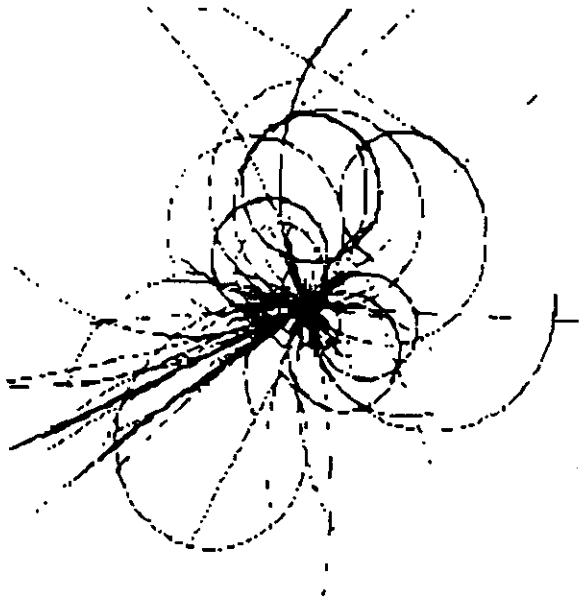
THE SUPERCONDUCTING SUPER COLLIDER LABORATORY



SSCL-N-698

NOTES

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SSCL-N-698

THEORY OF SURVEY, ALIGNMENT AND ORBITS.

This is an error theory. I will reserve the term "error" for random, independent mistakes in construction, adjustment, and survey. Errors are reasonably gaussian with an r.m.s $\langle \delta \rangle$.

Quad displacement and calculated adjustment, orbit distortion and correction dipole strengths, which result from errors, are also random and gaussian; but, from point to point around the ring, they are <u>not</u> independent. We will be particularly interested in the largest distortion and largest correction around the ring. The expectation for these values will be expressed as a many-ring distribution, found numerically by choosing many random sets of errors.

The correlation around a ring is examined by considering the harmonic structure. The expectation for harmonic amplitudes of an independent variable is the same for all harmonics.

I assume a simple FODO structure with φ_0 phase/cell. I use 60°, 72° and 90° to show the phase dependence. I use the notation where a betatron oscillation is reduced to a simple sine wave: $y = (\beta_0/\beta)^{1/2} \times$ actual disp., $y' = dy/d\varphi = (\beta_0\beta)^{1/2} \times$ actual angle, β_0 is repetitive β_{max} in the FODO cell.

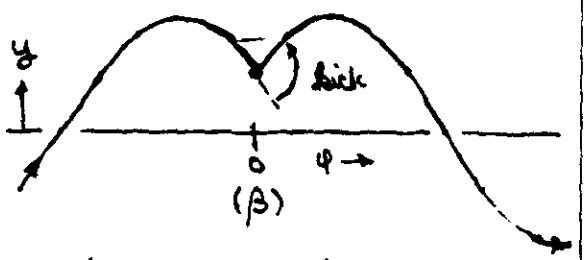
Typically we have 800 quads in 400 cells (N), with quad spacing of 120 M. (400 feet) and $\nu \approx 80$, $\beta_0 \approx 400$ M.

SINGLE QUAD DISPLACED u.

Let $b = p(\beta\beta_0)^{1/2}$

Then the kick is ub

and the orbit $y = \frac{ub}{2\sin\pi\nu}\cos(\varphi - \pi\nu)$.



(φ can be read forward or backward but not across $\varphi = 0$).

For focussing quads $b_0 = 2(1 + \sin\varphi_0/2)/\cos\varphi_0/2$, def. $b_1 = -2$.

Let $y = a_0/2 + \sum_k a_k \cos(ik\varphi/\nu)$, then $a_k = \frac{ub}{\pi}\frac{\nu}{\nu^2 - k^2}$.

FOR 2N QUADS (N SIMPLE CELLS)

Let u be random, and independent, with rms. $\langle u \rangle$. The orbit for a particular set of errors is the linear sum of the effects of individual u's. The many ring expectation is

$$\langle y \rangle = \frac{\langle b \rangle}{2\sin\pi\nu}\sqrt{\frac{N}{2}}\,\langle u \rangle. \qquad \langle b \rangle = (b_0^2 + b_1^2)^{1/2}$$

and $$\langle a_k \rangle = \langle b_k \rangle = \frac{\langle b \rangle}{\pi}\sqrt{\frac{N}{2}}\,\frac{\nu}{\nu^2 - k^2}\langle u \rangle, \qquad \text{or}$$

$$\langle a_k \rangle = \langle b_k \rangle = \frac{2\sin\pi\nu}{\pi}\,\frac{\nu}{\nu^2 - k^2}\langle y \rangle.$$

The orbit is dominated by a harmonic k_0, the nearest integer to ν, and has k_0, rough, oscillations. At one point the harmonic k_1 (next nearest) adds and gives a "peak-peak". The expectation for many rings is: (numerical exp.)

50 % have peak-peak $< 2\langle y \rangle$

90 % $< 2\sqrt{2}\,\langle y \rangle$.

RESPONSE TO HARMONIC DISPLACEMENTS

The displacements from surveys are random but never independent. We use harmonics, thus let $U = u_J \cos iJ\pi/N + v_J \sin iJ\pi/N$. Let me express $b_i = (f + g\cos i\pi)\cos i\pi$, $i=0$ at b_0, to include the alternation from b_0 to -2 to b_0 etc., then one finds

$$U_i b_i = g(u_J \cos iJ\pi/N + v_J \sin iJ\pi/N)$$
$$+ f(u_{N-J} \cos i(N-J)\pi/N - v_{(N-J)} \sin i(N-J)\pi/N).$$

We get <u>two</u> harmonics of kicks from each harmonic of U!

An orbit harmonic k is produced when $J=k$ or $N-J=k$:

$$a_k = \frac{N}{\pi}(g u_k + f u_{N-k}) \cdot \nu/(\nu^2-k^2)$$
$$b_k = \frac{N}{\pi}(g v_k - f v_{N-k}) \cdot \nu/(\nu^2-k^2).$$

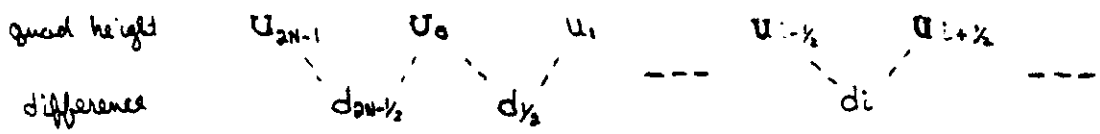
φ_0	b_0	b_1	$\langle b \rangle$	f	g
60°	3.46	−2	4.00	2.73	.73
72	3.93	−2	4.41	2.96	.96
90	4.83	−2	5.23	3.41	1.41

Example: $N = 400$, $\varphi_0 = 72°$ ($\nu = 80$). Significant distortion only when k near 80, or displacement harmonics near 80 and 320, the latter being 3 times more important. The wavelength for 320 is only 2½ quad spaces. A short-range precision survey gives the best control of orbit distortion.

SHORT-RANGE PRECISION SURVEYS

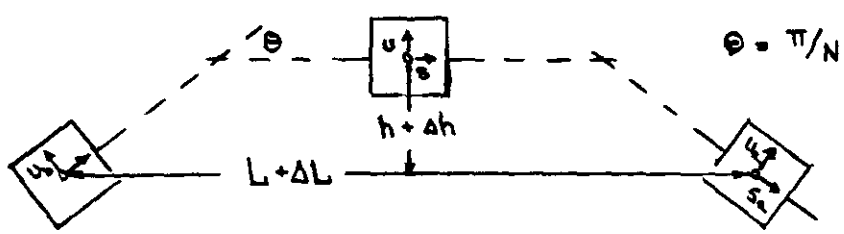
VERTICAL Levelling provides a 1st difference (gravity gives a direction).

Only height differences between adjacent quads are used, organized thus:



$$d_i = U_{i+1/2} - U_{i-1/2}\,, \qquad i = 1/2,\ 3/2,\ \cdots\ 2N-1/2\,.$$

HORIZONTAL. A 2nd. difference is necessary. Measure the deviations of the offset of a quad from the line joining its neighbors.



$$\Delta L = (S_a - S_b)\cos\theta + (U_a + U_b)\sin\theta$$

$$\Delta h = U - \tfrac{1}{2}(U_a + U_b)\cos\theta + \tfrac{1}{2}(S_a - S_b)\sin\theta$$

let $z = \Delta h - \frac{1}{2}\Delta L \tan\theta$, then

$$z_i = -\tfrac{1}{2}\tfrac{1}{\cos\theta}U_{i-1} + U_i - \tfrac{1}{2}\tfrac{1}{\cos\theta}U_{i+1}$$

(Hopefully corrections for ΔL are not necessary, but one must check)

Both sets of equations have det. = 0! The vertical set does not determine absolute height — missing zero harmonic; the horizontal does not determine the ring center — no 1st harmonic. Solve by harmonics.

VERTICAL SOLUTION: let $\vartheta = \pi/N$, there are $2N$ quads.

$$d_i \sin ik\vartheta = -d_{i-1/2} \sin ik\vartheta + d_{i+1/2} \sin ik\vartheta$$

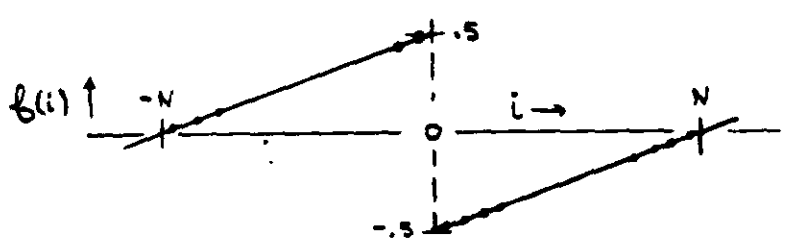
$$= -d_{i-1/2} \cos(i-\tfrac{1}{2})k\vartheta \cdot \sin k\vartheta/2 - d_{i-1/2} \sin \cos$$

$$- d_{i+1/2} \cos(i-\tfrac{1}{2})k\vartheta \sin k\vartheta/2 + d_{i+1/2} \sin \cos$$

sum for all i. Harmonic $U_k = -\dfrac{1}{2\sin k\vartheta/2} \dfrac{1}{N} \sum_i d_i \sin ik\vartheta$. (similar for V_k)

Take U at $i=0$ as typical, the solution for (U) (brackets mean computed)

is $(U) = \sum_{i=1/2}^{2N-1/2} d_i f_i$. $f_i = \sum_{k=1}^{N-1} - \dfrac{\sin ik\vartheta}{2N \sin k\vartheta} = \frac{1}{2}(i-N)/N$



If we make measurement errors $\langle\delta\rangle$ r.m.s., then

$\langle U_k\rangle = \langle V_k\rangle = \dfrac{1}{2\sin k\vartheta/2} \dfrac{1}{\sqrt{N}} \langle\delta\rangle$ which is excellent for high harmonics, but blows up for very low-ones.

For $\varphi_0 = 60°, 72°, 90°$;

$\nu = N/6, N/5, N/4$. Then for

k near ν, $\langle U_k\rangle, \langle V_k\rangle = (1.9, 1.6, 1.3)\ \langle\delta\rangle/\sqrt{N}$ and for

k near $N-\nu$, $\langle U_k\rangle, \langle V_k\rangle =$ —— .5 —— $\langle\delta\rangle/\sqrt{N}$.

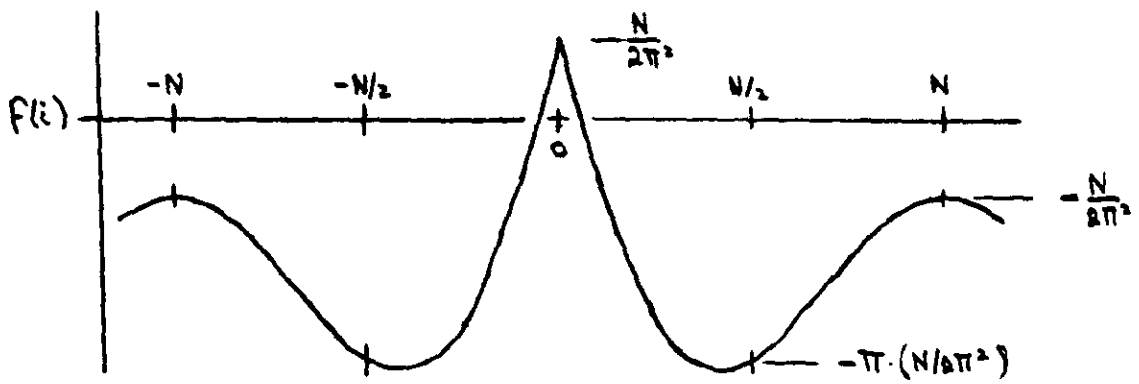
These proportions are very consistent with the relative importance of ν and $N-\nu$ harmonics in generating orbit distortion.

HORIZONTAL SOLUTION.

Same method as vertical.

harmonic $u_k = \frac{\cos\vartheta}{\cos\vartheta - \cos k\vartheta} \frac{1}{N} \sum_i z_i \cos ik\vartheta$ (no 1st harm.)

let $(U)_0 = \sum_i f_i z_i$. $f_i \to \frac{N}{2\pi^2}\left(\cos i\vartheta + 2(i\vartheta - \pi)\sin i\vartheta\right)$ large N.



$\Sigma f_i z_i$ involves detailled balancing of large numbers!

The error $\langle u_k \rangle = \langle v_k \rangle = \frac{\cos\vartheta}{\cos\vartheta - \cos k\vartheta} \frac{1}{\sqrt{N}} \langle \delta \rangle$, large for low k.

For $\varphi_0 = 60°, 72°, 90°$ $\langle U_\nu \rangle = (7.5, 5.2, 3.4)\ \langle \delta \rangle / \sqrt{N}$

$\langle U_{N-\nu} \rangle = .5\ \langle \delta \rangle / \sqrt{N}$

The orbit error is heavier on the lower (ν) harmonics. This proportion can be improved by a longer survey — three quad spaces with two offsets - but the degradation of $\langle \delta \rangle$ makes the overall performance about the same. The shorter survey is less work, and its analysis is much easier, so it is preferred. Both short surveys give excellent control of the significant harmonics for relatively little effort.

SMOOTHING FUNCTIONS — AN IMPORTANT STEP.

We must omit low harmonics from the solution, particularly where the general site survey is more accurate. Small errors in the short-range, precision survey can generate large, meaningless, low harmonic adjustments. If carried out correctly, these adjustments would not seriously distort the orbit, but small errors in making these large difficult adjustments certainly would. My choice is to cut at $k = N/10$.

Adjustments should not depend on far-away measurements. I include two tables of factors for combining close measurements — the SMOOTHING FUNCTIONS — which I managed to terminate in a way that produces the desired cut-off with negligible wobble in the response to higher harmonics. Note that the number of measurements used is small and independent of the ring size (N). Developing these tables was not easy!

For an initial, coarse adjustment the 1st column of the vertical function table, and 1st and 2nd columns of the horizontal table are sufficient. The coarse adjustment can start once a string of 27 quads is installed. The fine adjustment requires a string of 50 coarse adjusted quads.

VERTICAL SMOOTHING FUNCTION

$$(U) = \sum_{-}^{*} f_i d_i \quad , \quad f(-i) = -f(i)$$

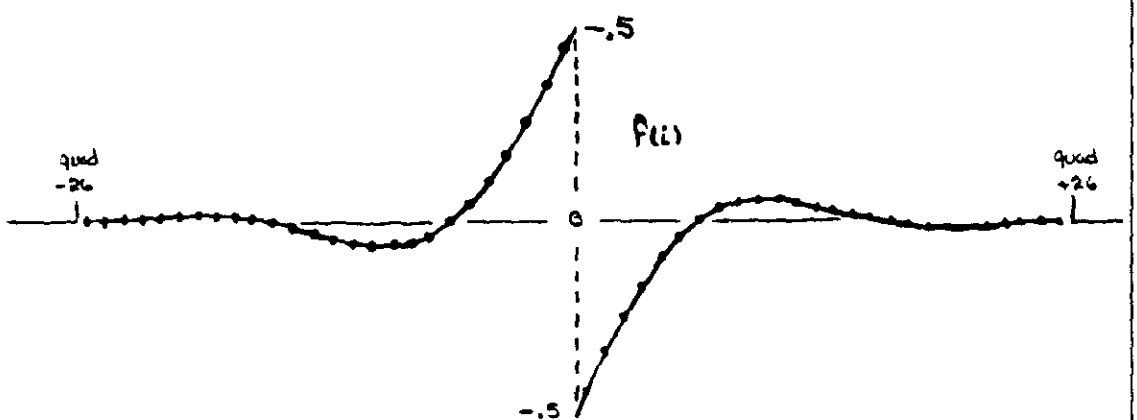
Let $U = u_k \cos jk\theta$, and find $(U)_j$

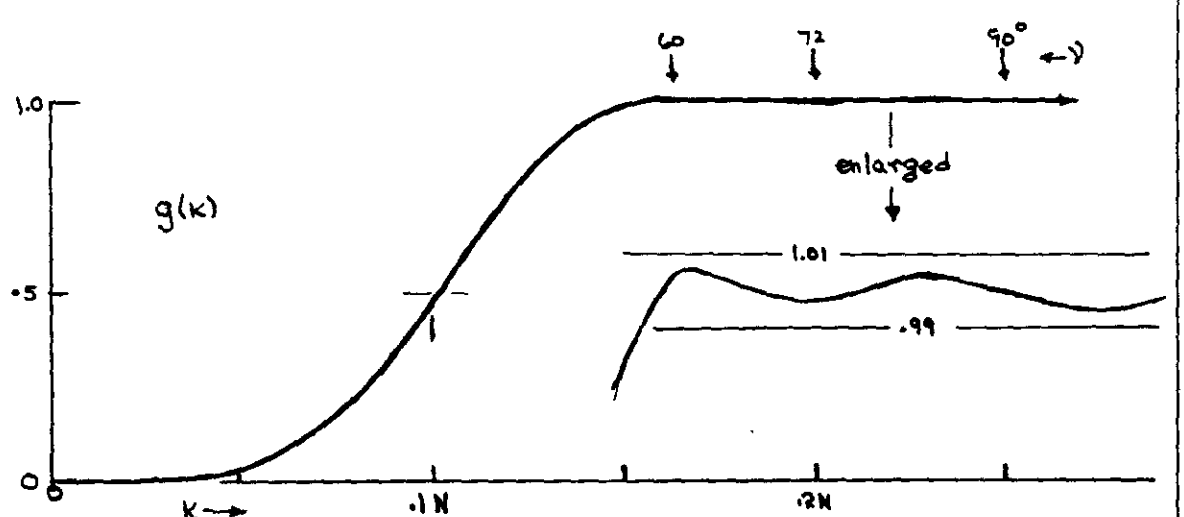
then calc. (u_k) and let $g(k) = (u_k)/u_k$.

We want $g(k<\frac{N}{20}) = 0$, $g(\frac{N}{10}) = 1/2$ and

$g(k>.15N) = 1$.

i	$f(i)$				
1/2	-.449	13/2	+.002	39/2	-.002
3/2	-.350	15/2	.033	35/2	-.007
5/2	-.257	17/2	.050	37/2	-.009
7/2	-.173	19/2	.057	39/2	-.010
9/2	-.101	21/2	.056	41/2	-.009
11/2	-.042	23/2	.048	43/2	-.007
		25/2	.038	45/2	-.004
		27/2	.026	47/2	-.003
		29/2	.014	49/2	-.001
		31/2	.005	51/2	-.001



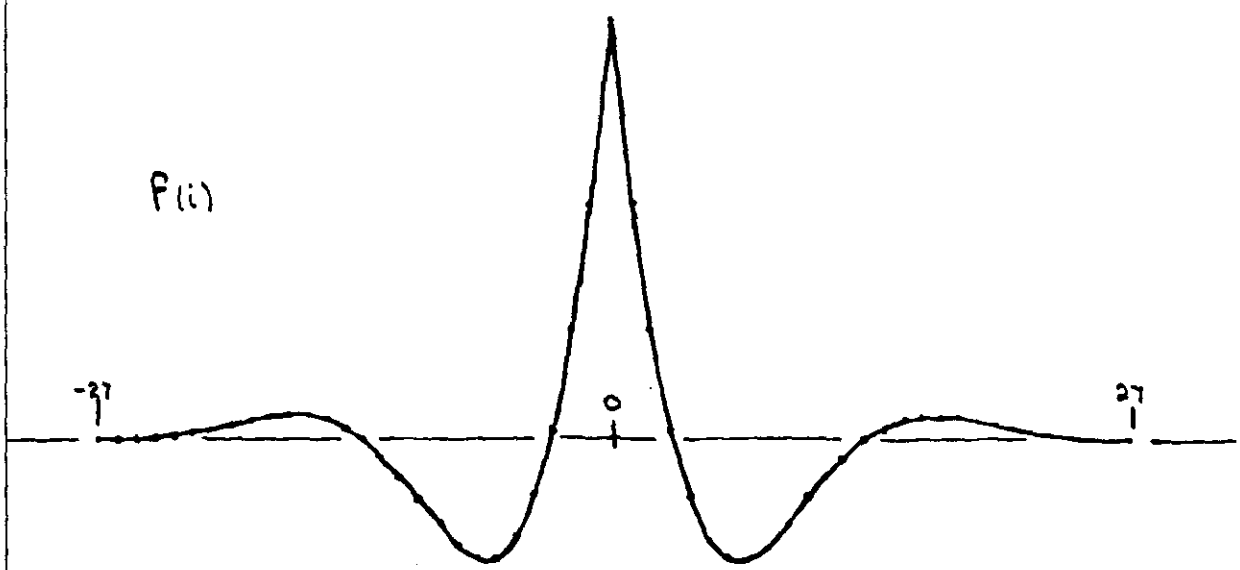


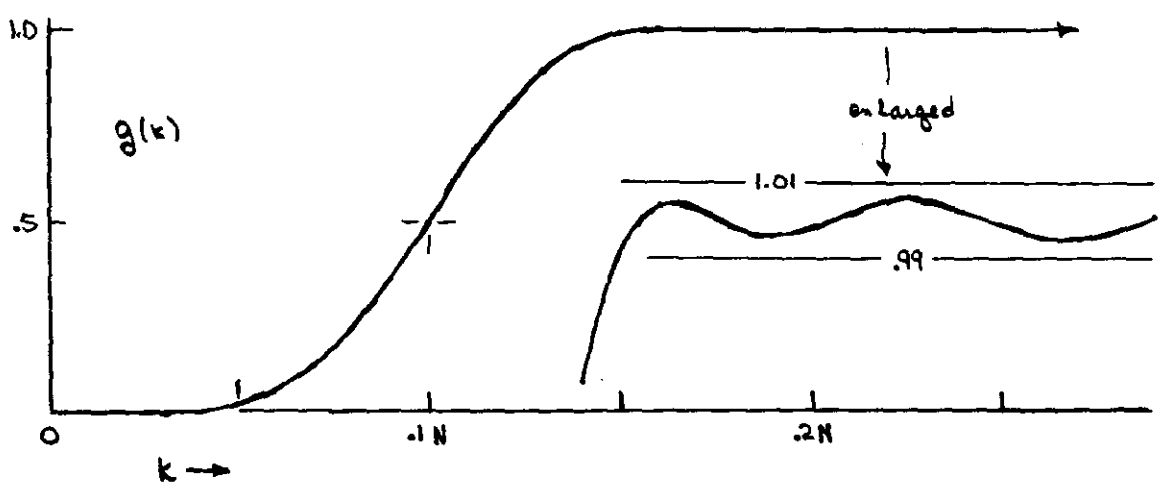
HORIZONTAL SMOOTHING FUNCTION

$f(i)$

$f(-i) = f(i)$

0	2.164	7	-.620	14	.061	21	.048
1	1.262	8	-.554	15	.102	22	.030
2	.556	9	-.449	16	.120	23	.017
3	.034	10	-.326	17	.121	24	.009
4	-.321	11	-.203	18	.110	25	.004
5	-.531	12	-.093	19	.092	26	.002
6	-.621	13	-.003	20	.070	27	.001

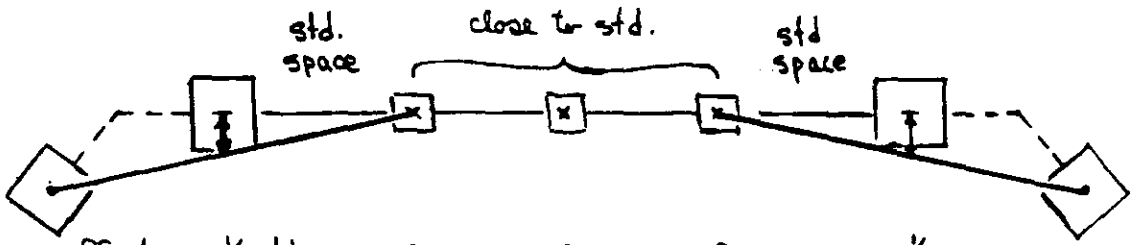




Rings with Straight Sections

The vertical survey and its analysis is not changed if the ring is not circular. The horizontal smoothing function, as given, can also be used for rings with straight sections, because we omit low harmonics. In both cases some care is needed to keep errors consistent.

One can probably assume that the special straight-section elements are not available at survey time; in any case arrange heavy concrete blocks as temporary survey stations so that they simulate normal cell spacing, as shown.



offset	~ ½ std.	o	o	o	~ ½
ΔL factor	~ ½ std.	o	o	o	~ ½

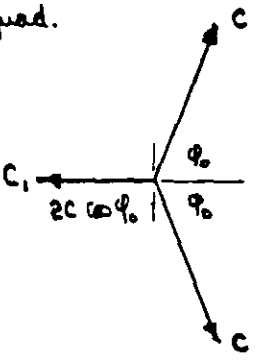
Proceed as before, except that new offsets and ΔL factors are used. Actually, this is the best way to lay out a straight line!

The calculated horizontal move is not perfect but it is well within survey precision. Most importantly the harmonic structure of adjustments and errors is not changed.

CORRECTION BY STEERING DIPOLES.

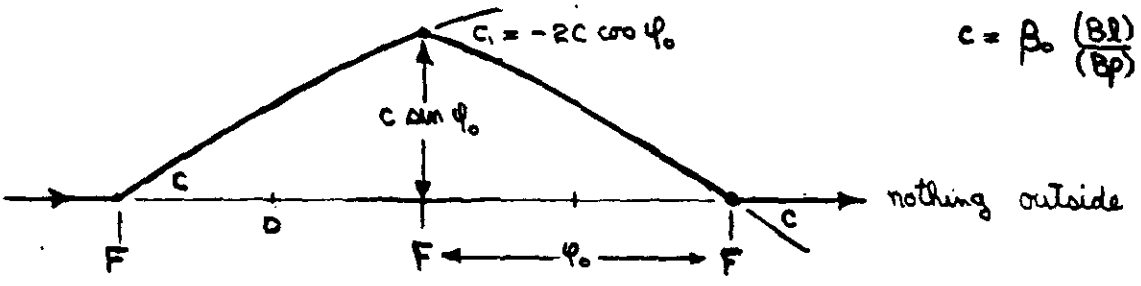
One dipole, one position detector at each F quad.

(In other plane at alternate quads.)



THE BUMP –

A local adjustment.



$$c = \beta_0 \frac{(B\ell)}{(B\rho)} \text{ corr.}$$

THE ARRAY – is a superposition of bumps,

Thus $\quad C_n = (y_n \cdot 2\cos\varphi_0 - y_{n-1} - y_{n+1}) / \sin\varphi_0 \,.$

Can readily be automated but, because of non-linear fields, one might need correction in steps – say 1/4 each time – after new measurements of the orbit.

For consistent operation one must routinely correct the orbit to much better than the aperture!

ORBITS AFTER SMOOTHING.

The precision short-range survey errors $\langle\delta\rangle$ produce a correlated u displacement. In addition there are simple $\langle u\rangle$ errors, primarily from the actual field center with respect to the survey mark on the outside of the cryostat (may change on repeated cool-down!). Let me write

$$\langle a_k\rangle = \langle b_k\rangle = \langle\epsilon_k\rangle \frac{\sqrt{N}}{\pi} \frac{\nu}{\nu^2-k^2} .$$

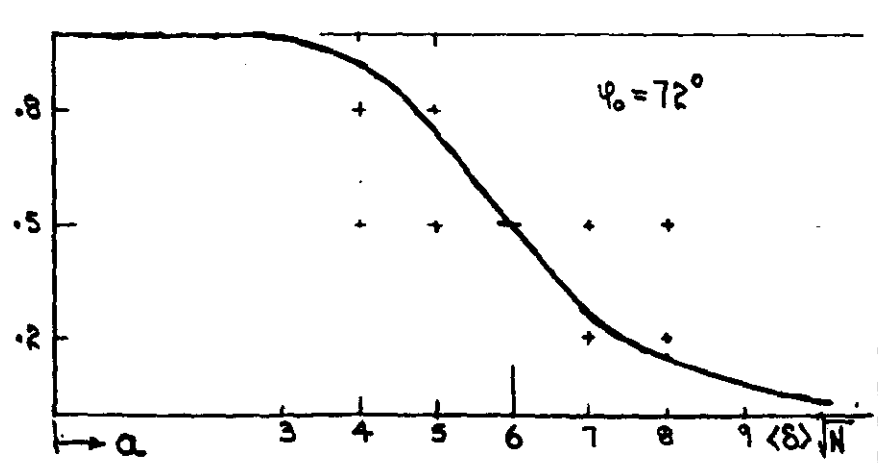
The contributions to $\langle\epsilon\rangle$ for <u>k near ν</u> are:

		60°	72°	90°
HORIZ. smoothing combined	from $q\cdot u_\nu$	5.45	5.03	4.81
	$q\cdot u_{N-\nu}$	1.46	1.64	2.00
		5.64 $\langle\delta\rangle$	5.29 $\langle\delta\rangle$	5.21 $\langle\delta\rangle$
VERT. smoothing combined	$q\cdot u_\nu$	1.41	1.55	1.84
	$q\cdot u_{N-\nu}$	1.42	1.56	1.85
		2.00 $\langle\delta\rangle$	2.20 $\langle\delta\rangle$	2.60 $\langle\delta\rangle$
BOTH from simple, random $\langle u\rangle$		2.83 $\langle u\rangle$	3.12 $\langle u\rangle$	3.70 $\langle u\rangle$

By comparison with simple $\langle u\rangle$, one expects peak-peak (50%) $= \langle\epsilon_\nu\rangle\sqrt{N}$

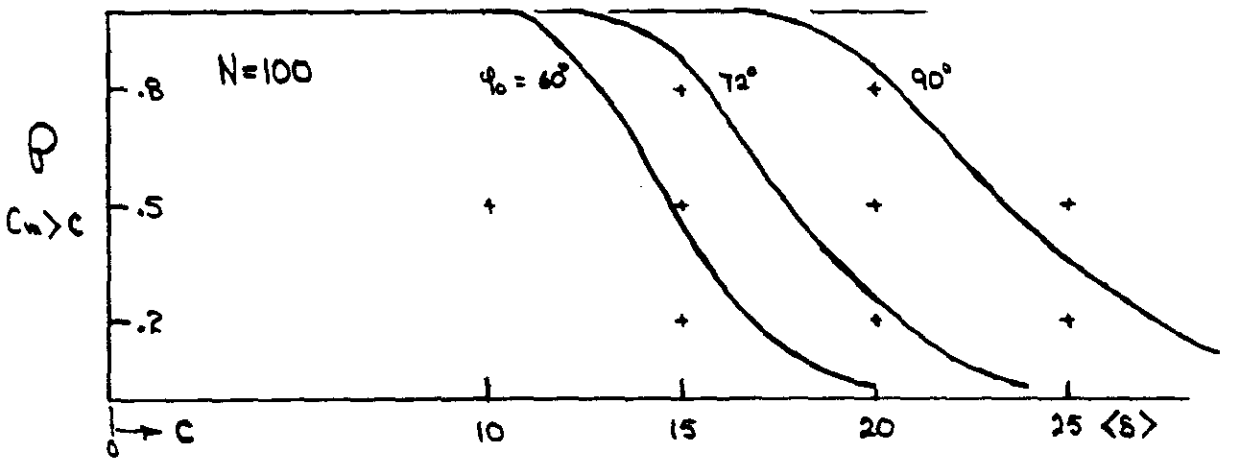
Numerical results for peak-peak after HORIZ. SMOOTHING, are a little larger:

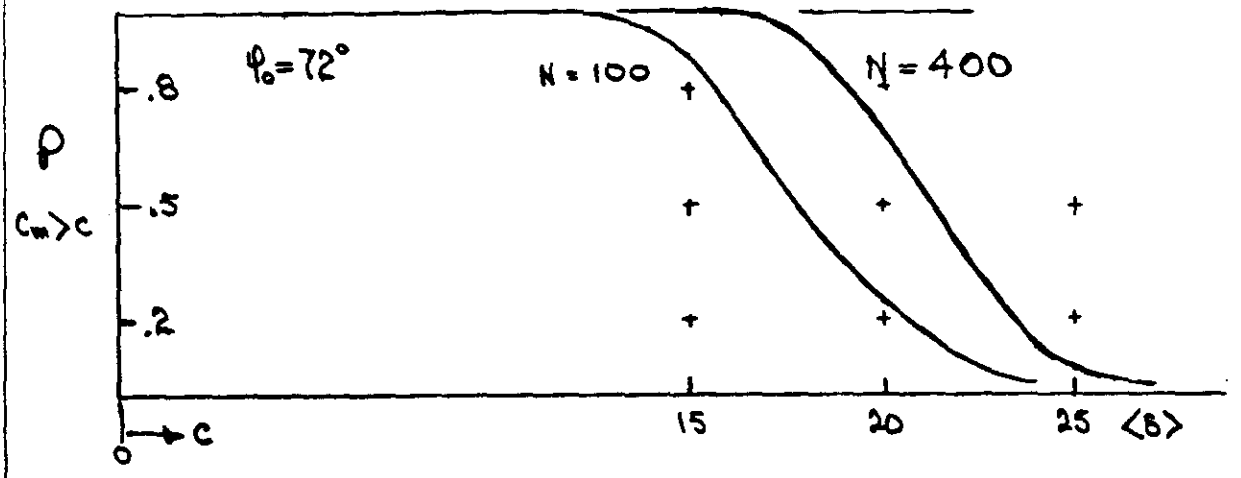
Probability of peak-peak > a



SIZE OF CORRECTION DIPOLES.

Dipoles at each F must correct for the F and nearby D displacements, as well as survey mark error and bend field error. The rms correction $\langle c \rangle$ does not depend on N, but c_{max} will. The correlations after smoothing are significant and one must resort to numerical distributions. For HORIZ. SMOOTHING, after survey error $\langle \delta \rangle$



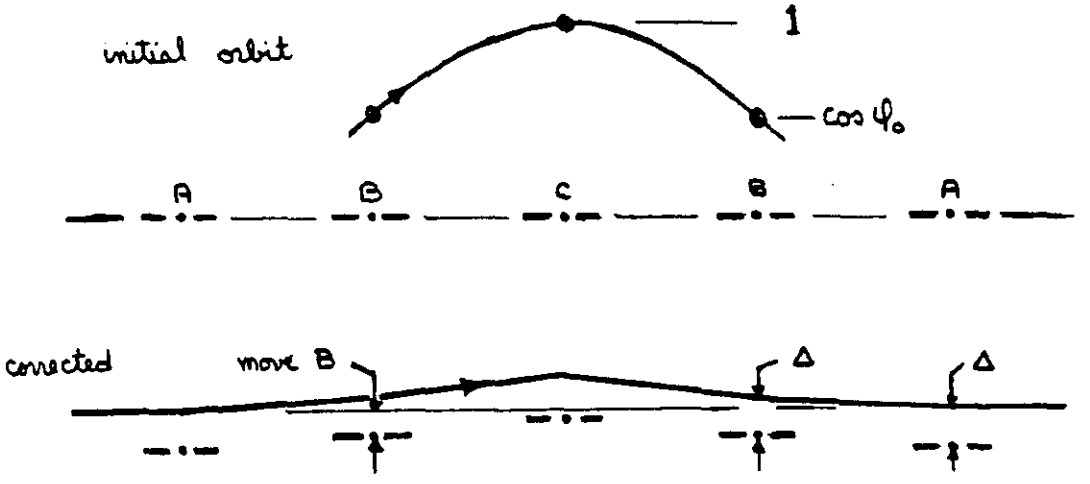


Let d = corr. dipole / half-cell bend. $d = c/\beta_0 \theta$

Now $\beta_0 \theta \approx 3M$. so typically $d_{max} = 10 \langle \delta \rangle$, δ in meters, with some allowance for the "other" errors.

CORRECTIONS BY QUAD MOVES

Consider the orbit with respect to the quads, then we do not have the equivalent of the simple bump. Example: correct typical orbit, as shown, at three places by moving 5 focussing quads; so that residual Δ is the same in the 5 quads and zero elsewhere.



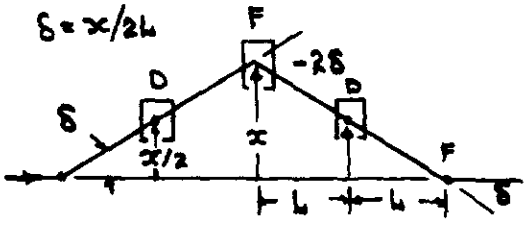
φ_0	moves A	B	C	corrected orbit Δ
60°	-.158	-.132	-.027	.158
72	-.105	-.189	-.053	.105
80	-.073	-.209	-.065	.073
90	-.037	-.215	-.073	.037

The quad moves are a substantial fraction of the correction! Intervening quads and dipoles must be realigned so even a single bump is a lot of work. Quad bumps can be used, sparingly, to relieve excessive demand on correction dipoles.

Reduction of Steering by Quad Moves.

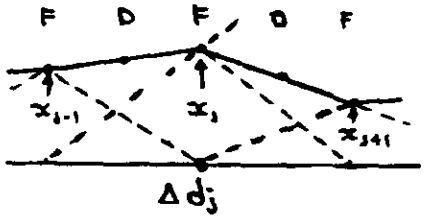
We start with the beam, at lower energy, accurately centered at each F quad by steering. The aim is to reduce the maximum steering by adjustments of quad positions derived from the pattern of steering.

Make a "triangular" move of an F, D's and bends. Steering is changed $\delta, -2\delta, \delta$. Beam is at same position relative to the elements,



so there are no focussing or non-linear effects. One can readily design combination moves to reduce steering peaks, however for a factor of 2 reduction in one plane about 1/5 to 1/4 of the ring must be moved.

The Grand Move.



Express the steering angle as $d\theta$, as before. Then for a set of moves x_j

$$\Delta d_j \, \theta L = \tfrac{1}{2} x_{j-1} - x_j + \tfrac{1}{2} x_{j+1}$$

Let $x_j = a_k \cos jk\alpha \qquad \alpha = 2\pi/N$

then $\Delta d_j \, \theta L = -a_k (1 - \cos k\alpha) \cdot \cos jk\alpha$,

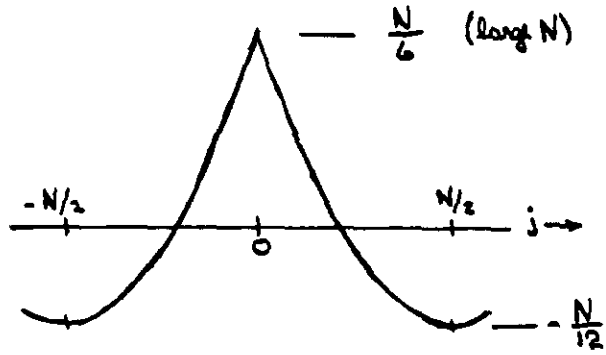
a low harmonic (k) of move (x) requires small steering ($1-\cos k\alpha$). A zero harmonic requires no steering. The solution for the set of x's which make the steering (d) zero is found by solving for harmonics and combining them for a typical move x_0. The zero harmonic is omitted. This is very similar to the horizontal survey solution.

The harmonic amplitude to make $d=0$ is

$$a_k = \frac{\theta L}{(1-\cos k\alpha)} \sum d_j \cos jk\alpha \text{ , then the typical } x$$

$$x_0 = \frac{a_{N/2}}{2} + \sum_{k=1}^{N/2-1} a_k = \theta L \sum_j d_j \underbrace{\left(\frac{\cos \pi j}{4} + \sum_{k=1}^{N/2-1} \frac{\cos jk\alpha}{(1-\cos k\alpha)} \right)}_{F_j}$$

$$F_j = \tfrac{1}{6}(N - 1/N) - |j| + j^2/N.$$



This exact solution has the same problem as the exact survey solution, and the same cure — omit low harmonics, which contribute very little to the steering. In fact, the <u>horizontal survey smoothing function</u>, $f(i)$ is just what we want;

$$x_j = \theta L \sum_{i=-27}^{27} f(i)\, d_i$$ where i is measured from position j. (The cut-off is at $K = N/20$ because $\alpha = 2\theta$). A numerical example follows.

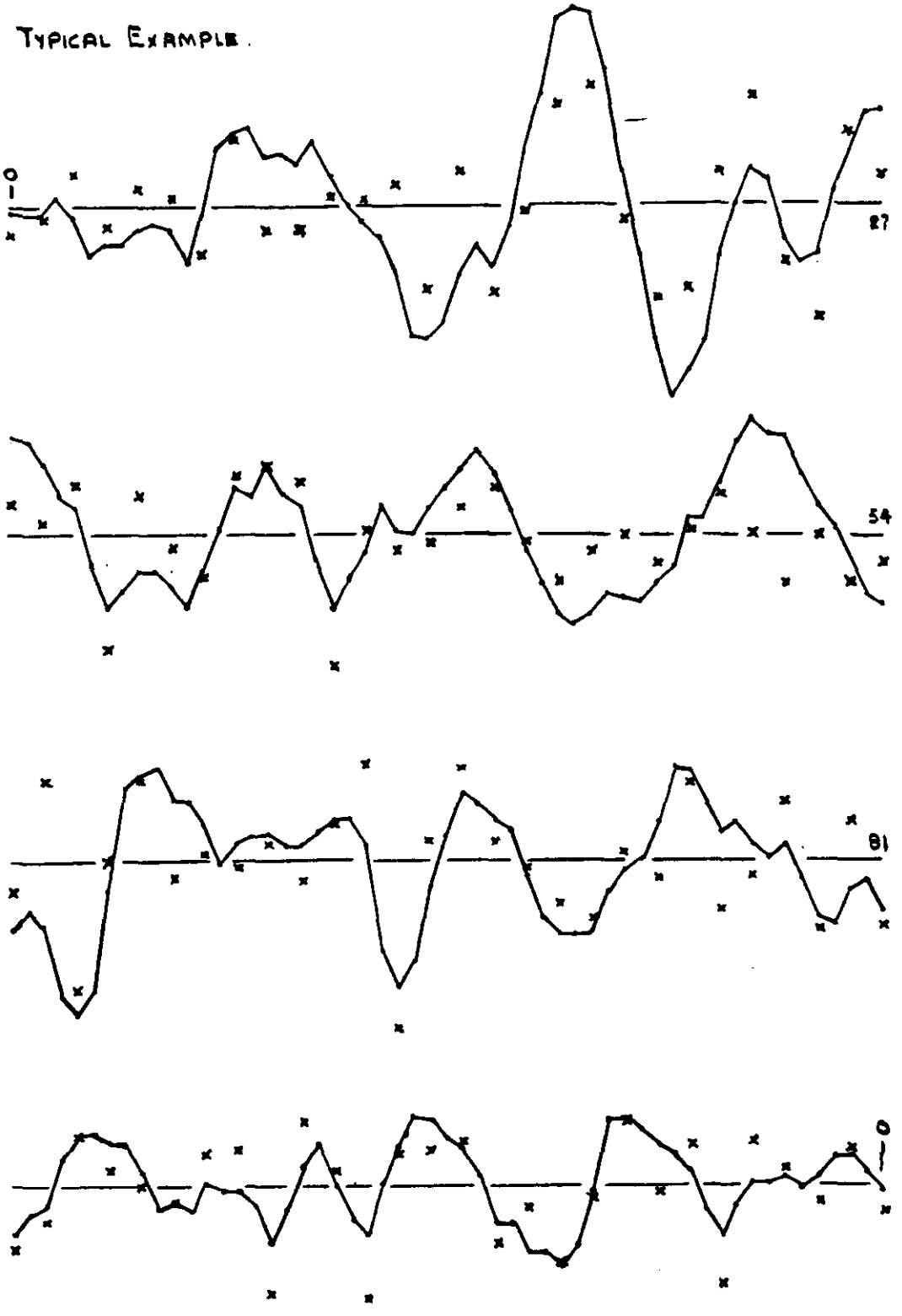
Given a horizontal precision survey with error $\langle\delta\rangle$, adjustment as prescribed by the smoothing function, centering of beam on F quads by steering angles $d_j\theta$, and finally a grand move as calculated above (also use horiz. $f(i)$ for vert. move), then:

the r.m.s. move will be $3\tfrac{1}{2}\langle\delta\rangle$, and

the maximum move will be $11\langle\delta\rangle$ (50%)

$13\langle\delta\rangle$ (80%).

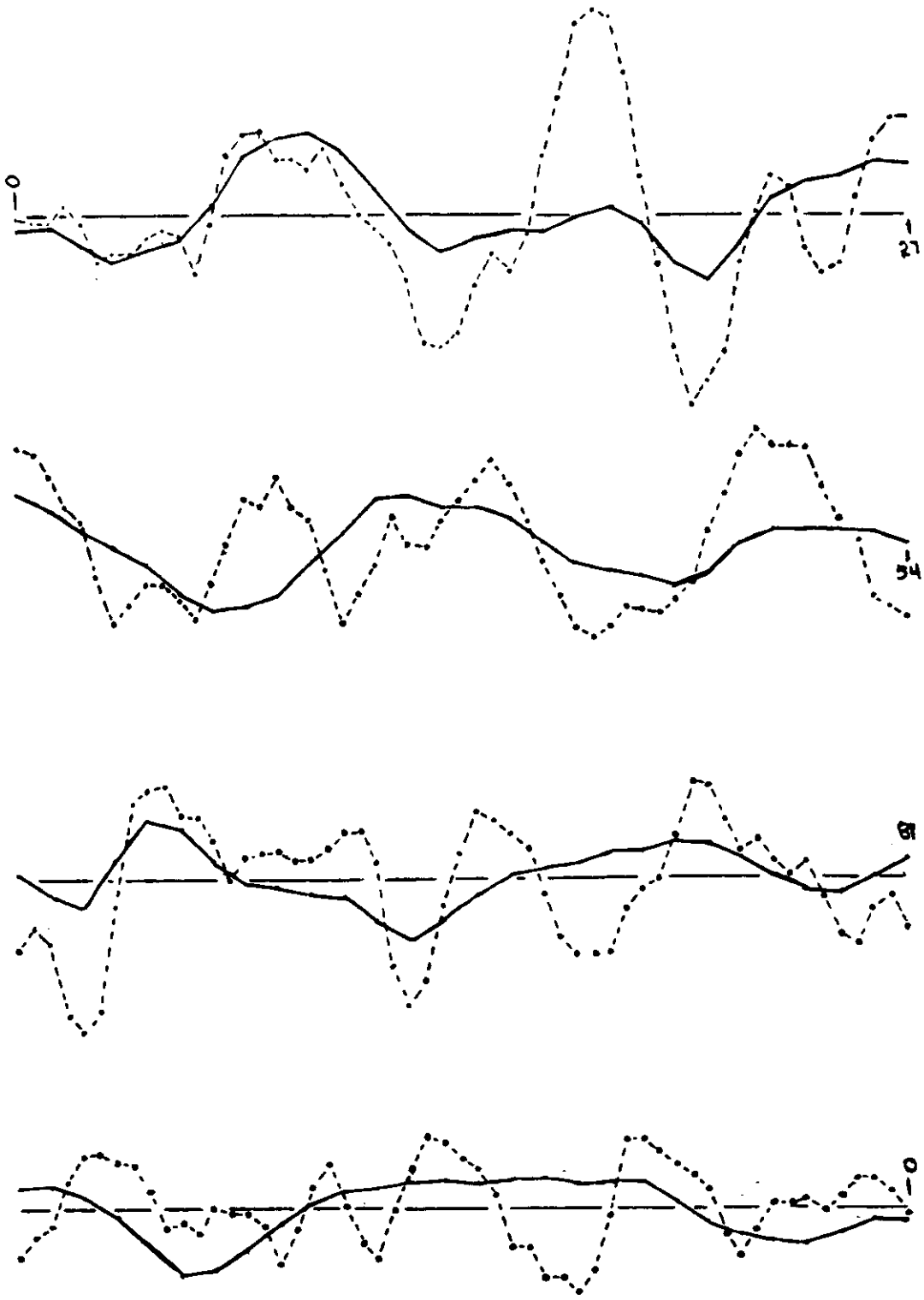
TYPICAL EXAMPLE.



QUAD. DISPLACEMENT (SOLID LINE). N = 108. $\Phi_0 = 72°$

STEERING STRENGTH (X) TO CENTER BEAM IN F QUADS.,

THE SIGN IS REVERSED, AND SCALE ADJUSTED TO EMPHASIZE CORRELATION.



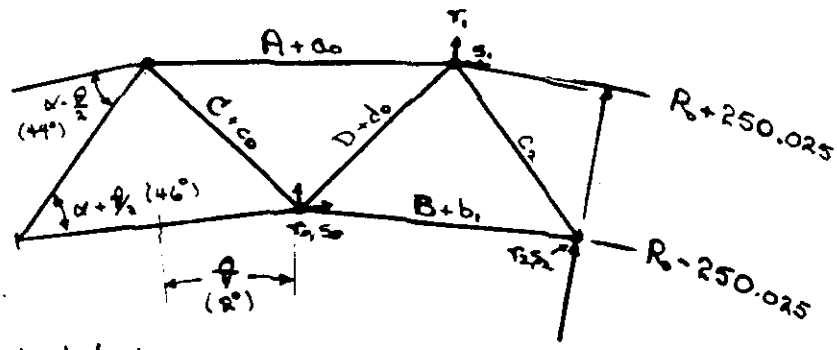
QUAD. DISPLACEMENT FROM IDEAL. BEFORE (DOTTED) AND AFTER (SOLID-F only) "SMOOTHING" USING STEERING STRENGTH. IN A REAL RING, THESE DISPLACEMENTS ARE UNKNOWN.

SURVEYING A 90 km. CIRCUMFERENCE CIRCLE (FLAT WORLD)

180 MONUMENTS – 2° SPACING, BUT ZIG-ZAG PATTERN.

choose $\alpha = 45°$

$A = 1017.248$
$B = 982.345$
$C = D = 707.071$



a_j, b_j, c_j, d_j are measured.

r_j, s_j and z_j (elev.) are calculated.

Harmonic Solution for r_j. Let $r_j = a_0 + \sum_{k=2}^{90} a_k \cos kj\vartheta + b_k \sin kj\vartheta$.

$$N(\cos\vartheta - \cos k\vartheta)\cdot a_k = \sum_{j\ \text{even}} \left(a_j \frac{\cos(\alpha + \vartheta/2)}{\sin(\alpha - \vartheta/2)} - (c_j + d_j)\frac{\cos\vartheta}{\sin(\alpha - \vartheta/2)} \right) \cos kj\vartheta$$

(N=180)

$$+ \sum_{j\ \text{odd}} \left(-d_j \frac{\cos(\alpha - \vartheta/2)}{\sin(\alpha + \vartheta/2)} + (c_{j+1} + d_{j-1})\frac{\cos\vartheta}{\sin(\alpha + \vartheta/2)} \right) \cos kj\vartheta .$$

(each c, d appears twice, once in the even sum, once in the odd)

Let the error in a be $\delta \cdot A$ etc. ($\delta = 10^{-6}$) and $\langle a_k \rangle$ = rms

For $\alpha = 45°$

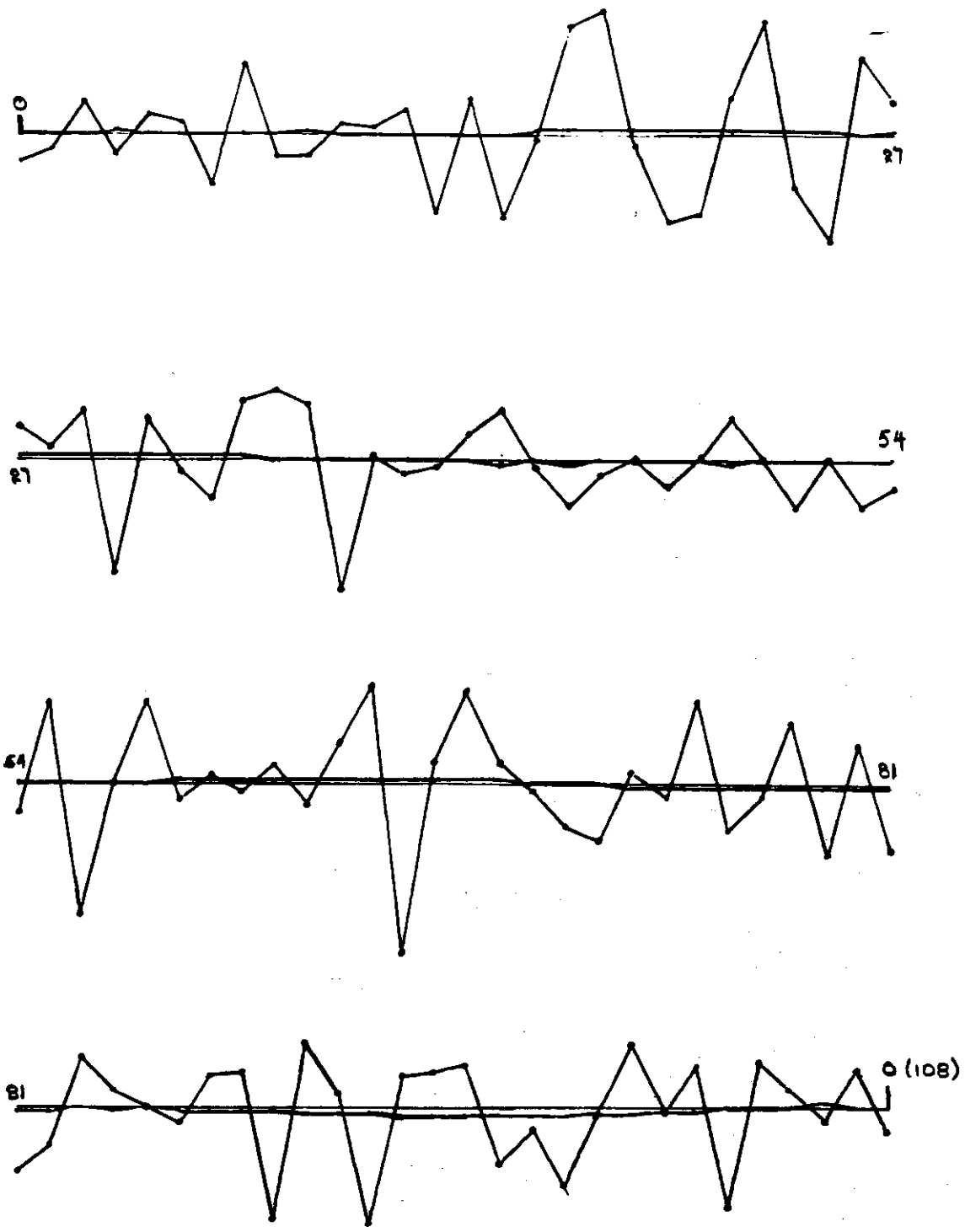
$$\langle a_k \rangle = \frac{1}{(\cos\vartheta - \cos k\vartheta)} \frac{\delta}{2\sqrt{N}} \left(A^2 + B^2 + \cos^2\vartheta\,(C^2 + D^2)\left(\frac{1}{\sin^2 44} + \frac{1}{\sin^2 46} - \frac{2\cos k\vartheta}{\sin 44 \cdot \sin 46}\right)\right)^{1/2}$$

$$\langle a_k \rangle = \langle b_k \rangle = 91.3 \times 10^{-6} \left(1 - \tfrac{2}{3}\cos k\vartheta\right)^{1/2} / (\cos\vartheta - \cos k\vartheta)$$ for a_0, a_{90} div. by $\sqrt{2}$

k	$\langle a_k \rangle$	$\langle b_k \rangle$
0	.0612	–
1	–	–
2	.0289	.0289
3	.0109	.0109
4	.0058	.0058
5	.0037	.0037
45	9×10^{-5}	9×10^{-5}
90	4×10^{-5}	—

$\langle a_k \rangle$	$\langle b_k \rangle$
.0117	–
.0165	.0165
.0165	.0165
.0117	–

} six radial measurments with $\delta = 2 \times 10^{-6}$.



STEERING COIL STRENGTH. BEFORE AND AFTER (small)

"SMOOTHING" QUAD DISPLACEMENTS, USING STEERING STRENGTH.

THE PROCEEDURE IS VERY EFFECTIVE AT REDUCING STEERING.