SSC-N-664

The Superconducting Super Collider

Recent SSC Dynamic Aperture Measurements from Simulations

Lindsay Schachinger and Yiton Yan

SSC Laboratory

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ABSTRACT

In this note, we describe in detail the ingredients for a recent set of dynamic aperture measurements for the SSC. Two configurations, both composed only of FODO cells, were tracked for 10,000 turns. The first configuration had six dipoles per half-cell and values for systematic multipole errors in the dipoles appropriate for an injection energy of 1 TeV, the second 4.5 dipoles per half-cell and a 2 TeV injection energy. All simulations included random multipole errors in the dipoles, quadrupole and dipole misalignments and rotations, and misaligned beam position monitors.

Introduction

Recently, we have begun the first systematic study of long-term stability in the SSC with many different sources of nonlinearity included in the simulations. The purpose of this note is to spell out clearly which nonlinearities and errors were included and with what strengths, how many appropriate corrections were made, and to present the results of the first set of simulations.

Two linear lattices have been studied so far. Both were entirely composed of FODO cells with 90° phase advance. The first we call BCDR, after,[3] since it is similar to that in the CDR. It employs bore-tube correctors in the dipoles to correct the systematic multipoles b_2, b_3, and b_4; it has six 17-meter dipoles per half-cell (for a half-cell length of approximately 115 m); the errors are appropriate for 1 TeV injection energy and 4 cm dipole aperture; and binning[4] is used to correct the random b_2. The second we call 4.5DIP; it has 4.5 17-meter dipoles per half-cell (for a half-cell length of approximately 89 m); it uses only the chromaticity sextupoles to correct the systematic b_2 and no correctors for systematic b_3 or b_4; the errors are appropriate for 2 TeV injection energy and 4 cm dipole aperture; and no realistic correction is applied in the simulations for random b_2; but it is assumed that one will be found which will reduce its value by a factor of 5, thus the random b_2 used is artificially reduced from its expected value by this factor.

The tracking is done for 10,000 turns (with one case extended for 100,000 turns), with synchrotron oscillations, using CRAY XMP in MFE. A post-Teapot tracking program, Ztrack, is developed for vectorized multi-particle tracking. Ztrack reads in a machine file generated from Teapot and assigns initial particle coordinates and then tracks the particles simultaneously element by element. One hundred and twenty-eight particles with initial amplitude equally distributed between 6 mm and 12 mm in x-coordinate are tracked for each case. Six cases, three seeds for each of BCDR and 4.5DIP, are studied. If a particle is lost, Ztrack will reduce the particle number after the lost particle information is stored.

Alternatively, one can choose to substitute a new particle for each particle as it is lost so as to maintain 64 or a multiple of 64 particles to take the best advantage of the CRAY XMP vector process.

The Lattices

The BCDR and the 4.5DIP lattices are given in detail in standard format in Appendices A and B. The linear BCDR lattice consists of 320 90° cells, each 229.592 m long, and the tunes are set to $Q_x = 81.285$, $Q_y = 82.265$. The 4.5DIP lattice consists of 426 90° cells, each 177.572 m long, and the tunes are set to $Q_x = 107.285, Q_y = 106.265$.

Steps in Preparing the Machines for Tracking

Several steps are involved in preparing a Teapot [2] file describing the machine ("machine file") for tracking on the CRAY. The first is the addition and correction of multipole errors, followed by the addition and correction of closed orbit errors. Finally, the tunes are set to their desired values, the chromaticities are set to zero and a final machine file is written which is then transferred to the CRAY where the RF cavity is added and particles are tracked. The steps up to the creation of the final machine file will now be described in some detail for both lattices which have been studied so far. We will include Teapot statements which show exactly what was included in each case.

1. Addition and correction of random multipole errors

After the linear lattice is defined, with the correct tunes and chromaticities, random multipole errors are added to the lattice. For the BCDR case, the errors added are:

```
! random errors
errors, HABNDM, sigb2=2.0, sigb3=0.30e2, sigb4=0.70e4, &
                siga2=0.6, siga3=0.70e2, siga4=0.20e4, &
                seed = XXX, cut = 6
errors, HABNDP, sigb2=2.0, sigb3=0.30e2, sigb4=0.70e4, &
                siga2=0.6, siga3=0.70e2, siga4=0.20e4, &
                seed = XXX, cut = 6
cewg, bin=7, scheme=bcdr
```

The multipole sigmas are given as a fraction of the bend field at one meter. All distributions are cut at six sigma (cut = 6). Though the bends are cut in half, each half gets the same random errors. This is accomplished by the command cewg, which sets the random errors in each half of a dipole equal, and also sets the bore tube correctors so that the random b_2 is corrected, using a binning [4][3] scheme with 7 bins.

For the 4.5DIP lattice, the random errors are:

```
! random errors
errors, HABNDM, sigb2=0.4, sigb3=0.30e2, sigb4=0.70e4, &
                siga2=0.6, siga3=0.70e2, siga4=0.20e4, &
                seed = XXX, cut = 6
errors, HABNDP, sigb2=0.4, sigb3=0.30e2, sigb4=0.70e4, &
                siga2=0.6, siga3=0.70e2, siga4=0.20e4, &
                seed = XXX, cut = 6
! this just sets errors equal in two half dipoles
cewg, bin=0, scheme=none
```

The difference here is that the sigma for the random b_2 is simply reduced from 2 to 0.4 units, assuming that some correction scheme will later be found. In this case the cewg command does nothing other than set the randomly generated errors equal in the two halves of each dipole.

2. Addition and correction of systematic multipole errors

For systematic multipoles, the multipole values differ between the two lattices studied. The BCDR lattice has systematic multipoles in the dipoles as follows:

```
! systematic errors
errors, HABNDM, sigb2=7.4, sigb3=0.1e2, sigb4=0.64e4, &
         sigb6=-0.13e8, systematic=1.0
errors, HABNDP, sigb2=-7.4, sigb3=-0.1e2, sigb4=-0.64e4, &
         sigb6=0.13e8, systematic=1.0
```

Here we see that the bore-tube correctors are set by setting the "errors" in one half dipole to cancel those in the other half dipole. HABNDM contains the b_2 bore-tube corrector, and HABNDP the b_3 and b_4 ones. Although the command defines "sigmas", the errors in this case are actually the same in each dipole of the given name since at the end of the **errors** command there is the keyword **systematic**.

For the 4.5DIP case, the systematic errors are:

```
! systematic errors
errors, HABNDM, sigb2=-3.0, sigb3=0.0e2, sigb4=0.20e4, &
 sigb6=-0.05e8, systematic=1.0
errors, HABNDP, sigb2=-3.0, sigb3=0.0e2, sigb4=0.20e4, &
 sigb6=-0.05e8, systematic=1.0
```

Here all the systematic errors have been reduced to the values for 2 TeV injection. It was decided for these initial studies to set b_3 to zero for simplicity, since the correct value is uncertain. In this case there are no bore-tube correctors, so both half of the dipoles have the same systematic errors. The chromaticity sextupoles are later used to correct the linear chromaticity.

3. Addition and correction of errors which produce closed orbit deviations

The next step is the addition of errors which produce closed-orbit deviations. These include random steering errors (siga0 and sigb0), dipole rotations, dipole misalignments, quadrupole misalignments and rotations, and BPM misalignments. These all have the same values for both the BCDR and 4.5DIP cases, as follows:

```
!add orbit errors
errors, habndm, habndp, &
        siga0 = 8.34e-4, sigb0 = 12.0e-4, &
        sigtheta = 0.85e-3, sigx = 1.4e-3, sigy = 1.4e-3, &
        seed = XXX, cut = 5
errors, hqf90a, hqd90a, &
        sigx = 1.4e-3, sigy = 1.4e-3, &
        seed = XXX, cut = 5
bpmerrors, bpm, &
        sigxwrtq = 0.1e-3, sigywrtq = 0.1e-3, &
        seed = XXX, cut = 5
```

Since the bends and quadrupoles are cut in half, the values for their errors have been multiplied by $\sqrt{2}$. For the BCDR case, the relative misalignment between the two dipole halves means that the bore-tube correctors are effectively misaligned, also. This results in a feed-down effect which is probably too big, since the bore-tube correctors could probably be aligned to better than 1.4 mm rms. However, analytical calculations show that this effect contributes less than 1% to the smear.[6] In future simulations this could be eliminated by adding the closed orbit errors before issuing the command **cewg** which sets the errors in both halves of the dipoles equal. This would only work for the dipoles, though. The quadrupoles could be put back together to form one, single, whole quadrupole.

The BPM errors are with respect to the adjacent quadrupole, so the total BPM misalignment is the sum of the misalignment of the adjacent quadrupole and the misalignment of the BPM with respect to that quad. The rms misalignment with respect to the adjacent quad is 0.1 mm.

The orbit is then corrected as described in the references.[5] A final rms orbit deviation (in both the horizontal and vertical planes) of 1 mm is requested of

the orbit correction program. The program then iterates on the orbit until this rms deviation is achieved. We have checked that the resulting orbit agrees very well with an analytic calculation, and thus the orbit is fully corrected (i.e. there is negligible residual betatron oscillation). All orbit deviations quoted are with respect to the reference (or design) orbit.

4. Retuning

Both lattices are then retuned, so that the tunes are as given earlier, and the linear chromaticities are set to zero using the chromaticity sextupoles. Before this is done, the linear chromaticities are a few units, and the tune shifts are on the order of 0.005, depending on the seed.

Tracking

Six final machine files, 3 seeds for each of the BCDR and 4.5DIP, were prepared on a Sun workstation and then transferred to a CRAY for long-term tracking using a post-Teapot vectorized tracking program named Ztrack. Ztrack reads in a machine file and a "namelist" command file. The machine file describes the final nonlinear lattice in detail, while the command file tells how one wants the tracking to be performed, such as how many particles, how many turns, how particle coordinates are initialized, how particles are handled once they get lost, etc. All six machines are studied for 10,000 turns with a RF cavity for synchrotron oscillation. For each machine, 128 particles are tracked with initial horizontal (x-axis) amplitudes equally distributed between 6 mm and 12 mm with respect to the closed orbit. The corresponding initial vertical (y-axis) coordinates are assigned such that

$$\frac{y}{\sqrt{\beta_y}} = \frac{x}{\sqrt{\beta_x}} ,$$

where β_x and β_y are the beta functions at injection. The initial momentum deviations are all zero. The initial time-of-flight deviations are all three times the

rms half bunch length; that is, the initial time-of-flight deviations are 16.7 cm for the BCDR and 11.8 cm for the 4.5DIP. After the particle coordinates are initialized, the particles are tracked turn by turn for 10,000 turns. On each turn, they are tracked first through the RF cavity to update the particle energy (synchrotron oscillation only, no acceleration) and then through each element in the machine file. Once a particle's amplitude gets too large, it is "lost". The number of turns survived and the initial amplitude are stored for each lost particle and the total number of particles will be reduced after sorting the particle coordinate arrays. A restart file that includes both the alive-particle and lost-particle information are dumped each 1,000 turns for protecting against machine hardware problems or for tracking beyond 10,000 turns, if necessary.

Results

Particle-survival charts for the 10,000-turn tracking are shown in Figures 1(a,b,c) for the three BCDR cases (identified as BCDR-a, BCDR-b, and BCDR-c hereafter) and in Figures 2(a,b,c) for the three 4.5DIP cases (identified as 4.5DIP-a, 4.5DIP-b, and 4.5DIP-c). Each point in the charts represents a particle with its initial amplitude in the x coordinate on the horizontal axis and turn number at loss on the vertical axis. The dynamic apertures vary somewhat from case to case. For the BCDRs, the 10,000-turn dynamic apertures are about 7.5 mm and 7 mm for BCDR-a and BCDR-b, respectively; but unknown for BCDR-c since all particles are lost before 10,000 turns are reached in this case. For the 4.5DIPs, the 10,000-turn dynamic apertures are 7 mm, 6 mm, and 6.5 mm for 4.5DIP-a, 4.5DIP-b and 4.5DIP-c, respectively.

Besides the dynamic aperture measurement, the spread (called survival spread hereafter) traced out by the points in each of the particle-survival charts is also very interesting. The left margins (as shown in the figures) of the survival spreads are almost straight for most of the cases. By extrapolation one would predict small dynamic apertures for 10,000,000 turns (life time of the SSC injection lat-

tice). To test such an extrapolation or to measure the dynamic aperture for 10,000,000 turns directly, one would simply track particles for 10,000,000 turns. However, this is essentially impossible with currently available computers. So the case 4.5DIP-b was chosen and an additional 64 particles, with initial amplitude spread equally between 5 mm and 6.4 mm in the x coordinate, were tracked for 100,000 turns. The particle-survival chart is shown in Figure 3. The dynamic apertures for 1,000 turn, 10,000 turn, and 100,000 turn are 7.3 mm, 6 mm, and 5.4 mm, respectively. The left margin of the survival spread is no longer straight, but rather curved to favor a larger extrapolated dynamic aperture at 10,000,000 turns.

Conclusions

With the supercomputer vector processor, we were able to track hundreds of particles element by element through the SSC lattice for 10,000 turns without much difficulty. The particle-survival charts as shown in Figures 1 and 2 display the dynamic aperture for less than 10,000 turns. By extrapolation one may also try to predict the dynamic apertures for 10,000,000 turns, but the results are not reliable. The 100,000-turn particle-survival chart shown in Figure 3 allows one to make a better prediction of the dynamic aperture at 10,000,000 turns, but one would always question the extrapolation reliability. Since 10,000,000 turn element by element tracking for the SSC lattice is essentially impossible, other tracking methods which are necessarily less accurate are being pursued. A brief description of such fast tracking modeling activity currently going on in the SSC Laboratory can be found in Reference 7. However, to make sure the results from fast tracking are reliable, we suggest that one compare the results with detailed element by element tracking results (perhaps up to 100,000 turn) such as those we have presented in this paper.

Acknowledgments

Many thanks to Vern Paxson, who not only designed and wrote the orbit-correction program, but also set up a system whereby these runs could be automated up to the transfer to the CRAY. Helpful discussions with Jack Peterson and Rae Stiening and support and encouragement from Alex Chao are much appreciated.

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6. Johan Bengtsson, private communication.
7. Y. Yan, "Supercomputing for the Superconducting Super Collider," SSC-239, 1989; also to be published in MFE.

Appendix A – BCDR Lattice

```
TITLE  !
CDR, Bore-tube correctors
! SSC 90 DEGREE CELL LATTICE, AUG 6, 1987
!  Values of PI and C, the speed of light
 PARA, PI        = 3.14159265358979
 PARA, CLIGHT    = 2.99792458D8
!  Energy in eV
 PARA, ENERGY    = 2.0D13
!  Magnetic rigidity (T-m)
 PARA, BRHO = 6.671281903963D4
!  Total number of horizontal dipoles
 PARA, NBENDS    = 3840
!  Dipole magnetic field length (metres)
 PARA, LBFIELD   = 16.54
!  Bending radius IN DIPOLES
 PARA, RHO       = (NBENDS * LBFIELD) / (2.0 * PI)
!  Dipole bend angle
 PARA, BANGLE    =  LBFIELD / RHO
!  Dipole field (Tesla)
 PARA, BFIELD    = BRHO / RHO
!  Magnetic length of arc quads
 PARA, LQFIELD   = 3.64*230.0/200.0
!  Overall length of spool pieces (m)
 PARA, LSPOOL    = 5.57
```

```
!  Magnetic length of spool piece elements (m)
 PARA, LSPOOLEL  = 0.001
!  Length of non-magnetic ends of quadrupoles and dipoles (m)
 PARA, LEND      = 0.4
 PARA, LBTOT = LBFIELD + 2.0*LEND
!  Beam position monitor length (m)
 PARA, LMONITOR  = 0.2
!  Half cell length
 PARA, LHCELL = LSPOOL + 6*LBTOT + (LQFIELD + 2*LEND + LMONITOR)
!  Nominal strengths of dipoles and quadrupoles
!  90 degree cell quadrupole strength (metres**-2)
 PARA, KF90F     =   3.0113080d-3
 PARA, KF90D     =   3.0315935d-3
!  Spool chromatic sextupole strengths (metres**-3)
 PARA, K2HSPOOL =  2.0*0.23563325419d1
 PARA, K2VSPOOL =  2.0*0.47186113779d1
!  Field free end, and beam position monitor ----------
 DEND:       DRIFT, L = LEND
 BPM:        MONITOR, L = LMONITOR
!  Dipole with field free ends ------------------------
 DIPOLE:     LINE(B) = (DEND, B, DEND)
!  Spool pieces
!  Spool piece, containing 3 Primary and 1 Secondary correction
!  elements, each of length LSPOOLEL
 HSPOOL:     DRIFT, L = 0.5 * (LSPOOL - 2*LSPOOLEL)
!  Horizontal and vertical dipole correctors-----------
 HCORR:      HKICK, L = LSPOOLEL, KICK = 0.0
 VCORR:      VKICK, L = LSPOOLEL, KICK = 0.0
```

```
!  Buses
 PARA,  MAINBUSA = 1.0
 PARA,  MAINHSA  = 1.0
 PARA,  MAINVSA  = 1.0
!  Magnetic elements
 QF90A:  MARKER
 QD90A:  MARKER
 HABNDM: SBEND, L = 0.5*LBTOT,  ANGLE = 0.5*MAINBUSA*BANGLE
 HABNDP: SBEND, L = 0.5*LBTOT,  ANGLE = 0.5*MAINBUSA*BANGLE
 HQF90A:     QUAD,  L = 0.5*LQFIELD, K1 =   MAINBUSA * KF90F
 HQD90A:     QUAD,  L = 0.5*LQFIELD, K1 = -(MAINBUSA * KF90D)
 HSEXTA:     SEXT,  L = LSPOOLEL,    K2 =   MAINHSA * K2HSPOOL
 VSEXTA:     SEXT,  L = LSPOOLEL,    K2 = -(MAINVSA * K2VSPOOL)
!  Physical elements
 SPOOLFA:    LINE = (HSPOOL,HCORR, HSEXTA, HSPOOL)
 SPOOLDA:    LINE = (HSPOOL,VCORR, VSEXTA, HSPOOL)
 CELLFFA:    LINE = (QF90A, HQF90A, BPM, DEND, SPOOLFA, &
             HABNDM,HABNDP,HABNDM,HABNDP,HABNDM,HABNDP, &
             HABNDM,HABNDP,HABNDM,HABNDP,HABNDM,HABNDP, &
                     DEND, HQD90A, QD90A, HQD90A, BPM, DEND, &
                     SPOOLDA, &
             HABNDM,HABNDP,HABNDM,HABNDP,HABNDM,HABNDP, &
             HABNDM,HABNDP,HABNDM,HABNDP,HABNDM,HABNDP, &
                     DEND, HQF90A)
ring: line = (320*cellffa)
use, ring
```

Appendix B – 4.5DIP Lattice

```
TITLE  !
4.5 dipoles/hcell, systematic b2 corrected w/ chromaticity sexts
! SSC 90 DEGREE CELL LATTICE, AUG 6, 1987
!  Values of PI and C, the speed of light
 PARA, PI        = 3.14159265358979
 PARA, CLIGHT    = 2.99792458D8
!  Energy in eV
 PARA, ENERGY    = 2.0D13
!  Magnetic rigidity (T-m)
 PARA, BRHO = 6.671281903963D4
!  Total number of horizontal dipoles
 PARA, NBENDS    = 3834
!  Dipole magnetic field length (metres)
 PARA, LBFIELD   = 16.54
!  Bending radius IN DIPOLES
 PARA, RHO       = (NBENDS * LBFIELD) / (2.0 * PI)
!  Dipole bend angle
 PARA, BANGLE   =  LBFIELD / RHO
!  Dipole field (Tesla)
 PARA, BFIELD    = BRHO / RHO
!  Magnetic length of arc quads
 PARA, LQFIELD   = 3.64*230.0/200.0
!  Overall length of spool pieces (m)
 PARA, LSPOOL    = 5.57
```

```
!  Magnetic length of spool piece elements (m)
 PARA, LSPOOLEL  = 0.001
!  Length of non-magnetic ends of quadrupoles and dipoles (m)
 PARA, LEND      = 0.4
 PARA, LBTOT = LBFIELD + 2.0*LEND
!  Beam position monitor length (m)
 PARA, LMONITOR  = 0.2
!  Half cell length
 PARA, LHCELL = LSPOOL + 4.5*LBTOT + (LQFIELD + 2*LEND + LMONITOR)
!  Nominal strengths of dipoles and quadrupoles
!  90 degree cell quadrupole strength (metres**-2)
 PARA, KF90F     =   3.86876935e-3
 PARA, KF90D     =   3.84854638e-3
!  Spool chromatic sextupole strengths (metres**-3)
 PARA, K2HSPOOL =  2.0*0.23563325419d1
 PARA, K2VSPOOL =  2.0*0.47186113779d1
!  Field free end, and beam position monitor ------------
 DEND:       DRIFT, L = LEND
 BPM:        MONITOR, L = LMONITOR
!  Dipole with field free ends --------------------------
 DIPOLE:     LINE(B) = (DEND, B, DEND)
!  Spool pieces
!  Spool piece, containing 3 Primary and 1 Secondary correction
!  elements, each of length LSPOOLEL
 HSPOOL:     DRIFT, L = 0.5 * (LSPOOL - 2*LSPOOLEL)
!  Horizontal and vertical dipole correctors -------
 HCORR:      HKICK, L = LSPOOLEL, KICK = 0.0
 VCORR:      VKICK, L = LSPOOLEL, KICK = 0.0
```

```
! Buses
 PARA,  MAINBUSA = 1.0
 PARA,  MAINHSA  = 1.0
 PARA,  MAINVSA  = 1.0
! Magnetic elements
 QF90A:  MARKER
 QD90A:  MARKER
 HABNDM: SBEND, L = 0.5*LBTOT,  ANGLE = 0.5*MAINBUSA*BANGLE, &
 HABNDP: SBEND, L = 0.5*LBTOT,  ANGLE = 0.5*MAINBUSA*BANGLE, &
 HQF90A:     QUAD,  L = 0.5*LQFIELD, K1 =   MAINBUSA * KF90F
 HQD90A:     QUAD,  L = 0.5*LQFIELD, K1 = -(MAINBUSA * KF90D)
 HSEXTA:     SEXT,  L = LSPOOLEL,    K2 =   MAINHSA * K2HSPOOL
 VSEXTA:     SEXT,  L = LSPOOLEL,    K2 = -(MAINVSA * K2VSPOOL)
! Physical elements
 SPOOLFA:    LINE = (HSPOOL,HCORR, HSEXTA, HSPOOL)
 SPOOLDA:    LINE = (HSPOOL,VCORR, VSEXTA, HSPOOL)
 CELLFFA:    LINE = (QF90A, HQF90A, BPM, DEND, SPOOLFA, &
             HABNDM,HABNDP,HABNDM,HABNDP, &
             HABNDM,HABNDP,HABNDM,HABNDP, &
             HABNDM, &
                     DEND, HQD90A, QD90A, HQD90A, BPM, DEND, &
                     SPOOLDA, &
             HABNDM,HABNDP,HABNDM,HABNDP, &
             HABNDM,HABNDP,HABNDM,HABNDP, &
             HABNDM, &
                     DEND, HQF90A)
!
ring: line = (426*CELLFFA)
use, ring
```

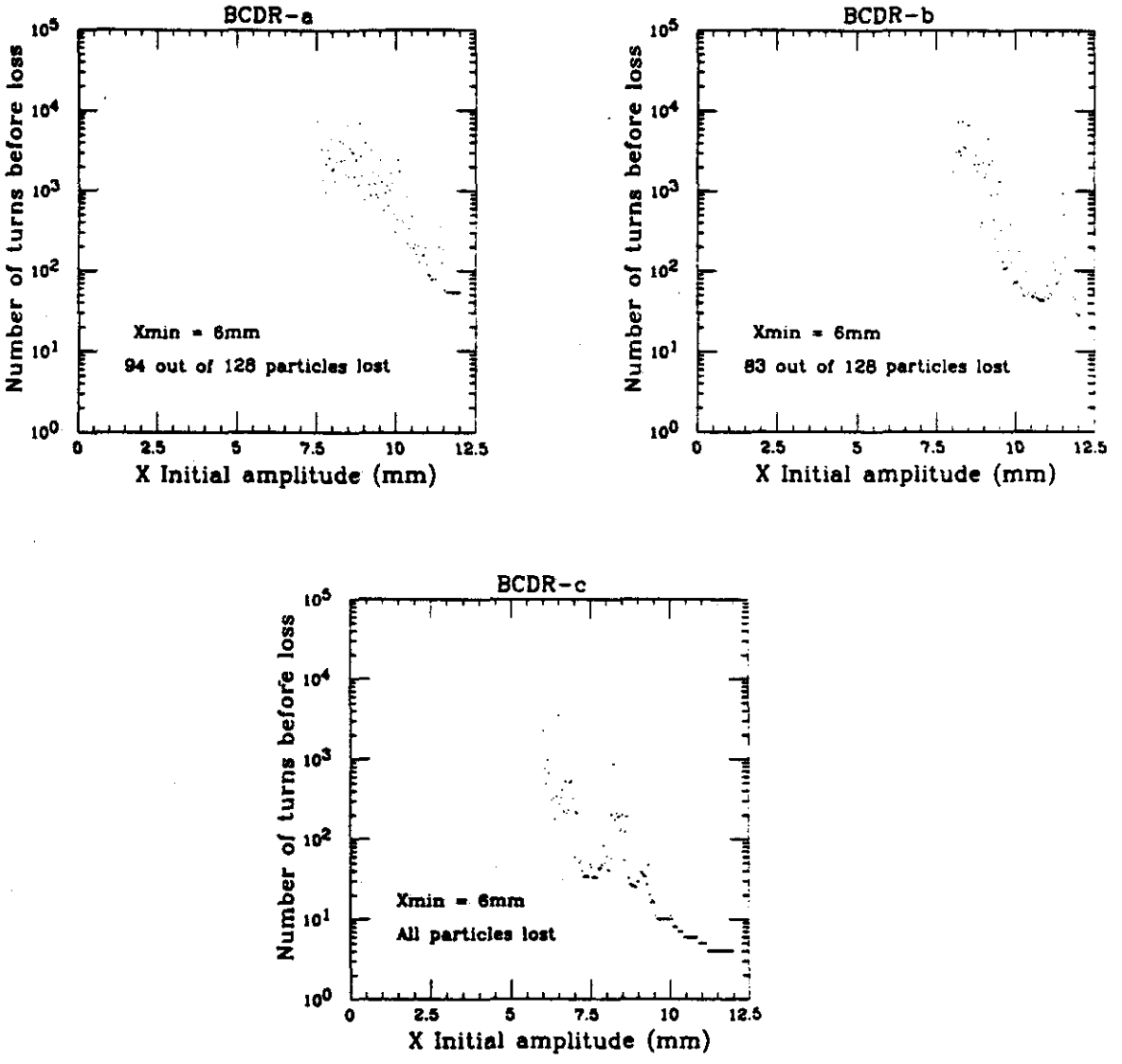

Figure 1. Particle-survival charts for 10,000-turn tracking of the BCDRs. The three charts (a), (b), (c) are for three tracking cases BCDR-a, BCDR-b, and BCDR-c that differ from each other only by different random generator seeds. Each point in the charts represents a particle with its initial amplitude in x coordinate on the horizontal axis and turn number at loss on the vertical axis.

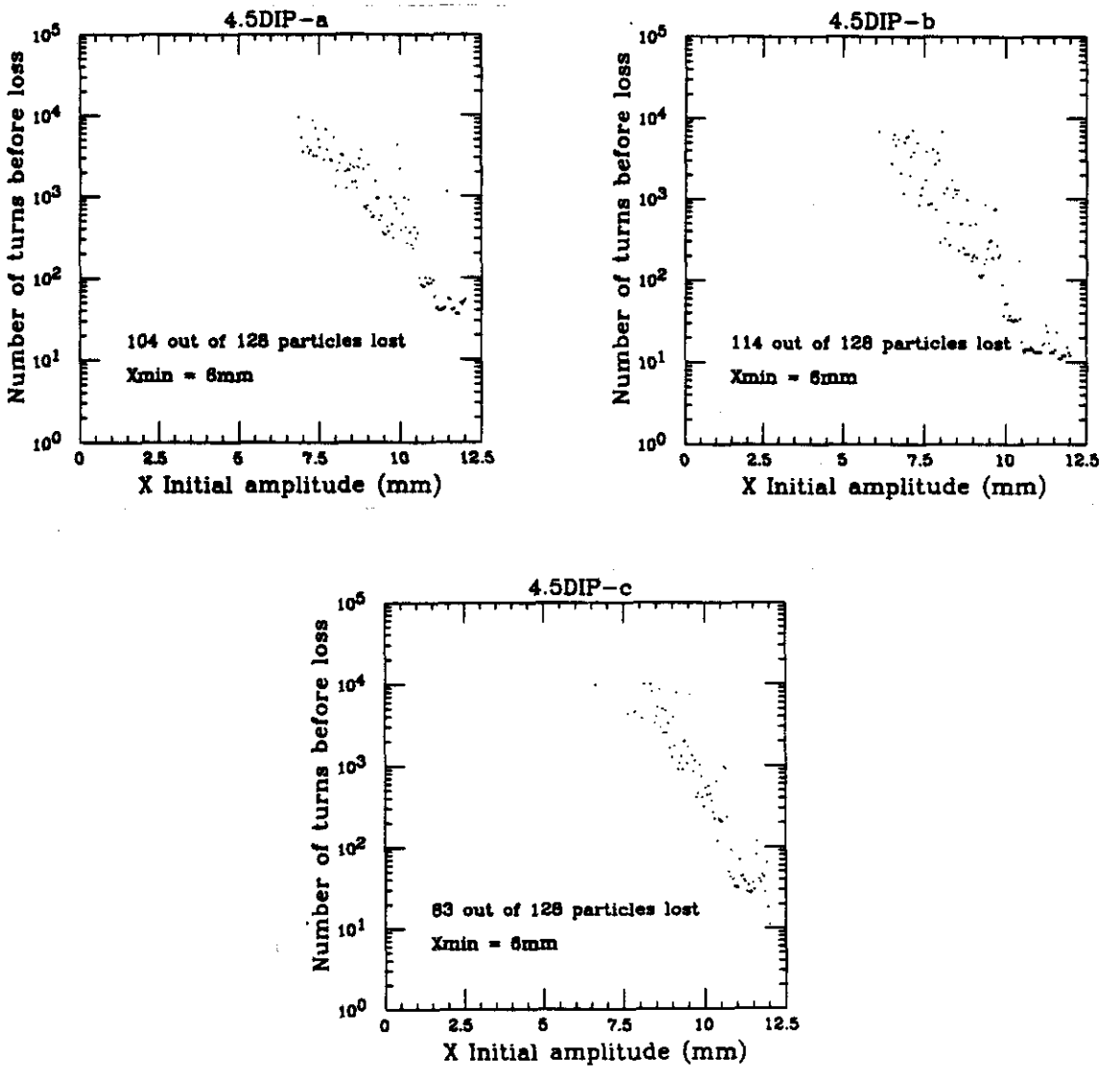

Figure 2. Particle-survival charts for 10,000-turn tracking of the 4.5DIPs. Similar to Figure 1, the three charts (a), (b), (c) are for three tracking cases 4.5DIP-a, 4.5DIP-b, and 4.5DIP-c that differ from each other only by different random generator seeds.

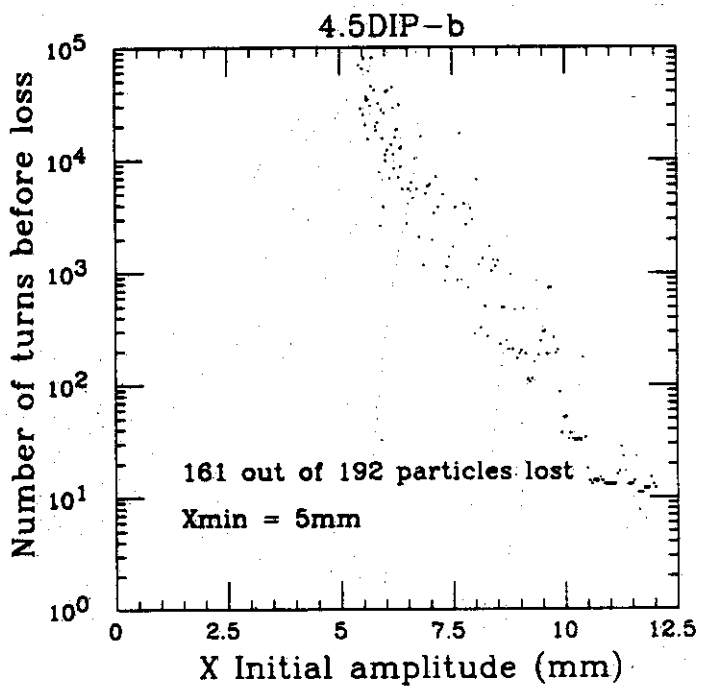

***Figure* 3. Particle-survival charts for 100,000-turn tracking of the 4.5DIP-b.**