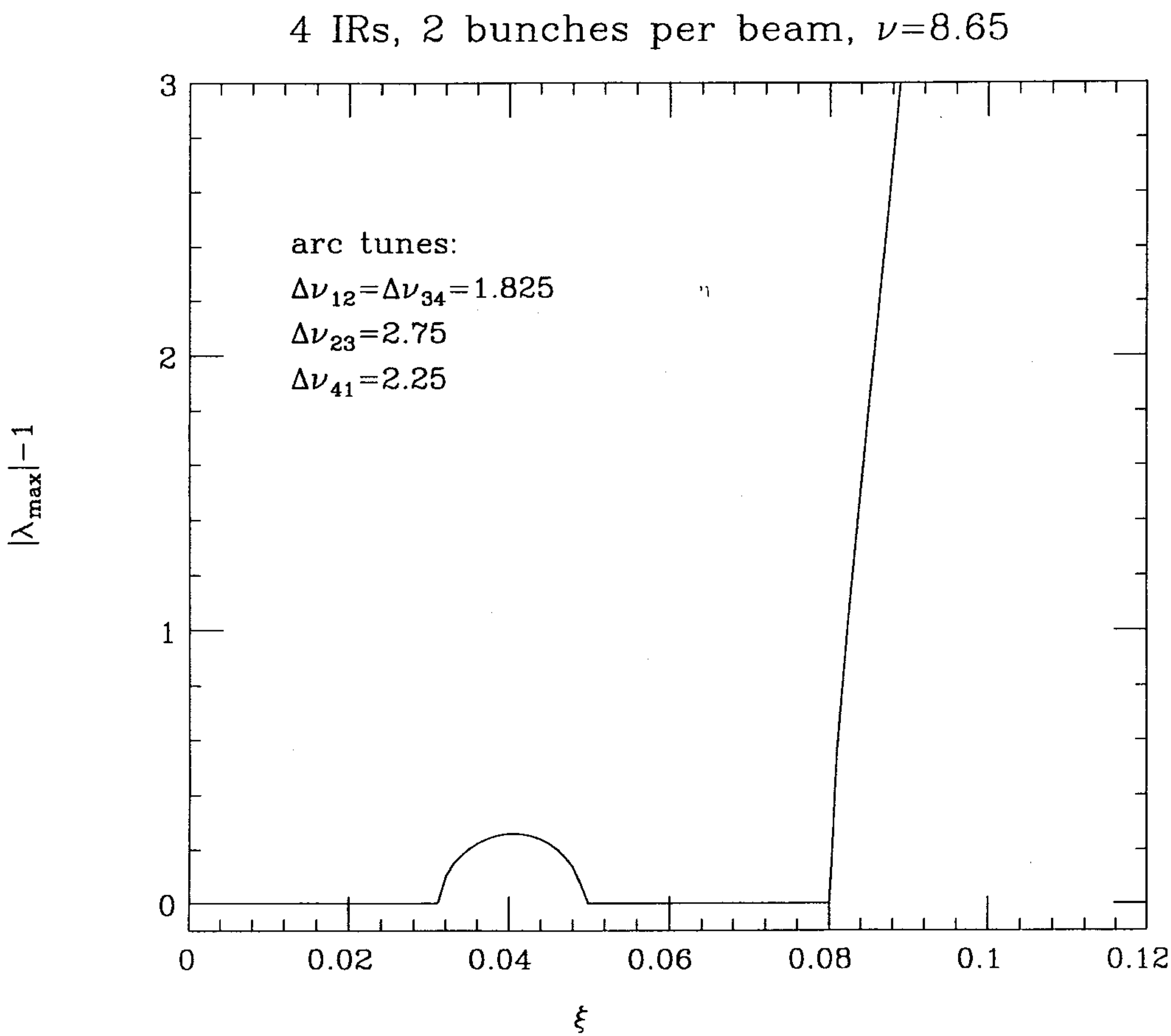


Figure 1



4 IRs, 2 bunches per beam, ν=8.65
arc tunes:
Δν12=Δν34=1.825
Δν23=2.75
Δν41=2.25
3
2
1
0
0
0.02
0.04
0.06
0.08
0.1
0.12
|λmax|−1
ξ

Figure 2

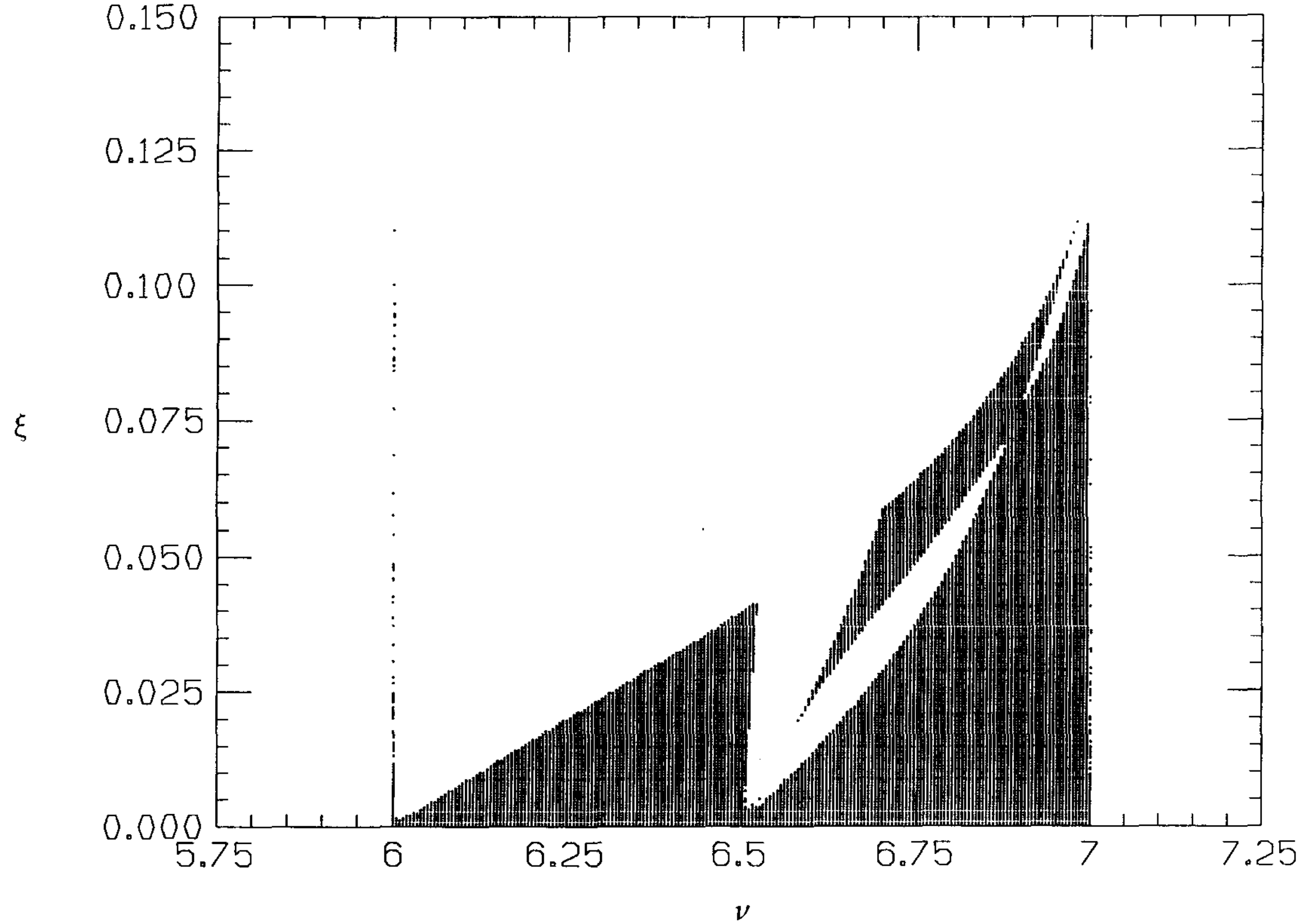
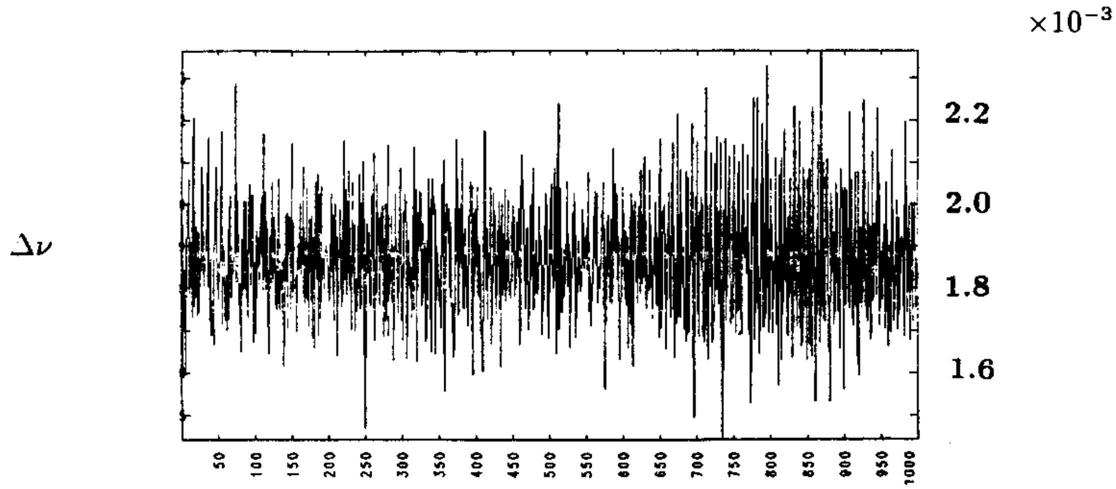


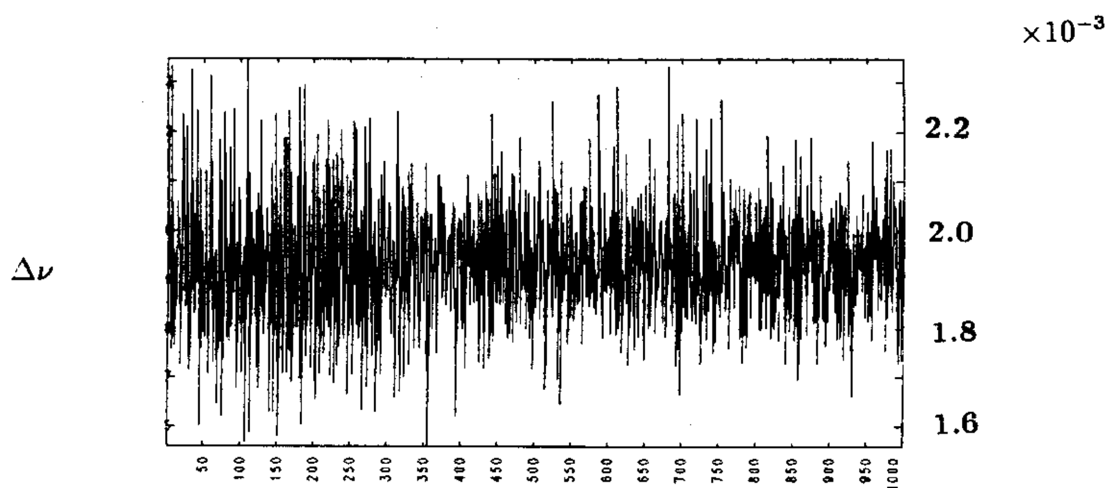
Figure 3

Beam 1



Rotations

Beam 2



Rotations

Figure 4

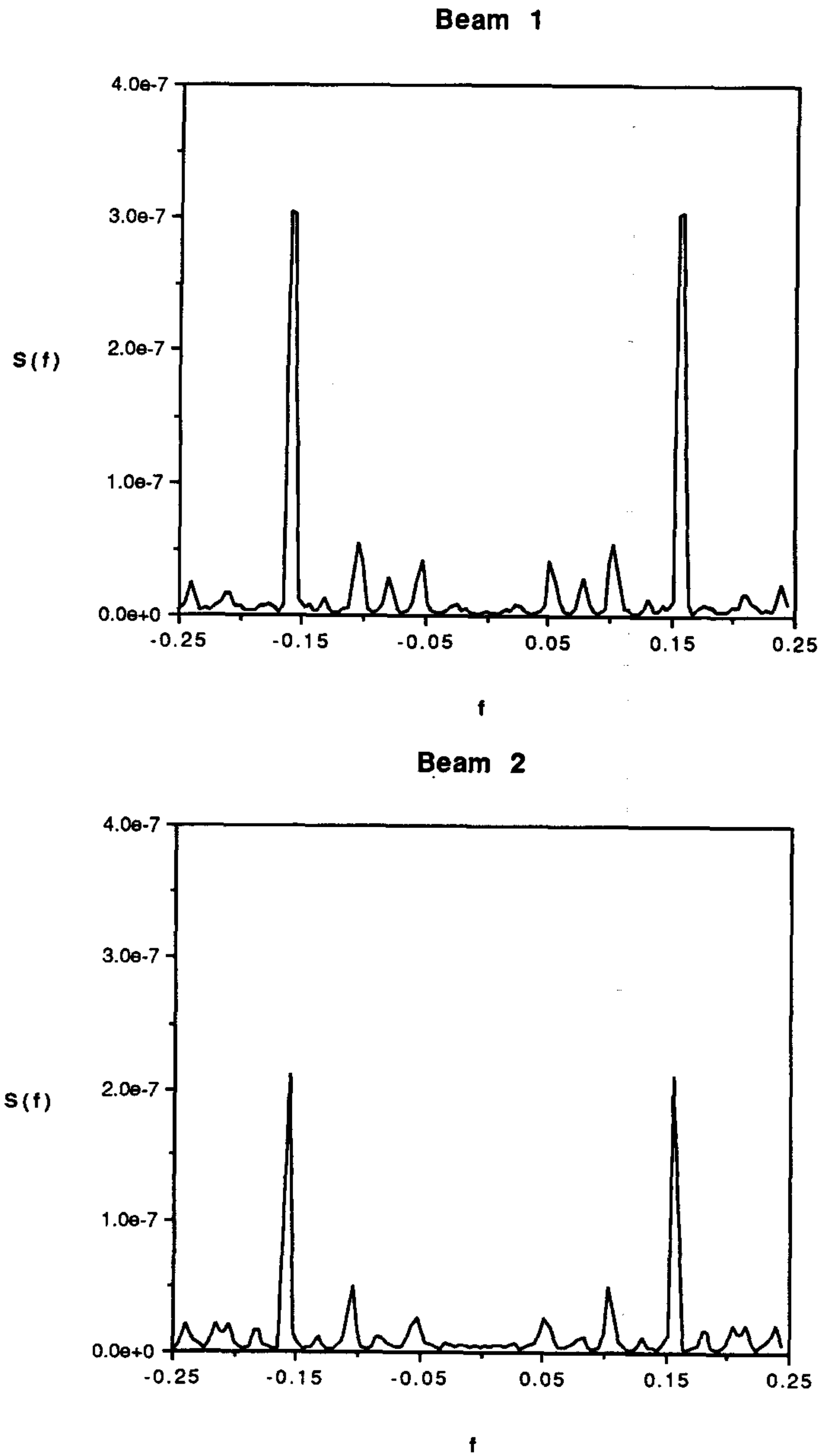
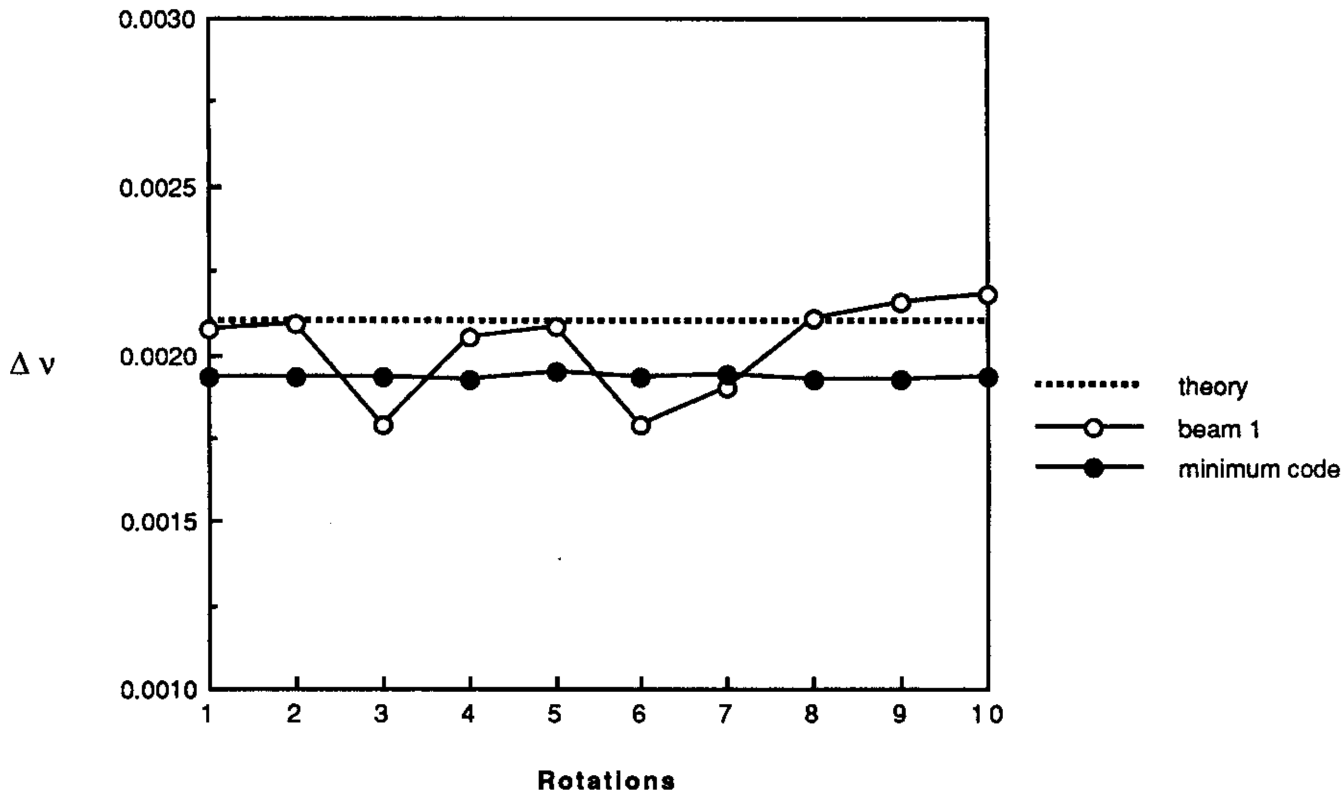

Beam 1
S(f)
4.0e-7
3.0e-7
2.0e-7
1.0e-7
0.0e+0
-0.25
-0.15
-0.05
0.05
0.15
0.25
f
Beam 2
S(f)
4.0e-7
3.0e-7
2.0e-7
1.0e-7
0.0e+0
-0.25
-0.15
-0.05
0.05
0.15
0.25
f

Figure 5



0.0030
0.0025
0.0020
0.0015
0.0010
Δ ν
1
2
3
4
5
6
7
8
9
10
Rotations
theory
beam 1
minimum code

Figure 6

Figure 7a

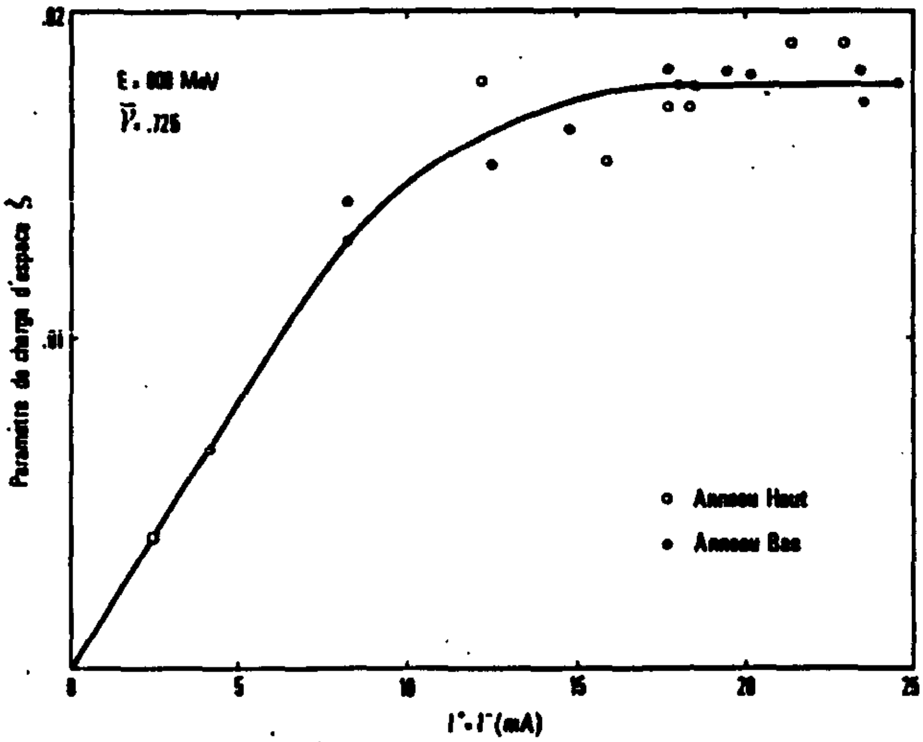


Figure 7b