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SSC APERTURE DETERMINATION

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1. INTRODUCTION

The choice of magnet aperture is an important issue for the SSC. On the one hand, a larger aperture is desirable because it allows more room for the beam for its various envisioned operations, and it provides a better magnet field quality which is critical for assuring the beam stability. On the other hand, a smaller aperture means much savings to the magnet cost. It is estimated that a change of d_C, the inner diameter of the SSC dipole inner coil, by 1 cm would translate to a change of approximately 140 M$ in magnet cost.

An extensive effort to optimize the aperture for the SSC has been carried out by the CDG since 1984. The main emphases during different periods since 1984 have been as follows:

(1) October 1984 - August 1985: An Aperture Task Force was formed. The main results were the defining of the aperture evaluation algorithm, an estimation of the SSC dipole multipole errors [SSC-7], and the preliminary aperture evaluation summarized in SSC-SR-1013. A tentative choice of d_C= 4 cm was made.

(2) September 1985 - April 1986: The previous results were formulated into the CDR. Extensive theoretical studies were performed, but experimental verifications of the proposed aperture criteria were still lacking, and only 60 degree cell lattices were examined carefully.

(3) May 1986 - June 1988: On the experimental front, studies were carried out at the Tevatron, and the data were analyzed. No evidence of need to modify the aperture criteria was observed. However, the experiments studied only the 1-dimensional case. On the theoretical front, more detailed studies, summarized in SSC-SR-1024, reconfirmed that the choice of d_C of 4 cm was optimal, but indicated a slight improvement if the cell phase advance was increased to 90 degrees.

(4) Present: On the experimental front, preparation is under way for 2-dimensional experiments at the Tevatron. On the theoretical front, the priority has been the design of correction schemes assuming 90-degree cells and d_C= 4 cm.

This note is an interim status report of the progress made so far and the continuing studies. What could still affect the aperture choice at the present stage of studies and how to deal with them also are discussed.

2. STATUS

The first step in aperture optimization is to develop an algorithm. Most of this was done by the Aperture Task Force in 1984-85. A slightly updated form is summarized below:

(1) The aperture need is analysed as a function of cell lattice structure, which is specified by the cell length L and phase advance per cell μ.

(2) A set of criteria is devised to judge whether an aperture is sufficiently linear for the various beam operations. An aperture is judged to be linear if particle motion inside it is sufficiently close to its unperturbed sinusoidal motion.

(3) The expected magnet field errors are analysed as functions of d_c.

(4) A combination of L, μ, and d_c is judged acceptable if criteria (2) are fuilfilled inside the corresponding needed aperture (1). Of all the acceptable combinations of L, μ, and d_c, the one with the minimum cost (using a cost model) is the optimal cell design choice.

(5) The suggested set of criteria (2) is to be verified by a set of experiments.

(6) For a given choice of L, μ, and d_c, an optimal correction scheme to deal with the various alignment and systematic and random field errors is to be designed.

In the following, items (1) to (5) are discussed. The important item (6) is discussed elsewhere by the Correction Element Working Group.

2.1 Needed Aperture

To judge whether the magnet aperture is acceptable, it is necessary first to analyze how much aperture is needed by the beam during its various stages of operation from injection to storage. For the SSC, it is envisioned that the maximum aperture need occurs at injection because of the larger beam size, the possible injection errors, and the need for beam diagnostics and beam correction at that time. [Note however that in the present design, there is no attempt to relax the aperture need for 20 TeV beams. This is because, during storage, there is an additional effect - the beam-beam effect - and because a much longer beam lifetime is necessary during storage.]

The beam requires an aperture for four reasons [SSC-SR-1024]:

– The beam size is taken to be $\sqrt{6}\,\sigma$ (σ is the rms beam size) or 95% of beam. With a nomalized emittance of $\varepsilon_N = \gamma\sigma_x\sigma_{x'} = 1.0$ mm-mrad, this gives 1.46 mm in a cell with L = 228.5 m and μ = 90 degrees (as in the 9/87 lattice, SSC-146). For different cell structures, it scales with $\sqrt{\beta_{max}}$.

- The maximum injection error is estimated to be 1.63 mm for the case of the 9/87 lattice (or 1.5 mm for the CDR lattice). This need also scales with $\sqrt{\beta_{max}}$.
- The rms closed orbit error at β_{max} is estimated to be 0.43 mm for the CDR lattice. The corresponding aperture need is taken to be 3 times the rms value, i.e. 1.29 mm [SSC-SR-1024]. Similarly for the 9/87 lattice, the need is 1.16 mm. In the range of interest, this need does not depend sensitively on β_{max}.
- Miscellaneous effects such as lattice mismatching, power supply ripple and noise accounts for 0.52 mm for the 9/87 lattice (or 0.5 mm for the CDR lattice). About half of this amount is envisioned to scale with $\sqrt{\beta_{max}}$ and the other half with η_{max}.

By adding up these four components linearly, one obtains the total 1-dimensional aperture need, which is a function of L and μ through their dependences on β_{max} and η_{max}. Figure 1 shows the result.

Note that the above analysis assumes the collider is dedicated for pp colliding beam operation. In particular, no provisions are made for p-pbar or fixed-target operations. Such provisions, if included, would increase the needed aperture.

Note also that the above aperture need is for on-momentum particles. After the beam is injected, it is envisioned that one may need to vary the beam momentum to measure chromatic effects. When doing so, the extreme momentum deviation of some particles may be as large as $\delta = 10^{-3}$, as analyzed in SSC-N-263. For this extreme case, the allowance for injection error is no longer needed because the beam has already been stored. Removing the injection contribution, the resulting off-momentum aperture need is shown in Figure 2.

From Figs. 1 and 2, the needed on-momentum and off-momentum apertures for the 9/87 lattice are 4.76 mm and 3.13 mm, respectively. These values are evaluated at a location with β_{max} of 388 m. To some extent, what counts from here on is the total needed aperture. Exactly how it is decomposed into the four differential needs is not too critical. Some trade-off among the four contributions could be envisioned, if desirable pending on more studies.

2.2 Aperture Criteria

The needed aperture estimated in section 2.1 must have the property that the motion of a particle inside the region (1) is stable for a large number (10^{7-8}) of turns, and (2) can be predicted sufficiently accurately using the designed, unperturbed, linear lattice during actual operations. For these reasons, a concept of "linear aperture" was introduced in SSC-22 and is followed in the SSC aperture analysis. This represents a more conservative approach than previous storage ring designs, and is justified by its large cost implications on the SSC.

Presumably, the requirements (1) and (2) mentioned above can be reached if the particle motion is sufficiently linear. In a storage ring with error multipoles, the motion of a particle with a small betatron amplitude is linear, i.e. it executes a simple harmonic motion with a unperturbed tune of ν_0 and its turn-by-turn phase space trajectory traces out a circle. As the amplitude is increased, its motion deviates from linearity in two ways. First, its tune deviates from ν_0 and second, its phase space trajectory is either distorted or smeared around a circle. In the linear aperture algorithm, it is suggested that particle motion inside the needed aperture be sufficiently linear by imposing two criteria:

- tune shift from ν_0 is less than ±0.005, and
- rms deviation of turn-by-turn phase space trajectory from a circle (this quantity is called the rms smear) is less than 6.4% (corresponding to a peak-to-peak smear of ±15%).

These criteria are not meant to be rigorous statements applicable for nonlinear dynamical systems in general. However, they are believed to serve as practical, conservative design criteria for the SSC error multipoles. The choice of the values 0.005 and 6.4% is somewhat arbitrary. The value 0.005 is consistent with the tune spacing between adjacent resonances of relatively low order, and is of the order of magnitude as a tolerable beam-beam tune shift. It also means the beam injection error can be damped in time before emittance grows by filamentation. The 6.4% value of smear is believed reasonable in terms of understanding the particle motion using perturbation theories (which has been demonstrated), and in terms of long lifetimes (which are supported by tracking simulations for 10^4 turns and limited tests with 10^6 turns). The full justification, however, has to include experimental studies, to be discussed in section 2.5. Figure 3 combines the requirements of the needed aperture and the linearity criteria.

Note that v_0 must be chosen to avoid low order resonances. It is in fact only after doing so that the linear aperture criteria are meant to be applicable. In practice this means the working point v_0 must be adjustable during operation over a reasonable range to search for an optimum.

In addition to the two linear aperture criteria mentioned above, there is a supplemental condition on the "dynamic aperture", which is defined here to be the stable amplitude obtained by simulating a small number (typically 400) of turns. The condition is that the dynamic aperture is to be at least 30% larger than the linear aperture. This is to assure some elbow room needed for fast, crude beam diagnosis during injection operations. It also has the purpose of providing safety margin for the linear aperture to indeed provide long beam lifetimes - the simulation of which is unpractical. Obviously the physical aperture due to the vacuum chamber pipe has to be larger than the dynamic aperture.

The magnet aperture d_c also affects the vacuum pipe radius, which in turn affects the impedance seen by the beam and therefore its collective instabilities. In the range of interest, however, this is a weak effect. For example, the parasitic heating varies from 0.7 kw for $d_c = 4.5$ cm to 1.1 kw for $d_c = 2.5$ cm (Ref. SSC-SR-1024), which are much smaller than the synchrotron radiation heating of 18 kw (totals of both rings).

2.3 Magnet Errors

The linear and dynamic apertures available to the beam in the SSC are limited by the magnetic quality of the dipole magnets. (The errors of the quadrupole magnets in the arc regions are not expected to be important. Errors of the triplet quadrupoles are important, but they do not impact directly on the dipole aperture issue.) The field error is specified by the multipole coefficients a_n and b_n with

$$\Delta B_y + i\,\Delta B_x = B_o \sum_n (b_n + i\,a_n)(x + i\,y)^n.$$

Both the systematic and the random components of the error multipoles are of importance. To estimate the random geometric errors in the SSC dipoles, the SSC Aperture Task Force analyzed existing data on superconducting magnets -- from some 800 Tevatron dipoles and about 12 CBA dipoles -- and projected them to the SSC design [SSC-7]. Although the coil sizes of these magnets were quite different (d_c = 7.6 cm for Tevatron, and 13.2 cm for CBA), the two sets of data were quite compatible, so that extrapolation to the SSC coil size (in the neighborhood of 4 cm) could be done with fair confidence. It was felt that these estimates should be conservative in that advances in magnet technology had not been factored into the extrapolation. Subsequent short models of the SSC dipoles also indicate that these estimates have been reasonable.

These projected rms (random) variations in the multipole strengths due to geometric effects are listed in Table 1. Two values are listed for the quadrupole rms's $\sigma(a_1)$ and $\sigma(b_1)$. The first values (in parentheses) are the raw extrapolated values; the second (smaller) values are those that seem achievable if the technique of coil centering within the iron yoke after initial measurement is used.

The geometric systematic errors also were extrapolated from the measurements on the Tevatron dipoles, and are listed in Table 1. Note that the projected values of the allowed multipoles (b_2, b_4, b_6, . . .) could be smaller for the SSC dipoles, since they reflect the cross section design in the Tevatron dipoles and not the more elaborate design in the SSC dipoles.

There are in addition three other sources of magnetic errors: persistent-current magnetization, yoke-saturation effects, and coil distortion due to magnetic forces. The persistent-current multipoles expected in the SSC dipoles at injection based on measurements of 13 short model magnets also are listed in Table 1. Yoke saturation effects can produce a change in the normal sextupole coefficient b_2 ranging from about +1 to -2 units, depending on the design of the iron yoke. A good estimate can not be made until the yoke design is complete. Coil distortion due to magnetic forces at high field strengths are estimated to produce a change in b_2 of about -1 unit. However, the exact value depends on the mechanical support of the magnet collars by the iron yoke, which is a design issue to be settled. Experimentally the sextupole strength due to mechanical distortion is difficult to distinguish from yoke-saturation effects because they both occur mainly at high field strengths. Measurements on prototype magnets will be used to establish the strength of the correctors needed.

Having projected the SSC dipole errors, a set of specifications was established as shown in Table II. It was established basically by taking the projected values and relaxing some of them (especially the higher order multipoles) to a tolerable level. This relaxation did not affect the linear aperture, and the effect on the dynamic aperture, although noticeable, is small.

Table II is the present updated set of dipole specifications. It must be emphasized that it is not the final specifications, and it is to be modified by studies taking into account of magnet data as they are accumulated. These modifications could be made by making trade-offs among the multipoles, or by changing the complexity of the correction schemes. It may also change if the lattice cell structure changes.

In the appendix, a more detailed discussion of the origin of the dipole magnet multipole specifications,including the specification on magnet ends, is given.

Table I

Projected Magnetic Errors in the SSC Dipoles

Except for the persistent-current multipoles, these strengths are scaled from measurements on the Tevatron dipoles. The persistent-current strengths are based on measurements of SSC model dipole magnets scaled to 6-μm filaments and 2750 A/mm^2 critical current at 4.2 K and 5 T. Units are 10^{-4} B_0 at 1 cm. (B_0 is dipole field strength.) n denotes the 2(n + 1) multipole.

	Skew coef. a_n		Normal coef. b_n		
			systematic		
n	systematic	random (rms)	geom.	persist.	random (rms)
1	0.21	(3.3)0.7	0.11	–	(1.6)0.7
2	0.11	0.61	0.45	−7.6	2.0
3	−0.07	0.69	−0.14	–	0.35
4	−0.05	0.14	−0.33	0.8	0.59
5	−0.02	0.16	−0.24	–	0.06
6	−0.04	0.03	1.57	−0.2	0.08
7	0.05	0.03	0.009	–	0.02
8	−0.01	0.01	−2.1	–	0.02

Table II
Tolerance Specifications for SSC Dipoles
Geometric Systematic and Random Multipole Errors

	Skew coef. a_n		Normal coef. b_n	
n	systematic	random (rms)	systematic	random (rms)
1	0.2	0.7	0.2	0.7
2	0.1	0.6	1.0	2.0
3	0.2	0.7	0.1	0.3
4	0.2	0.2	0.2	0.7
5	-	0.2	0.04	0.1
6	-	0.1	0.07	0.2
7	–	0.2	0.1	0.2
8	–	0.1	0.2	0.1

2.4 Optimization

It turns out that the main effect of the random multipoles is to contribute to the smear, while the systematic multipoles give rise to tune shifts. Our strategy has been to deal with the random multipoles first. The cell parameters L and μ and magnet aperture d_c are thus chosen to meet the smear criterion. Having determined L, μ and d_c, the tune shift due to systematic multipoles are dealt with by the design of a correction system. In this report, only the optimization using random multipoles and the smear criterion is discussed. We are particularly making the following assumptions of the correction system design in the rest of the report:

(1) The systematic multipoles and the tune shift criterion are under control.

(2) The effective random b_2 has been reduced by a factor of 5 either by magnet sorting or by a binning technique.

(3) The effects due to random a_1 and b_1 errors are compensated for (by a proper distribution of quadrupole and skew quadrupole correctors) so that they do not contribute to the smear.

An extensive search for an optimal combination of L, μ and d_c was carried out in SSC-SR-1024. The parameters range considered was as follows: L = 160 to 260 m (corresponding to 8 - 14 dipoles per cell with each cell 17 m long), μ = 60 and 90 degrees, and d_c = 3.5 to 5 cm. For each case studied, the linear aperture determined by the condition rms smear = 6.4% is calculated. Depending on the sample of the random multipoles, this linear aperture has a mean value and a certain statistical spread. To allow for sufficient confidence level, the "achieved" linear aperture is taken to be 2 sigma below the mean. This achieved linear aperture is then compared with the needed aperture of the beam. A case is judged acceptable if the achieved aperture is larger than the needed one.

A cost model is then devised. The cost of the SSC is modeled as a function of L, μ and d_c. The cost is calculated for all the acceptable cases previously identified. In principle, the optimal design is the one that gives the minimal cost. But a broad optimum was found and a range of cases were identified to be close to the optimum for the SSC design. These are:

10 dipoles/cell, 60 degrees, 4 cm aperture (the CDR case)

8 dipoles/cell, 60 degrees, 3.5 cm aperture

10 dipoles/cell, 90 degrees, 3.5 cm aperture

12 dipoles/cell, 90 degrees, 4 cm aperture (the 9/87 case)

Among these alternatives, the 9/87 case was judged to be slightly preferred over the others and was recommended as the optimal combination of parameters for the SSC.

Figure 4 shows the linear and the short-term dynamic apertures for the 9/87 case in terms of a quantity $A_{tot}=(A_x^2+A_y^2)^{1/2}$, where A_x and A_y are evaluated at the β_{max}=388m in their respective planes. The solid curve represents the mean linear aperture obtained by a perturbation calculation. The agreement with simulations (error bars represent statistical spreads) is quite reasonable. Perturbation theory of course does not predict dynamic apertures.

Table III summarizes the results. As promised, the "achieved" linear aperture (2 sigma below the mean) is above the needed aperture. The dynamic aperture determined from 400 turn tracking is >30% larger than the linear aperture, as intended.

Table III

Comparison of the linear, the dynamic and the needed apertures for the 9/87 lattice.
The aperture values refer to A_{tot}. For reference, the rms beam size in the A_{tot} unit is 0.85 mm.

	dynamic	linear	needed	
$\delta = 0$	15.0 ± 0.5	10.6 ± 2.0	6.6	mm
$\delta = \pm 10^{-3}$	13.2 ± 0.8	9.6 ± 0.9	4.4	mm

2.5 E778

As mentioned before, an experiment to verify the SSC aperture criteria has been initiated. A collaboration with Fermilab, Cornell, SLAC and CERN resulted in the Experiment E778 at the Tevatron. So far, 15 shifts were taken in March-May 1987 and 12 shifts taken in February 1988. The main purposes are: (1) to check if predictions made by the theoretical tools agree with experiments, and (2) to see if the aperture determined to be acceptable using the suggested SSC criteria indeed allows satisfactory beam operations. These two purposes led to two types of E778 experiments.

In one type, a beam scraped down in emittance is first kicked to a desired amplitude while a set of intentional sextupoles are turned on to mimic the more nonlinear SSC environment. The subsequent turn-by-turn beam orbit signals at two adjacent position monitors are taken and analysed to give the smear and the tune shift corresponding to the kicked amplitude. These results are compared with those obtained by the theoretical tools used in the SSC design. The agreement of this type of experiments has been excellent. Figures 5 and 6 show some samples of these results. The SSC criterion that smear = 6.4% at 4.7mm amplitude corresponds to a sextupole setting of 12 amperes. The criterion that tune shift = 0.005 at 4.7mm corresponds to a sextupole setting of 14 amperes.

In the second type of experiment, the beam is injected with the sextupoles turned on. The point is to observe any possible difficulties during injection operation. It was found that the injection operation did not suffer from the sextupoles until their setting is set to about 30 amperes or more, which is beyond that corresponding to the SSC criteria.

Based on these studies, it was concluded that so far there has been no experimental evidence that the SSC aperture criteria need to be modified. As a consequence, the 4 cm aperture choice seems adequate.

However, experiments so far have concentrated on the 1-dimensional case only. The beam was kicked horizontally and only the horizontal smear and horizontal tune shift have been studied. A closer examination of the aperture criteria will have to be 2-dimensional -- although the experiment and data analysis would be more involved. For this reason, another round of shifts in spring of 1989 at the Tevatron is being proposed. Basically the idea is to perform similar experiments for the 2-D case.

3. REMAINING ISSUES

So far we have a self-consistent aperture evaluation algorithm and the 4 cm choice is consistent with the algorithm. But several considerations must be remembered before drawing that conclusion. These are listed below:

(1) The 2-dimensional E778 will have to be designed in more detail, to be approved by Fermilab and to be executed, hopefully in the spring of 1989. A confirmation of the aperture criteria in the 2-D case would mean one more strong support for the present aperture evaluation procedure. If the results are otherwise, the criteria will have to be modified accordingly. There has been some aperture experiments being performed at SpS. It would be a good idea to follow up on their results and draw useful information for our purposes.

(2) A safety margin has been explicitly included in the aperture algorithm. The "achieved" linear aperture has been taken to be the mean minus 2 sigma. But is this safety margin sufficient as suggested? Some format to examine this issue would be very important.

(3) Magnets are assumed to meet their specifications as listed in the parameters list, and particularly the multipole specifications listed in Table II. Note that those specifications refer to the total errors, regardless of their sources including measuring errors and temperature variations. Errors due to quadrupole magnets and dipole ends must be included.

(4) The rest of the collider systems, particularly the correction schemes, are assumed to function as designed. The orbit and injection errors and the beam emittance are under control so that the beam stays inside the specified region, i.e. 4.7 mm in x and y. The random quadrupole and skew quadrupole components are controlled by a distribution of quadrupole and skew quadrupole correctors. The random sextupole components are reduced by a factor of 5 by a binning or sorting technique. Finally, all systematic multipoles are to be controlled by a correction scheme distributed in the arcs.

(5) The present aperture assumes that we are not considering the possibility of later upgrades like p-pbar or slow spills for fixed-target operation.

Having listed the considerations above, the question is: what if it is found at a later time that it is desirable or necessary to make the design more conseravtive than the present one, and yet it is "too late" to change d_C? To address that question, one must note that the magnet aperture is not the only parameter that determines the issue. To some extent, the SSC can still be made to function smoothly with a fixed d_c by changing other parameters. The penalty is of course the design is no longer the optimal, leading to cost increases. These alternatives are listed below:

(1) A shorter cell means smaller needed aperture and a reduction in the nonlinear effects of the error multipoles. Thus shortening the cell has an effect similar to enlarging the magnet aperture. The limitation here is that as cells shorten, the chromaticity sextupoles become strong, especially for 90 degree cells. If much shorter cells are to be considered, we may want to go back to 60 degree cells.

(2) If random multipoles become a problem (e.g. exceeding their specifications), one may consider introducing more extensive sorting/binning schemes. At present, only the random b_2 components are corrected by binning. This leaves the possibility of sorting available for any random multipoles other than b_2 without introducing additional correctors. If the random b_2 exceeds its specification, more bins could help.

(3) If persistent current effects are more serious than expected, raising injection energy could be an alternative.

(4) Tighter injection errors, orbit errors and smaller beam emittance would help to relax the needed aperture.

To make the design more conservative, one must analyse what is the first limiting bottleneck to be encountered during operation. Depending on the result of the analysis, the answer could be increasing d_c, shorter cells, higher injection energy, or a different correction scheme. The present design attempts to make these various considerations self-consistent, and hopefully optimized. A similar effort will be needed in the continuing aperture studies.

Appendix

In this appendix, we attempt to summarize the origin of the multipole tolerance specifications for the dipole magnets. These specifications represent a compilation of results of several studies. Not all material in this appendix are relevant to the aperture studies discussed in the text, but they are provided for information nevertheless.

The tolerance values of the random and systematic multipoles, given in table II, refer to an integrated multipole strength in units of 10^{-4} B_0 at 1 cm radius. The integration is over the entire magnet, including the ends, and the tolerance values refer to this integral divided by the magnetic length (16.54 m) of the magnet.

As mentioned in the text, the random multipoles in Table II meets the proposed linear aperture requirements. They were basically those listed in Table I, which was obtained by projecting from the Tevatron and CBA magnets, except that

- a_1 and b_1 tolerances are set at a lower level than the projected values, expecting a better handling of these magnet errors by the SSC dipoles, and
- higher order multipoles ($a_{6, 7, 8}$ and $b_{5, 6, 7, 8}$) were rounded off upwards to 0.1 or 0.2 units. Which higher multipoles were rounded off to 0.1 and which to 0.2 units were somewhat arbitrary (compare tables I and II) and trade-offs among them could be made. The

rounding off does not change the linear aperture, and changes the dynamic aperture only slightly. The rounding off is motivated by the fact that magnet measurements may not have the accuracy better than 0.1 or 0.2 units.

The systematic multipole specifications in Table II were based on several inputs. As mentioned in the text, in the present SSC design, the systematic multipoles were to be controlled by the multipole correctors. Their tolerance specifications come from the following considerations:

- If possible, the systematic multipole specification should be larger than a fraction of the random rms specification. This is anticipating that, during magnet production, one may be forced to determine the systematic multipole values based on a relatively small sample of dipoles. If the sample has 25 dipoles, for example, the systematic multipole can be determined only to a statistical accuracy of 1/5 of the rms random multipole values. This observation led to the values of systematic $a_{1,2,3}$ and $b_{1,3,4}$.
- The specification for b_2 is somewhat more relaxed than that determined by the above statistical rule (it is 1/2 the rms random counterpart) and is somewhat tentative. This is because there is a sextupole corrector scheme that handles the persistent current sextupoles, which has a much larger values at injection. The limitation to 1 unit systematic b_2 is also influenced by considering the corrector strength at 20 TeV.
- It is assumed that proper multipole correctors are available in the SSC design to compensate for the a_1, b_1 and b_4 effects.

- Whether there needs to be a systematic b_3 correction scheme in the SSC design depends on whether the magnets meet the systematic b_3 tolerance specified.
- The systematic skew multipoles must be such that they do not cause too large a tune shift or linear coupling coefficient, including off momentum particles. [SSC-N-163] By splitting the x- and y-tunes by ± an integral, these tolerances can be relaxed, yielding tolerance values of $a_{2,\ 3,\ 4}$ in Table II.
- The systematic tolerances of $b_{5,\ 6,\ 7,\ 8}$ were obtained by the condition that tune shifts with amplitude and momentum be less than ±0.005 in the needed aperture region. The large systematic b_6 and b_8 in the projected values (Table I) were assumed to be specific to the Tevatron magnets and are correctable for the SSC. [Note the $b_{5,\ 6,\ 7,\ 8}$ values in SSC-N-183 were for 60 degree cells. The change to 90 degree cells relaxes these tolerances to those given in Table II.]

In Table IV, we have listed the specifications associated with the magnet ends. These specifications have not been considered to impact on the aperture issue. In this table, all values have the unit of "unit-meters" because they refer to integrated values. The numbers in the column labelled "2 ends + body, systematic" are just those systematic values of Table II multiplied by 16.54 m.

To make sure the random multipoles due to magnet ends are negligible compared with the magnet bodies, we require that the magnet ends contribute less than 1/3 of the rms random multipole tolerances given in Table II. The numbers under "each end, rms" in Table IV are thus obtained by taking 1/3 of the random values of Table II, multiplying by 16.54m, and dividing by 2 because there are two ends. The only exception is for b_2, which is obtained by the same recipe, but starting not with the specified 2.0 units in table II, but with o.4 units specified with binning taken into account.

The tolerances of systematic multipoles in the magnet ends are obtained by a separate consideration. [SSC-N-406] Having specified for the "body + 2 ends, systematic" case, these "each end, systematic" specifications are to make sure to avoid a situation in which large errors in magnet bodies are compensated by large errors of opposite sign in the ends. Although this is allowed to some extent, there exists tolerances on how much this compensation does not introduce subtle beam dynamics defects. The criteria used leading to these specifications in Table IV is that the quantities $\Sigma_{1,2,3,4,5,6}$ are all smaller than 0.02, where Σ_n is the sum of all resonance widths of the n-th order (in tune units) at an amplitude of 1 cm.

Table IV.

Specifications of Multipole Errors in the Dipole Magnet Ends

$$(A_n \text{ or } B_n) = \int^{16.54m} ds\,(a_n \text{ or } b_n)$$

∫ ds multipole	random tolerance each end (rms)	systematic tolerance each end	systematic tolerance 2 ends + body
A1	1.9	3.	3.3
A2	1.7	6.	1.7
A3	1.9	1.	3.3
A4	0.6	1.4	3.3
A5	0.6	2.	-
B1	1.9	3.	3.3
B2	1.1	14.	17.
B3	0.8	5.	1.7
B4	1.9	6.	3.3
B5	0.3	0.2	0.66

FIGURE CAPTIONS

Fig. 1. The needed aperture (1-D, on-momentum) as a function of cell structure. The cases for CDR and 9/87 lattices are indicated.

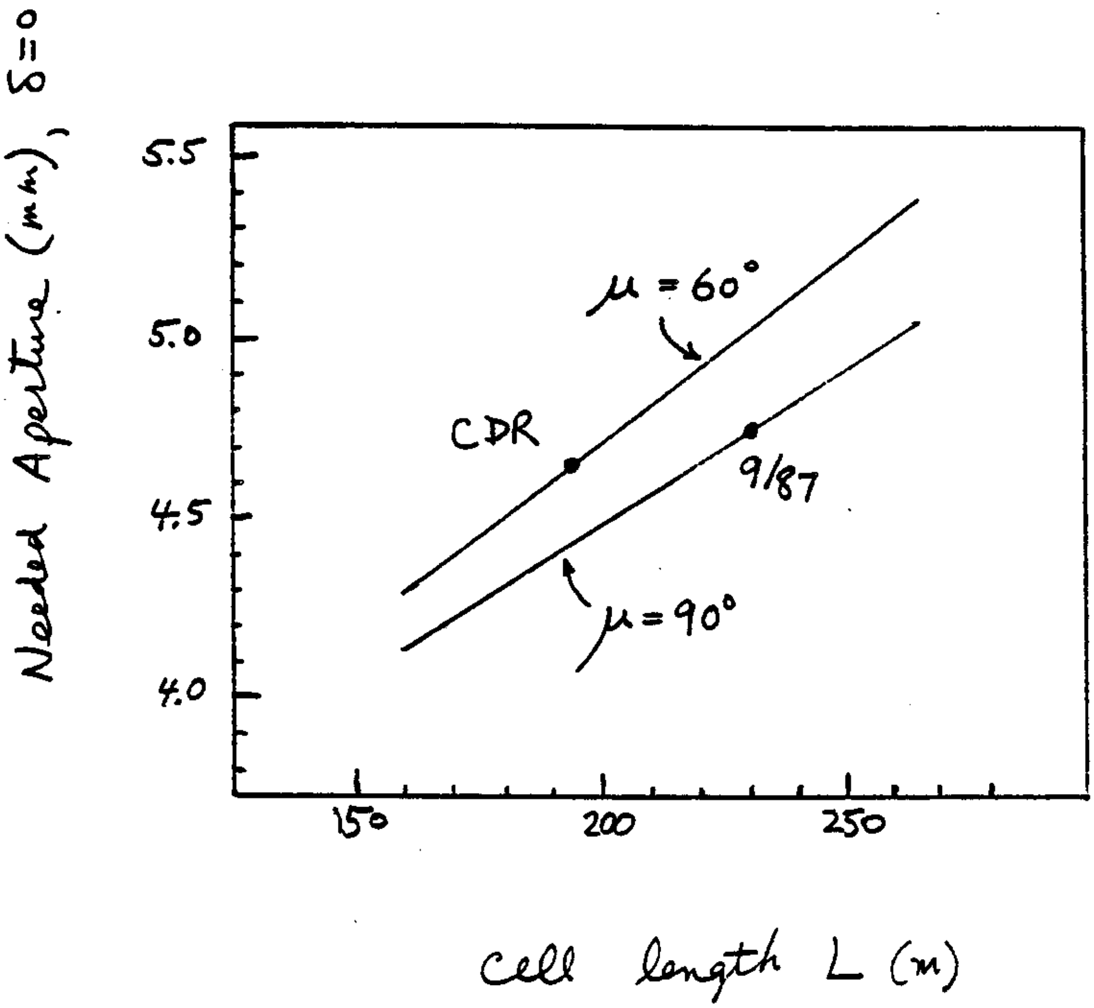
Fig. 2. The needed aperture (1-D, off-momentum) as a function of cell structure.

Fig. 3. The SSC design requires that the tune shift < 0.005 and the rms smear $< 6.4\%$ in the shaded regions. This figure shows the requirement for the 9/87 lattice and for $\delta = 0$ and $\pm 10^{-3}$. The amplitudes A_x and A_y are evaluated at $\beta_{max} = 388$ m.

Fig. 4. Linear and dynamic apertures for the 9/87 lattice

Fig. 5. RMS smear as a function of sextupole strength. The 5, 8 and 10 kv labels refer to the kicker strength and 10 kv corresponds to a kick amplitude of 4.5 mm. Solid curves are theoretical predictions. Diamonds are E778 data points.

Fig. 6. Tune shift with betatron amplitude with sextupoles set at 25 amperes. Solid curve is the theoretical prediction and crosses are E778 data.



Needed Aperture (mm), δ=0
5.5
5.0
4.5
4.0
μ = 60°
CDR
9/87
μ = 90°
150
200
250
cell length L (m)

Figure 1

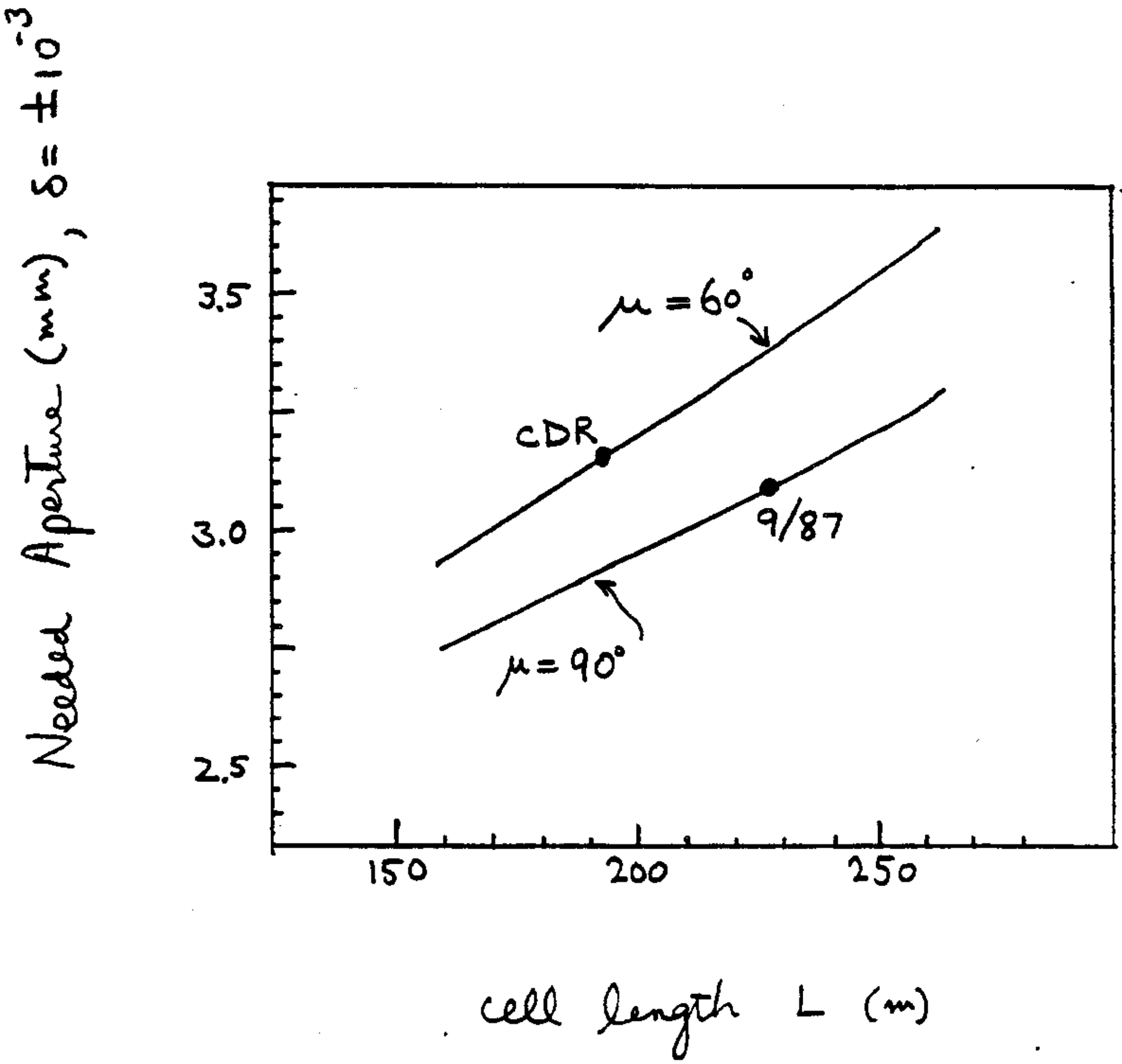
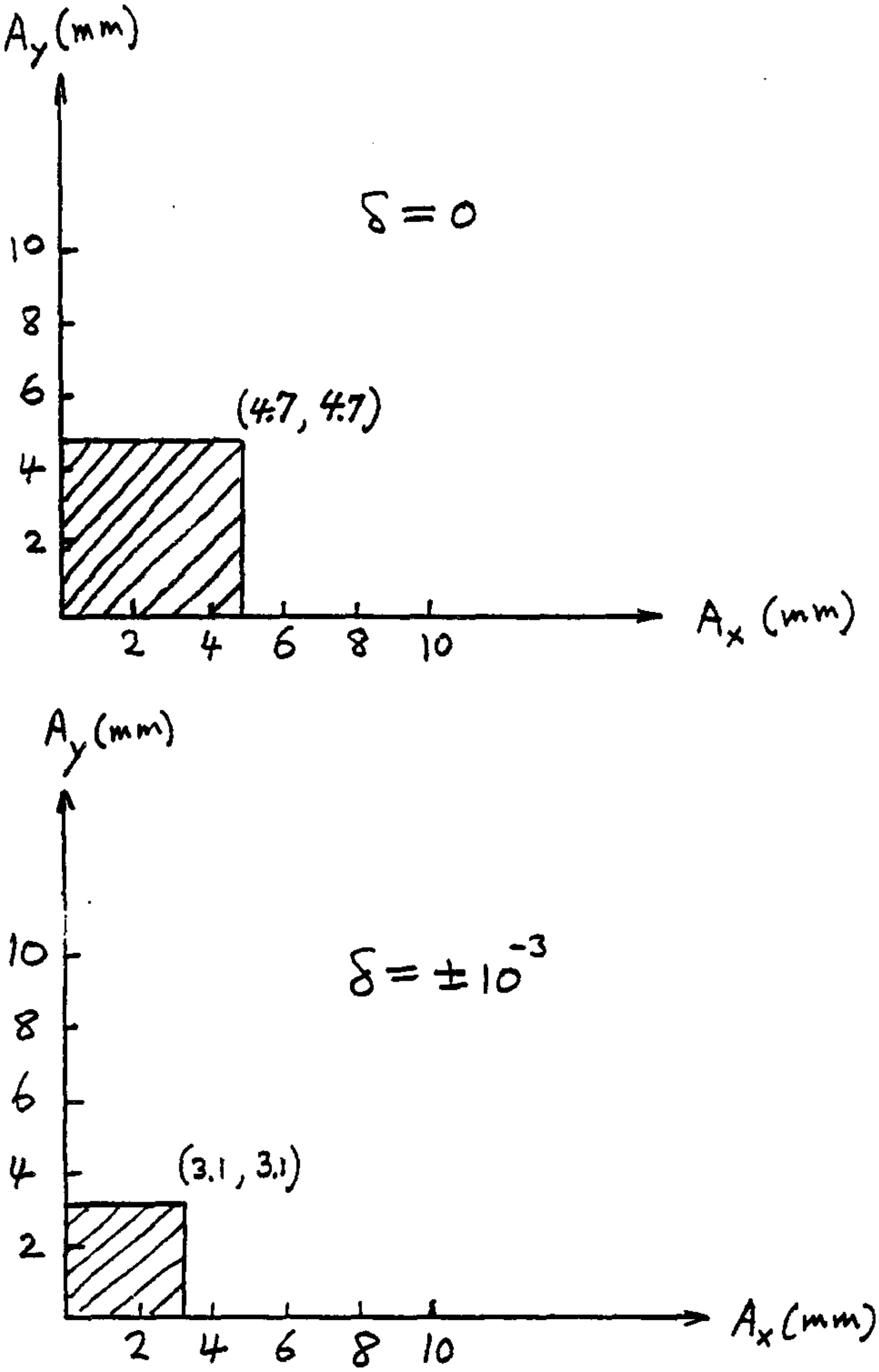
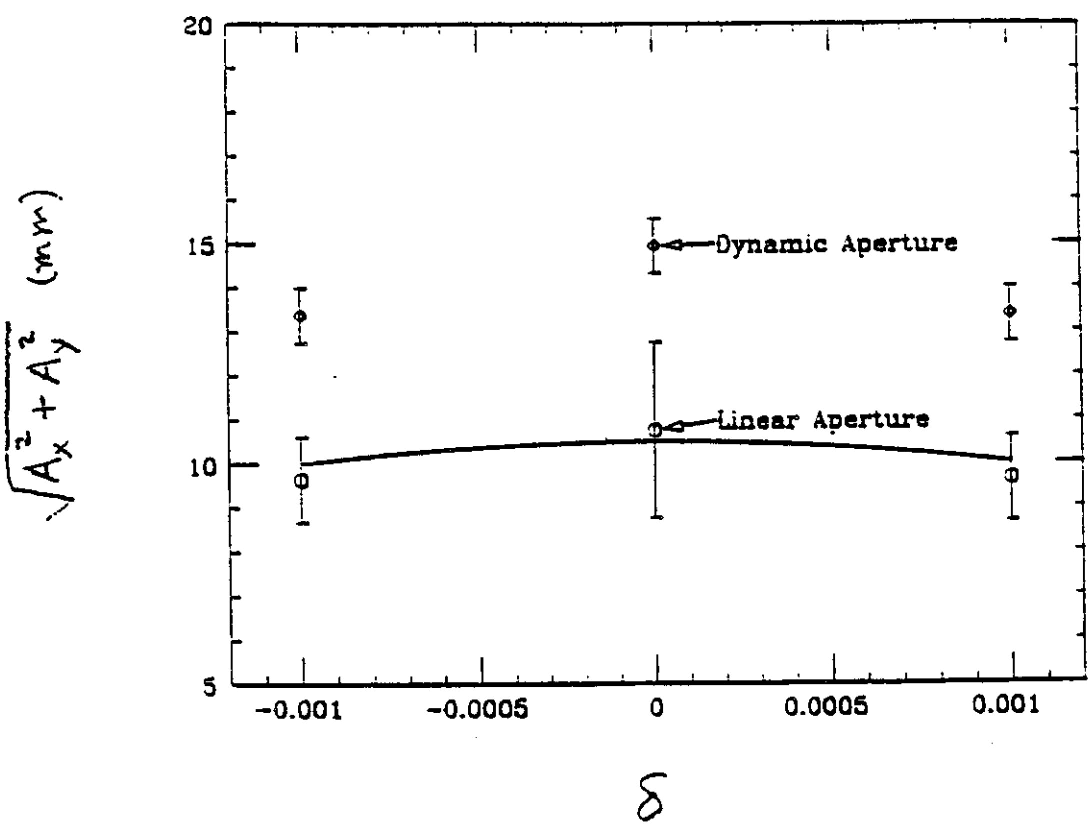

Needed Aperture (mm), δ = ±10⁻³
3.5
3.0
2.5
150
200
250
μ = 60°
μ = 90°
CDR
9/87
cell length L (m)

Figure 2



A_y (mm)
δ = 0
10
8
6
4
2
(4.7, 4.7)
2
4
6
8
10
A_x (mm)
A_y (mm)
δ = ±10^-3
10
8
6
4
2
(3.1, 3.1)
2
4
6
8
10
A_x (mm)

Fig. 3



Dynamic Aperture
Linear Aperture
20
15
10
5
-0.001
-0.0005
0
0.0005
0.001
$\sqrt{A_x^2 + A_y^2}$ (mm)
δ

Fig 4

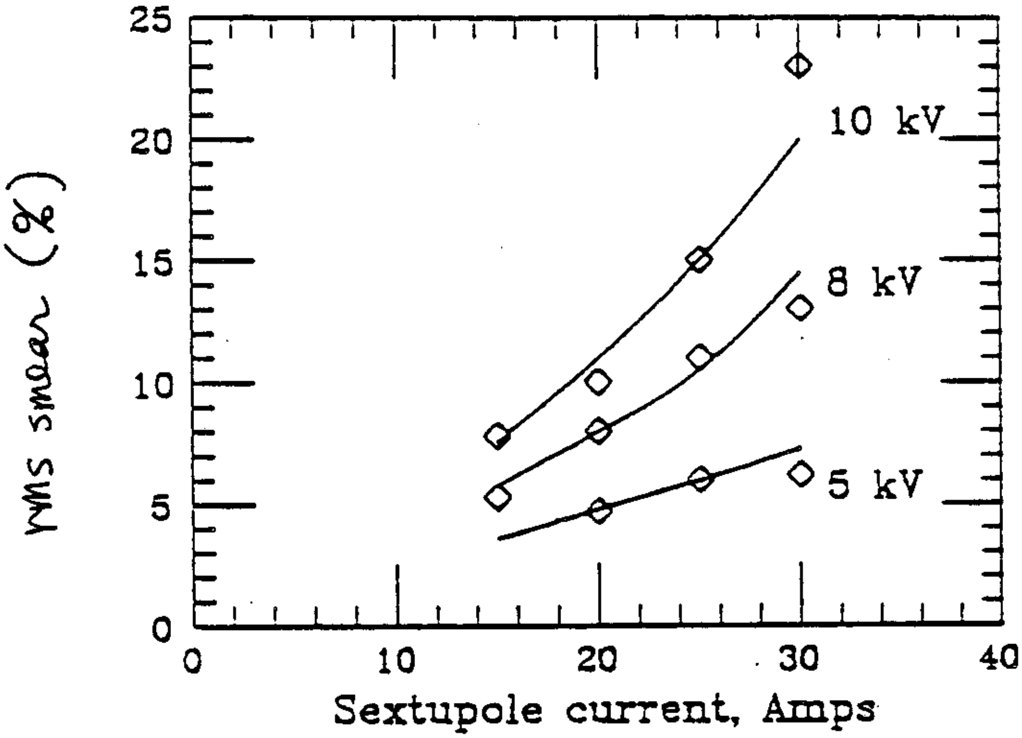

rms smear (%)
Sextupole current, Amps
10 kV
8 kV
5 kV
0
5
10
15
20
25
0
10
20
30
40

Fig. 5

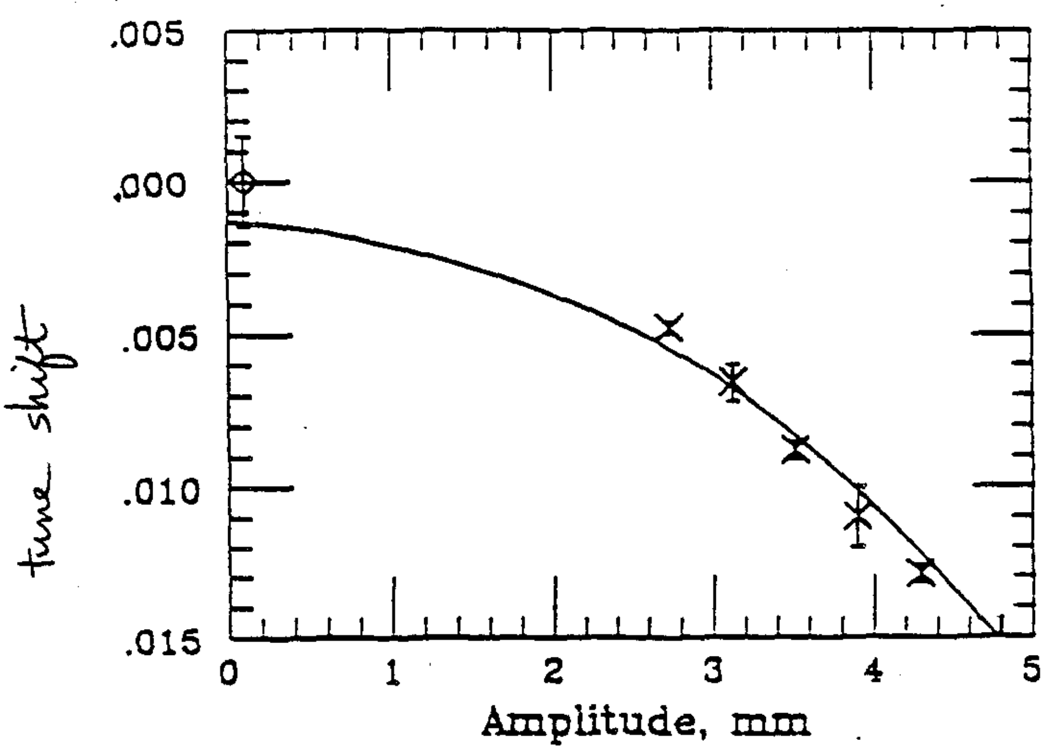


Fig. 6