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ASYMMETRIC NONLINEAR FIELD CORRECTION WITH 5-DIPOLE HALF-CELLS

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SUMMARY

The modification of (F,C,D) correction for use in a 5-dipole half-cell lattice is described; "Simpson's Rule" corrector strengths are perturbed to match the asymmetric geometry. First-Order correction of nonlinearities is similar to the symmetric case. Second-Order correction is degraded by 10-20%. Tuneability is also degraded to a similar degree, and closed-orbit sensitivity is magnified by a similar degree of loss of quasi-locality. As the correction method reduces nonlinear effects by orders of magnitude in both cases, this degradation due to asymmetry should be tolerable.

INTRODUCTION

(F,C,D) correctors[1,2,3] have been found to be extremely effective in the compensation of nonlinearities. The method requires insertion of nonlinear elements in the center (C) of half-cells, which would require the existence of an inter-dipole slot at the half-cell centers. Currently lattices with 4, 5, and 6 dipoles/half-cell are being considered. While the half-cell center is accessible in 4 and 6 dipole lattices, a five-dipole lattice has a dipole at the center and the corrector must be displaced (see Figure 1).

Figure 1 shows a suggested geometry for corrector placement in a 5-dipole cell. A C corrector is placed on one side of the center dipole, which means that it is displaced from the desired central location by ~10% of a half-cell length (9 m in the 90 m SSC lattice). Correctors are also placed near the quadrupoles. Figure 1 shows correctors on both sides of the quads; in practice these would be combined in units on either side of the quads. There would be only two physical elements per half-cell.

First-Order tune shifts depend on integrals of powers of betatron and dispersion functions (β_x, β_y, η) and all of these functions are reflection-symmetric about the quadrupoles in a matched FODO lattice. Assuming this reflection symmetry, "the three-point" system (Figure 1) is equivalent to a four-point half-cell correction system when averaged over a full cell, with virtual correctors at L=0, 0.4L, 0.6L, L. This 4-point system (Figure 2) has a Simpson's Rule - like integration rule, which is exact through third-order. The weighting rule is:

$$f_D = f_F = \frac{11}{72}, \qquad f_{C_1} = f_{C_2} = \frac{25}{72}$$

Thus integration-rule correction strengths of the correctors would be:

$$S_F = S_D = \frac{22}{72} B_n L = 0.30555... B_n L$$

$$S_C = \frac{50}{72} B_n L = 0.69444... B_n L.$$

These values are not greatly different from the $(\frac{1}{3}, \frac{2}{3})$ values of Simpson's Rule; the geometry is a relatively small perturbation of the Simpson's Rule geometry.

Tables (1 & 2) show correction of first-order octupole and decapole tune-shifts, in a reference Supercollider FODO lattice (L=90m, ϕ=90° Lattice). Correction by ~2 orders of magnitude is obtained for both symmetric and asymmetric cases. The asymmetric case provides slightly better first-order correction in this idealized matched lattice. This is expected since, to first-order, the asymmetric 2-3 (5-dipole) case uses a 4-point integration rule, which should be better than the 3-point rule, although the "improvement" is small.

Unlike the symmetric case, the 2-3 case does not provide complete "quasi-local" cancellation [4] of multipole content at the half-cell level. The degree of asymmetry implies loss of quasi-locality at ~10% level. This implies that second-order effects and closed-orbit sensitivity will be enlarged at that level.

In Table 3 we show calculations of second-order sextupole tune shifts for the two cases. With symmetric correction the tune shifts are ~20% smaller, implying a ~10% larger linear aperture or nonlinear "tolerance." This agrees with the above qualitative discussion.

Closed orbit and lattice perturbation sensitivity should be enlarged by the partial loss of quasilocality. With quasilocality, only the "local" perturbations (within half-cells) contribute ; "global" (multicell) perturbations do not contribute and these are expected to be an order of magnitude larger, the ~10% loss of quasilocality implies global errors at that level contribute. The net effect should be similar to the local perturbation level and this is not expected to be large, particularly for b_3 and b_4. Further study, including particle tracking studies for long-term stability, would be useful to confirm this *ad hoc* discussion.

Separable F, C, D tuneability is somewhat degraded by the asymmetric placement, and this capability is particularly useful for the octupoles. The two C correctors per cell could, in principle, be separable but in practice are not clearly separable and should be run in series. The average of the displaced C correctors is close to the effect of the centered C. The critical factors for octupole tune shift are β_x^2, $2\beta_x\,\beta_y$, β_y^2, and the separability factor

$$S = \left\langle \frac{2\beta_x(C)\,\beta_y(C)}{\beta_x^2(C)} \right\rangle,$$ averaged over C_1 and C_2 locations, is reduced by only 7%.

Adequate (F, C, D) separability is maintained.

The ability to correct random multipole content quasilocally[3] is also slightly degraded by the asymmetric placement but the correction is still possible, degraded by only $\lesssim$ 10%.

Tables 4, 5, 6 show similar calculations for a variant lattice, a weaker-focusing case with ϕ=72°. Advantages are shorter, cheaper quads and ~40% superior ability to control chromaticity. While linearity before correction is somewhat degraded, the linearity after correction is the same. The additional case shows the general applicability of the correction method, in symmetric or asymmetric forms, and suggests a possible design variation.

SUMMARY

Asymmetric placement of the Center (C) corrector in (F, C, D) correction to fit a 5-dipole half-cell has been considered. While some correction features are degraded at the ~10% level from the symmetric case, correction by orders of magnitude is still obtained. The correction is very desirable in either case and will greatly increase the linear aperture.

REFERENCES

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3. E. Forest and J. M. Peterson, Proc. 1988 European Particle Accelerator Conference.

4. D. V. Neuffer and E. Forest, Phys. Lett. **A135**, 197 (1989).

TABLE 1
CORRECTION OF OCTUPOLE*

($L=90$, $\theta = 0.4431°$, $\phi = 90°$, $b_4 = 10^{-4}\ cm^{-3}$)

[0.5 cm, $\frac{\Delta p}{p} < 0.001$, $\Delta\nu \lesssim \pm 0.005$]

Corrector Conditions

Symmetric (F, C, D) Geometry	$\Delta\nu_{max}$	Correction Factor	"Tolerance" (Units)
(0,0,0)	0.136	1	0.037
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.0019	76	2.6
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.0026	52	1.9
Chromaticity Optimized $(.1627, \frac{2}{3}, .1633)$	0.00127	107	3.9
Overall Optimized (.166, .6545, .166)	<0.00025	>550	>20
2 - 3 Asymmetric (F, $\bar{C}$, D) Geometry			
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.0050	27	1.0
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.0015	90	3.3
Chromaticity Optimized $(.1496, \frac{50}{72}, .15)$	0.0010	135	5.0
Overall Optimized (.152, .684, .1525)	<0.0002	>700	>20

*L is the half-cell length, θ is the bend per half-cell, ϕ is the phase advance/cell. The "Correction Factor" is the ratio of maximum corrected to maximum uncorrected tune shifts within the design aperture. "Tolerance" is the maximum b_n for which the maximum tune shift is $\leq \pm 0.005$. A unit is $\bar{b}_n = 10^{-4}\ cm^{-n}$.

TABLE 2
CORRECTION OF DECAPOLE

($L=90$, $\theta = 0.4431°$, $\phi = 90°$, $b_4 = 10^{-4}\ cm^{-4}$)

[0.5 cm, $\frac{\Delta p}{p} < 0.001$, $\Delta\nu \lesssim \pm 0.005$]

Corrector Conditions

Symmetric (F, C, D) Geometry	$\Delta\nu_{max}$	Correction Factor	"Tolerance" (Units)
(0,0,0)	0.65	1	0.077
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.00217	30.0	2.3
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.00174	37.4	2.9
Chromaticity Optimized			
$(.1627, \frac{2}{3}, .1633)$	0.000196	332	2.6
Optimized			
(.159 .661, .169)	<0.00003	>1800	>170
2 - 3 Asymmetric F, $\bar{C}$, D) Geometry			
(0,0,0)	0.65	1	0.077
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.0043	15.1	1.2
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.00174	37.4	2.9
Chromaticity Optimized			
$(.1454\ \frac{50}{72}, .155)$	0.00051	127	9.8
Optimized			
(.1467, .6893, .155)	<0.00003	>1800	>170

TABLE 3
SECOND-ORDER SEXTUPOLE TUNE SHIFTS
(L=90,m $\theta = 0.4431$, $\phi = 90°$ Lattice, $b_2 = 10 \cdot 10^{-4}$)
[0.5 cm, $\frac{\Delta p}{p} < 0.001$, $\Delta\nu \lesssim \pm 0.005$]

Corrector Conditions	$\Delta\nu_{max}$	"Tolerance"
F, D only	0.040	3.5
Symmetric F, C, D; $C = \frac{2}{3}$	0.0056	9.45
Symmetric F, C, D; C = 0.6	0.0051	9.9
2 - 3 Asymmetric F, C, D; $C = \frac{50}{72}$	0.0070	8.45
2 - 3 Asymmetric F, C, D; C = 0.6	0.0063	8.9

TABLE 4
CORRECTION OF OCTUPOLE
(L=90 m, θ = 0.4431, ϕ=72°; $b_3 = 10^{-4}$)

$[0.5\text{ cm}, \frac{\Delta p}{p} < 0.001, \Delta\nu \lesssim \pm 0.005]$

Corrector Conditions

Symmetric (F, C, D) Geometry	$\Delta\nu_{max}$	Correction Factor	"Tolerance" (Units)
(0,0,0)	0.236	1	0.021
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.00115	205	4.3
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.00313	75	1.6
Chromaticity Optimized			
$(.1644, \frac{2}{3}, .1659)$	0.00065	360	7.7
Overall Optimized			
(.166, .663, .166)	*<0.0002*	*>1200*	*>25*
2 - 3 Asymmetric F, $\bar{C}$, D) Geometry			
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.0050	47	1.0
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.00092	255	5.4
Chromaticity Optimized			
$(.151, \frac{50}{72}, .1521)$	0.00051	460	9.8
Optimized			
(.152, .6915, .152)	<0.0002	>1200	>25

TABLE 5
CORRECTION OF DECAPOLE
(L=90 m, θ = 0.4431, ϕ =72°, $b_4 = 10^{-4}$)

$[0.5 \text{ cm}, \frac{\Delta p}{p} < 0.001, \Delta\nu \lesssim \pm 0.005]$

Corrector Conditions

Symmetric (F, C, D) Geometry	$\Delta\nu_{max}$	Correction Factor	"Tolerance" (Units)
(0,0,0)	0.134	1	0.037
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.0020	67	2.5
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.0022	61	2.3
Chromaticity Optimized			
(.1611, $\frac{2}{3}$, .1702)	0.00043	310	12
Optimized			
(.1614 .6677, .1668)	<0.00005	>2700	>100
2 - 3 Asymmetric F, $\bar{C}$, D) Geometry			
$(\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$	0.0050	27	1.0
$(\frac{11}{72}, \frac{50}{72}, \frac{11}{72})$	0.0016	84	3.1
Chromaticity Optimized			
(.1483, $\frac{50}{72}$, .1556)	0.00035	380	14
Optimized	<0.0002	>700	>20

TABLE 6
SECOND-ORDER SEXTUPOLE TUNE SHIFTS
(L=90, $\theta = 0.4431°$, ϕ=72)
$[0.5\ \text{cm}, \frac{\Delta p}{p} < 0.001, \Delta\nu \lesssim \pm 0.005]$

Corrector Conditions	$\Delta\nu_{max}$	"Tolerance"
F, D only	0.040	3.5
Symmetric F, C, D; $C = \frac{2}{3}$	0.0091	7.4
Symmetric F, C, D; C = 0.6	0.0079	8.0
Asymmetric F, C, D; $C = \frac{50}{72}$	0.0110	6.7
Asymmetric F, C,D; C = 0.6	0.0090	7.45

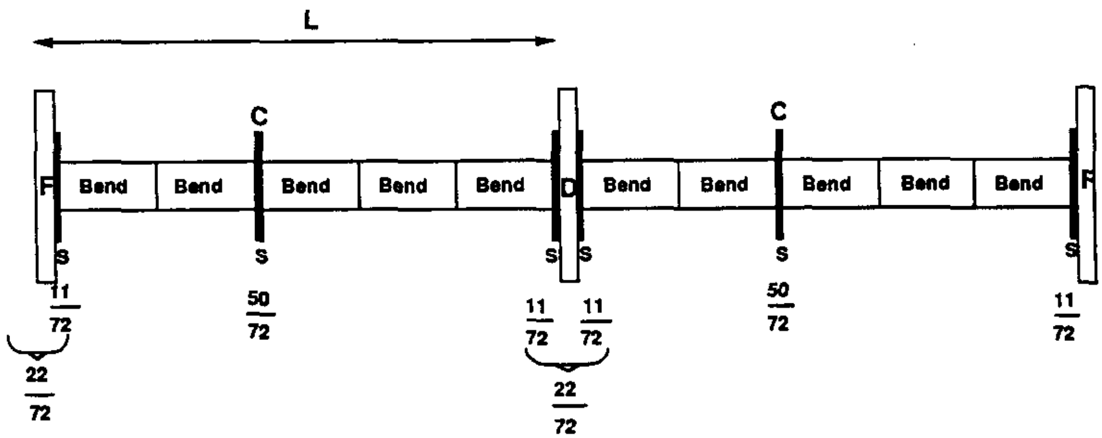

Figure 1

An SSC cell with 5 dipoles/half-cell and displaced center correctors. Correctors on opposite sides of quads would be combined in units on either side.

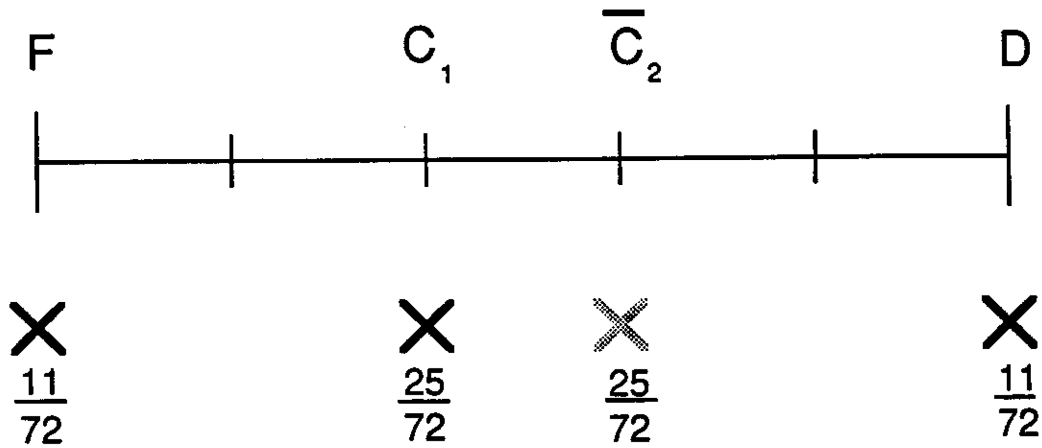

Figure 2

Reflection of the center corrector location about the D quad obtains virtual corrector locations F, C_1, C_2, D. Integration Rule corrector strengths for this virtual geometry are also shown.