The Resonance Correction Scheme

For the Low Energy Booster

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Abstract

We describe the resonance correction scheme that may be implemented on the Low Energy Booster of SSC. This scheme will correct: (1) the quadrupole resonances 2vx=33 and 2vy=33; (2) the skew quadrupole coupling and sum resonances vx-vy=0 and vx+vy=33; (3) the sextupole third integer resonances 3vx=50, vx+2vy=50 and 3vx=49, vx+2vy=49; (4) the skew sextupole third integer resonances 3vy=50, 2vx+vy=50 and 3vy=49, 2vx+vy=49. Additionally, the tune and the chromaticity remain unchanged.

The configuration we chose depended partly on the hardware available. 72 quadrupole correctors are in 72 main quadrupoles. 72 sextupole correctors are in 72 main sextupoles. 36 skew quadrupole correctors and 72 skew sextupole correctors are located in some of the correction trim coil assemblies. We hope the configuration we chose be as efficient as possible. Besides, we hope the number of power supplies required be kept to a minimum to reduce the total cost.

Introduction

The Low Energy Booster of SSC is required to accelerate high intensity protons. It leads to a fast cycling machine (10 Hz) with a large space tune shift. This large tune shift can cause particles to cross several second and third order resonances as shown in the tune diagram Fig. 1. In order to reduce the possibility of beam loss, we need to correct the following four classes of resonances: (1) the quadrupole resonances 2vx=33 and 2vy=33; (2) the skew quadrupole coupling and sum resonances vx-vy=0 and vx+vy=33; (3) the sextupole third order resonances 3vx=50, vx+2vy=50 and 3vx=49, vx+2vy=49; (4) the skew sextupole third order resonances 3vy=50, 2vx+vy=50 and 3vy=49, 2vx+vy=49.

In order to devise a correction scheme properly, we have to solve the following problems:

1. How to produce the expected harmonic term?

We know that for the resonance mvx+nvy=p, the harmonic compenent exp(ipθ) is its driving term. We hope the correction scheme produce the desired harmonic compenent to cancel that one excited by errors.

As an example, we consider the case of resonance 2vx=m. How can we produce the desired harmonic compenent exp(imθ)? We can prove that the following arrangement of correctors

$$K_{pq} = f_p^m * K_q \qquad p=1,2,\ldots,H$$

$$f_p^m = \cos(m*2\pi(p-1)/H)$$

would produce the harmonic compenent exp(imθ) and its alias harmonics Hk$\pm$m, for k=0,$\pm$1,$\pm$2,....., and H is the superperiodicity of the machine, K_{pq} is the strength of the q'th corrector in p'th superperiod, K_q is the normalized strength of the q'th corrector in p'th superperiod. In fact the conclusion above is true for any resonances.[1]

2.How to arrange the correction scheme more effective

a) How to arrange the correctors of one set more effective

Under some simplied assumptions about the machine lattice, C.J.Gardner proved [2] that if we put a set of correctors K1, K2, K3 and K4 in positions s1, s2, s3 and s4

where $\beta_{x1} = \beta_{x2} = a$, $\beta_{x3} = \beta_{x4} = A$, $\beta_{y1} = \beta_{y2} = B$, $\beta_{y3} = \beta_{y4} = b$,

and $\phi_x(s_j) = \phi_y(s_j) = \phi_j$, here ϕ_j is normalized betatron phase advances defined $\phi_j = \frac{1}{\nu}\psi_j = \frac{1}{\nu}\int_0^{s_j}\frac{ds}{\beta}$

to correct the resonances 2vx=p and 2vy=p simultaneouesly, the effectiveness of the correctors in producing the desired corrections is proportional to $|\sin(p(\phi_2-\phi_1))|$, $|\sin(p(\phi_4-\phi_3))|$ and AB-ab.

It's also true for correction resonances 3vx=p and vx+2vy=p simultaneouesly. So we know that corrector positions for which either p($\phi_2-\phi_1$) or p($\phi_4-\phi_3$) is an integral multiple of π must therefore be avoided. The optimum positions are those for which p($\phi_4-\phi_3$) and p($\phi_2-\phi_1$) are odd multiple of $\pi/2$. Besides two correctors should be placed at horizontal beta maximums and two at vertical beta maximums.

Now in the LEB each of the six superperiods is composed of nine almost identical FODO cells. Therefore the results mentioned above are available.

For the correction resonances vx-vy=0 and vx+vy=33, we note the two resonances of f_p^m for m=0 and for m=33 are orthogonal. (i.e. f_p^m of m=0 doesn't contribute to m=33 resonance and vice versa) Then we can easily prove that the effectiviness of a set of correctors is proportional to $\sin[(\psi_{x2}+\psi_{y2})-(\psi_{x1}+\psi_{y1})]$ for resonance vx+vy=33 and to $\sin[(\psi_{x2}-\psi_{y2})-(\psi_{x1}-\psi_{y1})]$ for resonance vx-vy=0 ,respectively. Of course it is also proportional to $\sqrt{aB}$ and $\sqrt{Ab}$. Here ψ_x and ψ_y are the betatron phase advances.

b) How to arrange sets of correctors more effective

At first, in order to reduce the number of power supplies we assume the strengths of sets are the same or only different in sign, i.e.

$$\vec{K1} = \begin{pmatrix} K_1 \\ K_2 \\ \vdots \end{pmatrix} = \pm \vec{K2}$$

Now we consider the resonance mvx+nvy=p. We define

$$\Delta\psi = (m\,\psi_x + n\,\psi_y)_2 - (m\,\psi_x + n\,\psi_y)_1$$

is the phase difference between the first set of correctors and the second set. It is obvious that when $\Delta\psi$ is integral multiple of 2π, or $\Delta\psi$ is odd multiple of π (just put $\vec{K2}=-\vec{K1}$), the total effection of these two sets of correctors are the most effective.

3.Estimate the strengths of correctors required

In order to solve this problem we have to estimate the resonance strengths excited by errors at first. The method here we use is:

a) We simulate the errors as kicks(the dipole errors as two kicks) where we assume a gaussian distribution of the random errors.

b) For a given resonance mvx+nvy=p, we calculate the resonance strengths excited by the random errors for 21 cases, i.e. use 21 random seeds.

c) For every case we calculate the corrector strengths requird for correcting. The limit of corrector strengths required depends on the maximum of 21 cases.

Quadrupole Resonance Correction

Here we want to correct the resonances 2vx=33 and 2vy=33.

According to the effective arrangement method mentioned above we place the 12 quadrupole correctors (3 sets) per one superperiod as shown in Fig. 2 (a) and with the following additional conditions:

$Q_c1=Q_c7=-Q_c5$, $\quad Q_c2=Q_c8=-Q_c6$,

$Q_c3=Q_c11=-Q_c9$, $\quad Q_c4=Q_c12=-Q_c10$

The actual corrector strengths can be determined with f_p^{33} given in the following table:

Superperiod	f_p^{33}	Actual corrector Strengths			
1	1	Q_c1,	Q_c2,	Q_c3,	Q_c4
2	-1	$-Q_c1$,	$-Q_c2$,	$-Q_c3$,	$-Q_c4$
3	1	Q_c1,	Q_c2,	Q_c3,	Q_c4
4	-1	$-Q_c1$,	$-Q_c2$,	$-Q_c3$,	$-Q_c4$
5	1	Q_c1,	Q_c2,	Q_c3,	Q_c4
6	-1	$-Q_c1$,	$-Q_c2$,	$-Q_c3$,	$-Q_c4$

Thus only four power supplies are required. Furthermore, the tunes are unaffected by the quadrupole correctors and only resonances of the 33'th harmonic and its aliases (i.e. 27'th, 21'st, 15'th, etc.) are excited.

In order to get the resonance strengths A_{33} (for $2\nu x=33$) and B_{33} (for $2\nu y=33$) , we consider the following random errors:

a) σ_1 =1 mm horizontal displacement of the eddy current sextupole ($B''=0.24T/m^2$) in dipole
b) σ_2 =1 mm horizontal displacement of the sextupole ($b_2=0.12$ @1 cm) in dipole
c) σ_3 =1 mm horizontal displacement of the main sextupoles
d) σ_4 =0.06% error in the strengths of QF, QD

For 21 random seeds, using program QER, we found that the errors in the strengths of QF, QD and the error of horizontal displacement of the main sextupoles are the most important. From 21 random seeds the worst case resonance strengths are

$Re(A_{33})=0.0119$ $Im(A_{33})=-0.00391$
$Re(B_{33})=0.00761$ $Im(B_{33})=-0.0146$

and the required corrector strengths (using program QCOR) are $[M^{-1}]$:

$Q_c1= 0.0000285$, $Q_c2=-0.000121$,
$Q_c3= -0.000717$, $Q_c4= 0.000597$ (M^{-1})

$|Q_c|<$ 0.21% of the main quadrupole strength for 0.8m long correctors.

Skew Quadrupole Resonance Correction

Here we want to correct the resonances vx+vy=33 and vx-vy=0.

Now we are going to place 6 skew quadrupole correctors (3 sets) per one superperiod as shown in Fig. 2 (c) and with the following additional conditions:

SQ1=SQ3=SQ5 , SQ2=SQ4=SQ6

In order to get the resonance strengths A_{33} (for vx+vy=33) and B_0 (for vx-vy=0) , we consider the following random errors:

a) σ_1 =1 mm vertical displacement of the eddy current sextupole (B''=0.24T/m^2) in dipole
b) σ_2 =1 mm vertical displacement of the sextupole (a_2=0.06 @1cm) in dipole
c) σ_3 =1 mm vertical displacement of the main sextupoles
d) σ_4 =0.6mrad rotation error of QF,QD

The calculation results (using program SQER) show that for the resonances vx+vy=33 and vx-vy=0 the rotation error of QF,QD and the error of vertical displacement of the main sextupoles play a main role. From 21 random seeds, in the worst case the resonance strengths are

$$Re(A_{33})=0.000793 , \quad Im(A_{33})=-0.00139 ,$$
$$Re(B_0)=0.00324 , \quad Im(B_0)=-0.0111$$

and the required corrector strengths are [M^{-1}]:

$(SQ1)_{33}$=-0.00182 , $(SQ2)_{33}$=-0.000547 for vx+vy=33

$(SQ1)_0$ =-0.00000435 , $(SQ2)_0$ =-0.00128 for vx-vy=0

If we want to correct these two resonances simultaneouesly, the actual corrector strengths can be expressed as

$$SQ = f_p^0 * (SQ)_0 + f_p^{33} * (SQ)_{33} ,$$

shown in the following table:

Superperiod	f_p^0	f_p^{33}	Actual Corrector Strength
1	1	1	$(SQ1)_0 + (SQ1)_{33}$, $(SQ2)_0 + (SQ2)_{33}$
2	1	-1	$(SQ1)_0 - (SQ1)_{33}$, $(SQ2)_0 - (SQ2)_{33}$
3	1	1	the same as superperiod 1
4	1	-1	the same as superperiod 2
5	1	1	the same as superperiod 1
6	1	-1	the same as superperiod 2

With this choice of f_p^0 and f_p^{33}, the skew quadrupole correctors can be grouped into four families. Two of the families will reside in superperiods 1, 3 and 5 while the other two families will occupy superperiods 2, 4 and 6. Thus only four power supplies are needed. Additionally, the correction scheme for vx-vy=0 will excite the alias harmonics ...-6, 6, 12, etc. and the vx+vy=33 correction scheme will excite ...-3, 3, 9,..., 21, 27, 39, etc. as well.

The maximum of the corrector strengths required is 0.00183 m^{-1}. For correctors of 20cm length this leads to

$$|SQ| < 2.09\% \text{ of the main quadrupole strength.}$$

Sextupole Resonance Correction

Here we want to correct the resonances 3vx=50, vx+2vy=50 and 3vx=49, vx+2vy=49.

We put 12 sextupole correctors (3 sets) per one superperiod as shown in Fig. 2(b) and with the additional conditions:

S1=S3=-S9 , S2=S4=-S10,

S5=S7=-S11 , S6=S8=-S12

In order to get the resonance strengths A_n (for 3vx=n) and B_n (for vx+2vy=n) , we consider the following random errors:

a) σ_1 =10% variation on the systematic value of the eddy current sextupole (B''=0.24T/m^2) in dipole
b) σ_2 =0.12 random sextupole (b_2=0.12 @ 1cm) in dipole
c) σ_3 =0.1% error of the main sextupoles

The calculation results (using program SEXER) show that the error of eddy current sextupole and b_2 in dipole are the main factors for the resonances. From 21 cases, we found the worst case resonance strengths excited by the errors to be

Re(A_{50})=-0.0432 Im(A_{50})=-0.0511
Re(B_{50})=-0.0973 Im(B_{50})=-0.0602

Re(A_{49})=-0.00479 Im(A_{49})=-0.0257
Re(B_{49})=-0.0376 Im(B_{49})=-0.0502

The required corrector strengths are [M^{-2}]:

for p=50 S1=-0.00358 S2=-0.0106
S5=-0.0142 S6= 0.0273

for p=49 S1=-0.00422 S2=0.00628
S5=-0.0119 S6=0.00429

If we want to correct these four resonances simultaneouesly, the actual corrector strengths can be expressed as

$$S = f_p^{49} * (S)_{49} + f_p^{50} * (S)_{50} \quad ,$$

shown in the following table:

Superperiod	f_p^{49}	f_p^{50}	Actual Corrector Strength	
1	1	1	$[(S1)_{49}+(S1)_{50}]$, $[(S5)_{49}+(S5)_{50}]$,	$[(S2)_{49}+(S2)_{50}]$, $[(S6)_{49}+(S6)_{50}]$
2	1/2	-1/2	$1/2*[(S1)_{49}-(S1)_{50}]$, $1/2*[(S5)_{49}-(S5)_{50}]$,	$1/2*[(S2)_{49}-(S2)_{50}]$, $1/2*[(S6)_{49}-(S6)_{50}]$
3	-1/2	-1/2	$-1/2*[(S1)_{49}+(S1)_{50}]$, $-1/2*[(S5)_{49}+(S5)_{50}]$,	$-1/2*[(S2)_{49}+(S2)_{50}]$, $-1/2*[(S6)_{49}+(S6)_{50}]$
4	-1	1	$-[(S1)_{49}-(S1)_{50}]$, $-[(S5)_{49}-(S5)_{50}]$,	$-[(S2)_{49}-(S2)_{50}]$, $-[(S6)_{49}-(S6)_{50}]$
5	-1/2	-1/2	THE SAME AS SUPERPERIOD 3	
6	1/2	-1/2	THE SAME AS SUPERPERIOD 2	

With the choice of f_p^{49} and f_p^{50}, the sextupole correctors can be grouped into 8 families. Four of the families will reside on superperiods 1, 3, and 5 with the half of the strength in superperiods 3 and 5. Similarly, other four families will reside on superperiods 2, 4 and 6 with half of the strength in superperiods 2 and 6. Thus, we need 8 power supplies (with the capability of driving some of the corrector at half the strength) for this sextupole corrector scheme. Note, the chromaticity remains unchanged. Additionally, this correction scheme will excite the following alias harmonics:

(1) Harmonic 49

....-1, 1, 5, 7,....41, 43, 47, 49, 53,.....

(2) Harmonic 50

....-2, 2, 4, 8,....42, 44, 48, 50, 54,.....

The maximum strength of correctors required for correcting all of the above resonances simultaneouesly is $|S|<0.0316\ m^{-2}$. This leads to (for 10cm long correctors)

$$|S| < 4.21\% \text{ of the main sextupole strengths.}$$

Skew Sextupole Resonance Correction

Here we want to correct the resonances 3vy=50, vy+2vx=50 and 3vy=49, vy+2vx=49.

We put 12 skew sextupole correctors (3 sets) per one superperiod as shown in Fig. 2(d) and with the following additional conditions

SS1=SS3=-SS9 , SS2=SS4=-SS10,

SS5=SS7=-SS11 , SS6=SS8=-SS12

In order to get the resonance strengths A_n (for 3vy=n) and B_n (for vy+2vx=n, n=50,49), we consider the following random errors:

a) σ_1 =0.06 a2, random skew sextupole in dipole
b) σ_2 =0.65mrad rotation error of the main sextupoles

From 21 cases, we found the worst case resonance strengths excited by the errors to be

$Re(A_{50})=0.00921$	$Im(A_{50})=-0.0124$
$Re(B_{50})=0.0260$	$Im(B_{50})=-0.00341$
$Re(A_{49})=-0.0470$	$Im(A_{49})=-0.000623$
$Re(B_{49})= 0.00460$	$Im(B_{49})=-0.00404$

and the required corrector strengths are $[M^{-2}]$:

for p=50	SS1=-0.00455	SS2= 0.00506
	SS5= 0.00492	SS6=-0.00351
for p=49	SS1=-0.000677	SS2=0.00219
	SS5=0.00548	SS6=0.0114

If we want to correct these four resonances simultaneouesly, the actual corrector strengths can be expressed as

$$SS = f_p^{49} * (SS)_{49} + f_p^{50} * (SS)_{50} \quad ,$$

shown in the following table:

Superperiod	f_p^{49}	f_p^{50}	Actual Corrector Strength	
1	1	1	$[(SS1)_{49}+(SS1)_{50}]$, $[(SS5)_{49}+(SS5)_{50}]$,	$[(SS2)_{49}+(SS2)_{50}]$, $[(SS6)_{49}+(SS6)_{50}]$
2	1/2	-1/2	$1/2*[(SS1)_{49}-(SS1)_{50}]$, $1/2*[(SS5)_{49}-(SS5)_{50}]$,	$1/2*[(SS2)_{49}-(SS2)_{50}]$, $1/2*[(SS6)_{49}-(SS6)_{50}]$
3	-1/2	-1/2	$-1/2*[(SS1)_{49}+(SS1)_{50}]$, $-1/2*[(SS5)_{49}+(SS5)_{50}]$,	$-1/2*[(SS2)_{49}+(SS2)_{50}]$, $-1/2*[(SS6)_{49}+(SS6)_{50}]$
4	-1	1	$-[(SS1)_{49}-(SS1)_{50}]$, $-[(SS5)_{48}-(SS5)_{50}]$,	$-[(SS2)_{49}-(SS2)_{50}]$, $-[(SS6)_{49}-(SS6)_{50}]$
5	-1/2	-1/2	THE SAME AS SUPERPERIOD 3	
6	1/2	-1/2	THE SAME AS SUPERPERIOD 2	

With f_p^{49} and f_p^{50} , we can see the skew sextupole correctors can be grouped into 8 families. Four of the families will reside on superperiods 1, 3, and 5 with the half of the strength in superperiods 3 and 5. Similarly, other four families will reside on superperiods 2, 4 and 6 with half of the strength in superperiods 2 and 6. Thus, we need 8 power supplies (with the capability of driving some of the corrector at half the strength) for this skew sextupole corrector scheme. Note, this correction scheme will excite the following alias harmonics:

(1) Harmonic 49

....-1, 1, 5, 7,....41, 43, 47, 49, 53,.....

(2) Harmonic 50

....-2, 2, 4, 8,....42, 44, 48, 50, 54,.....

The maximum strength of correctors required for correcting all of the above resonances simultaneouesly is $|SS|<0.0150\ m^{-2}$. This leads to (for 10cm long correctors)

$$|SS| < 2.0\% \text{ of the main sextupole strengths.}$$

Conclusion

We present a scheme for correcting: (1) half integer stop bandwidth resonances excited by stray quadrupole fields with four power supplies; (2) sum and coupling resonances excited by stray skew quadrupole fields with four power supplies; (3) third integer stop bandwidth resonances excited by stray sextupole fields and stray skew sextupole fields with eight power supplies respectly.

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References:

[1] S. Tepikian, " The Resonance Correction Scheme For The AGS Booster", ADD Booster Tech. Note #149, Sept., 1989

[2] C. Gardner, "Effective Placement of Stopband Correction Elements in an AGS Lattice", AGS/AD/Tech. Note #321, May, 1989

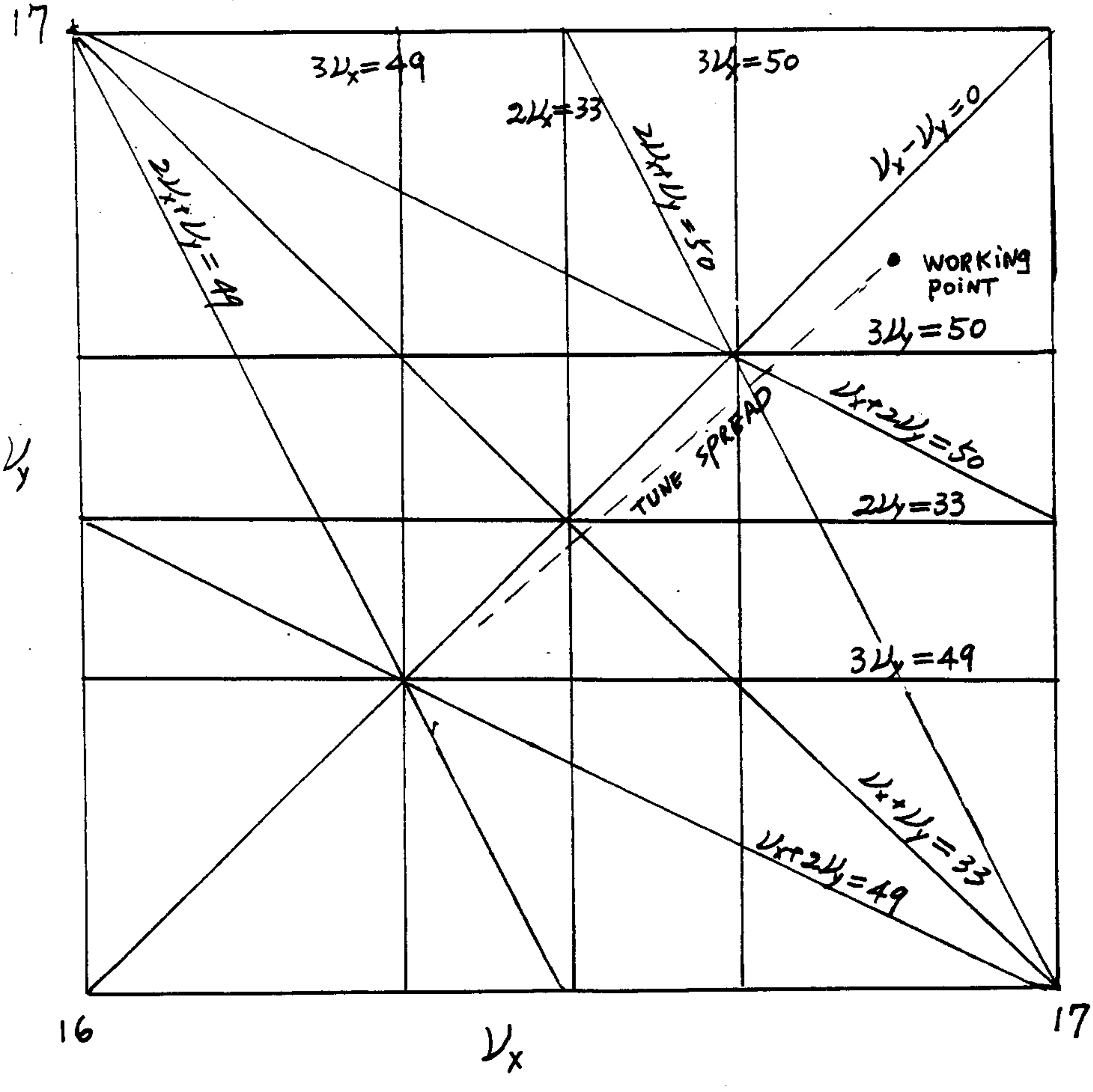

Fig. 1 Tune Diagram with the Expected Space Charge Tune Spread

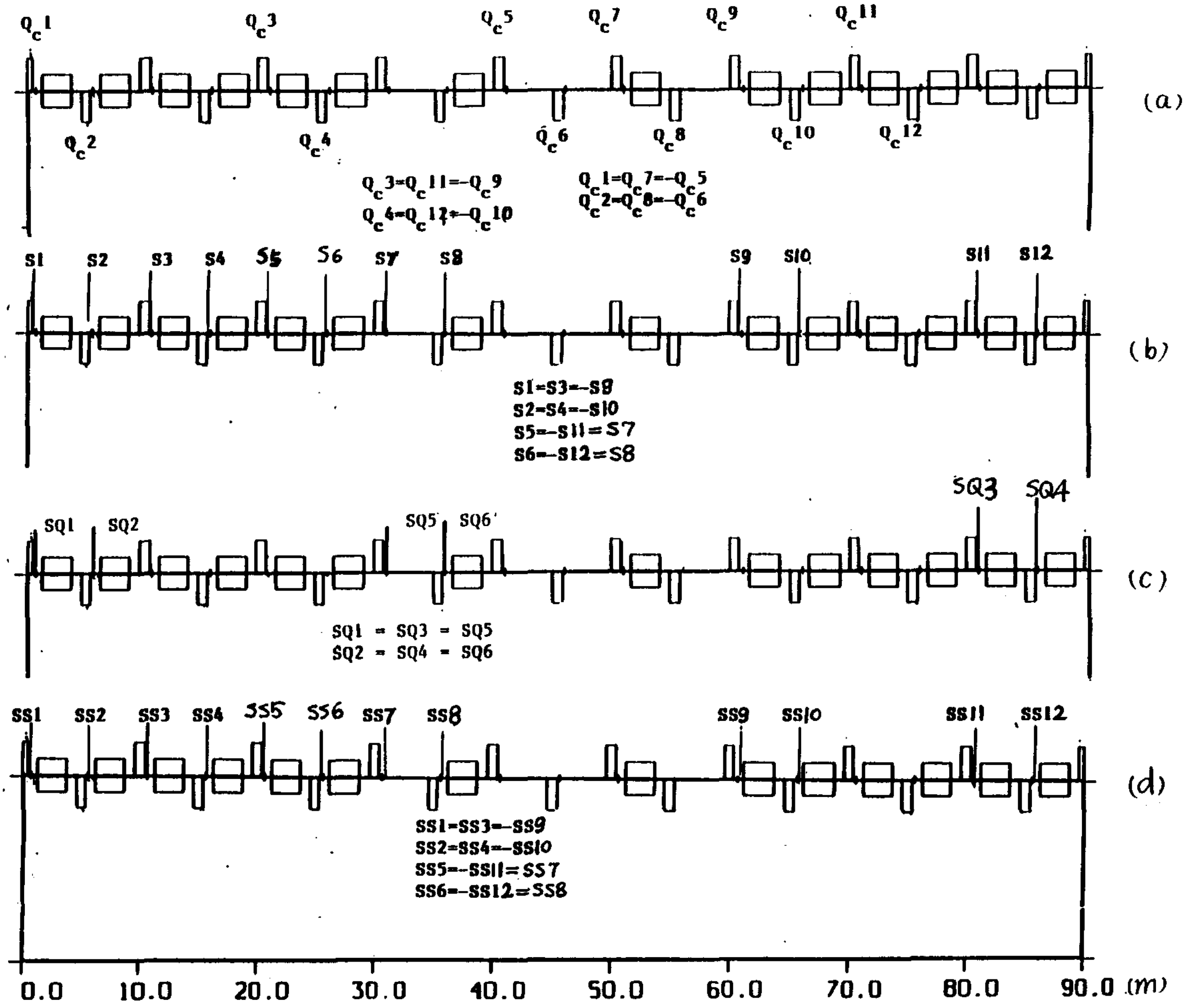

Fig. 2 THE ARRANGEMENT OF THE CORRECTORS