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Comment on Prediction of Cold Multipoles from their Warm Counterparts

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Abstract

Several notes have recently been circulated concerning correlation between room temperature and cryogenic measurements of low-order multipole moments in SSC magnets.[1,2] I will comment here on the experimental results, as well as on the issue of how large a fraction of all magnets must be measured in their cold state to assure, just from the standpoint of statistical sampling alone, that the final magnet population has tolerances within those specified by the SSC design.[3]

Nomenclature

Let us assume that we have both cold and warm measurements of some unique multipole in a sub-sample of N magnets. Let us label any doublet of these measurements for magnet i as c(i) and w(i), with i = 1,..,N. We can define the means and standard deviations for this subset of magnets as usual:

$$\bar{c} = \frac{1}{N} \sum_{i=1}^{N} c(i) \quad , \quad \bar{w} = \frac{1}{N} \sum_{i=1}^{N} w(i)$$

$$\sigma_c^2 = \frac{1}{N-1} \sum_{i=1}^{N} [c(i)-\bar{c}]^2 \quad , \quad \sigma_w^2 = \frac{1}{N-1} \sum_{i=1}^{N} [(w(i)-\bar{w}]^2 \tag{1}$$

where σ_c and σ_w contain contributions from both the measurement error and the actual scatter due to differences in the construction of the magnets. (Error due to measurement will presumably be quite small.) We can also define differences between the cold and warm values as follows:

$$d(i) = c(i) - w(i) \tag{2}$$

with the mean in the difference, and its standard deviation given by:

$$\bar{d} = \bar{c} - \bar{w} \quad , \quad \sigma_d^2 = \frac{1}{N-1} \sum_{i=1}^{N} [d(i)-\bar{d}]^2 \tag{3}$$

the latter can be rewritten as follows:

$$\sigma_d^2 = \frac{1}{N-1} \sum_{i=1}^{N} [\{c(i)-\bar{c}\} - \{w(i)-\bar{w}\}]^2$$

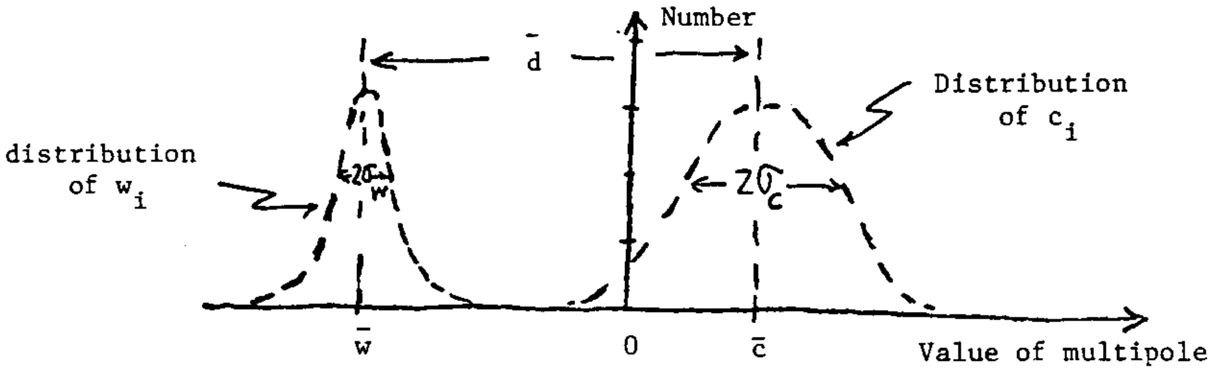
$$\sigma_d^2 = \sigma_c^2 + \sigma_w^2 - \frac{2}{N-1} \sum_{i=1}^{N} [c(i)w(i) - w(i)\bar{c} - c(i)\bar{w} + \bar{c}\,\bar{w}] \tag{4}$$

If the c(i) and the w(i) are randomly dispersed about their means, that is, if their difference is only correlated on the average, then we get the simpler (and hopefully incorrect!) result:

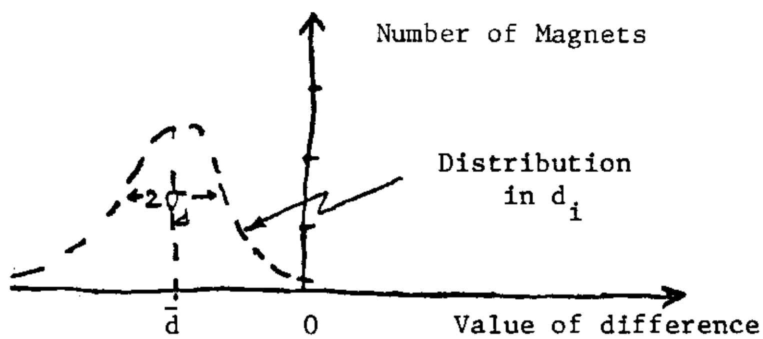
$$\sigma_d^2 = \sigma_c^2 + \sigma_w^2 \tag{5}$$

In case of correlations, we have to obtain σ_d from the defining formula (3).

Consequently, thus far we have the following picture:



and for the difference we have:



Predicting Values of Individual Cold Multipoles

Assuming that the difference measurements are Gaussian distributed, the relation between any warm measurement and its predicted cold value for some j^{th} magnet (beyond the initial N) can be written as follows:

$$c(j) = w(j) + \bar{d} + r*\sigma_d \tag{6}$$

where r is a random variable of zero mean and of variance unity. The square in the uncertainty for any predicted c(j) is then just:

$$\sigma_j^2 = \sigma_0^2 + \frac{\sigma_d^2}{N} + \sigma_d^2 \tag{7}$$

where σ_0 is the measurement error on the multipole for the warm magnet (this can hopefully be made quite small: $\sigma_0 << \sigma_d$), σ_d is the standard deviation of the previous difference measurements, and $\sigma_d/\sqrt{N}$ is the uncertainty on $\bar{d}$. From the last result, it is clear that, as long as $N \geq 5$, the size of the initial sample of magnets (N) will have little impact on the confidence for predicting any individual c(j) from its measured warm counterpart w(j). The best we can do is state that $c(j) = w(j) + \bar{d}$, with an uncertainty of σ_d.

Relation to Tolerances

Let us now review what we have obtained, and relate that to the tolerances desired for all the magnets. From the N doublet-values, we can determine $\bar{c}$, $\bar{w}$, $\bar{d}$, and their uncertainties; the latter estimated by $\sigma_c/\sqrt{N}$, $\sigma_w/\sqrt{N}$ and $\sigma_d/\sqrt{N}$. In addition, for Gaussian errors, we can estimate the uncertainties in the σ_i as $\sigma_i/\sqrt{2N}$. The value of $\bar{c}$ corresponds to a sampling of the average systematic multipole $\bar{c}_F$ for the entire final magnet population, while σ_c corresponds to a sampling of the random variation in the total population of multipoles (σ_F). The SSC must construct magnets such that the entire population satisfies the following criteria:

$$-t_s < \bar{c}_F < t_s \qquad \text{and} \qquad \sigma_F < t_r \tag{8}$$

where t_s and t_r are, respectively, the allowed upper limits on the systematic and random tolerances for that multipole.

Of course, we do not know $\bar{c}_F$ or σ_F, but we have their estimates and uncertainties from our sample of N measurements:

$$\bar{c}_F = \bar{c} \pm \sigma_c/\sqrt{N} \qquad \text{and} \qquad \sigma_F = \sigma_c \pm \sigma_c/\sqrt{2N} \tag{9}$$

Consequently, our goal is to assure that:

$$-t_s < \bar{c} \pm n\sigma_c/\sqrt{N} < t_s \qquad \text{and} \qquad \sigma_c(1 \pm n/\sqrt{2N}) < t_r \tag{10}$$

where n is the number of standard deviations (corresponding to some level of confidence we will demand) for compliance of $\bar{c}_F$ with t_s and σ_F with t_r. The above can be rewritten as follows:

$$\bar{c} + n\sigma_c/\sqrt{N} < t_s \qquad \text{and} \qquad \sigma_c(1 + n/\sqrt{2N}) < t_r \tag{11}$$

(The probability distribution is one sided for the limit on the random tolerance, but essentially double sided for the systematic. That is, for a given n, the level of confidence should be about a factor of two smaller for random deviations relative to systematic ones. This statement is clearly correct for the case that $\bar{c} \sim t_s << \sigma_c \sim t_r$.)

Once estimates of $\bar{c}$ and σ_c become available from the data, then, given t_s and t_r, we can evaluate the number of magnets (N) required to reach any desired level of confidence (specified by n) in predicting that the final sample of magnets will be within the required tolerances. It is clear, however, that certain broad conditions must be satisfied, independent of the value of N. Unless, for example, $\bar{c} < t_s$ and $\sigma_c < t_r$, no value of N will help assure final compliance! Rearranging the above expressions, we can write lower limits on the N values as follows:

$$N > [n\sigma_c/(t_s-\bar{c})]^2 \qquad \text{and} \qquad N > [n\sigma_c/\sqrt{2}(t_r-\sigma_c)]^2 \tag{12}$$

<u>Example</u>

As an example, let us see whether we can use the recent cold measurements of the skew a_2 values for six DSS magnets[1] to gauge the approximate values needed for N. The experimental values are: $a_2 = 0.01 \pm 0.12$, $\sigma_a = 0.30 \pm 0.09$; the desired values of the tolerances are[3] $t_r = 0.6$ and $t_s = 0.1$. Putting these quantities into our two expressions for N, we obtain the approximate values of $N > 180$ and $N > 8$ in Expressions (12) for the outrageous level of confidence that only about one magnet out of 15,000 will fail the prescribed criteria (corresponds to n=4). This is highly comforting, but appropriate only for a_2. Using the measurements obtained for b_2, we get far less gratifying results. The measured parameters are: $b_2 = -2.2 \pm 1.1$ and $\sigma_b = 2.6 \pm 0.8$; the tolerances here are $t_s = 1.0$ and $t_r = 2.0$ (prior to binning). If these were the final SSC magnets, they would not all be deemed acceptable for SSC operation, and would possibly suggest need for modification of the construction procedures.

There is, of course, a general flaw in the present conclusions, because we have ignored the uncertainties in the values of the above experimental quantities. That is, we used N = 6 measurements to obtain data, which we decided would be sufficiently reliable (for the case of a_2) only for $N > 180$. Consequently, we must conclude that the present sample is not large enough to use in deciding on the eventual value needed for N at our chosen 0.00006 level of confidence. Had the DSS measurements been based on a sample of about 400 magnets, and if the same parameters were found to apply (with all uncertainties reduced by a factor of about $\sqrt{400/6} = 8$), then the conclusions would have been defendable. Consequently, from this vantage point, establishing what number of magnets has to be measured in the cold state must be regarded as an iterative procedure. (Start with 100 to get orientation!) We can, of course, also make certain assumptions about limits to expect for any $\bar{c}$ or σ_c, and thereby get an estimate for the required N. For example, since the criterion based on t_s appears more constraining, it is not entirely illogical to assume that we can obtain a reasonable estimate for N by using $\sigma_c = t_r$, and $\bar{c} = 0.0$ in the first expression (ignoring the second one!). This yields, $N > 64$ for b_2 and $N > 576$ for a_2. This is the kind of a priori estimate that was sought in Ref. [2], but that, we believe, cannot be fully justified.

Tracking Quality

Another issue of importance is how to assure on the basis of warm measurements that magnet production is on keel. Having measured N magnets both warm and cold, and determined the correlations between warm and cold multipoles, we now measure the next batch of M magnets in just their warm state, and wish to know with some degree of confidence that the process has not deteriorated. We will assume that the correlations will remain reliable, although the absolute values of (both) parameters may deteriorate in time. From the reported DSS measurements[1], there is absolutely no doubt that correlations between warm and cold multipoles are exceedingly strong. The reported values of $\bar{d}$ and σ_d for b_2 are an order of magnitude smaller than the separate cold or warm values! Unfortunately, here again one must rely on results of measurement, and not much can be said a priori other than has already been mentioned in the above discussion, which we enlarge upon below.

We have already found Expressions (6) and (7) for the values of the extracted cold quantities from their warm measurements. For a new set of M of these quantities, we can write:

$$\bar{c}_M = \bar{w}_M + \bar{d} \text{ , with uncertainties related by } \sigma^2/M = \sigma_w^{\prime 2}/M + \sigma_d^2/N \quad (13)$$

where we have assumed that the new set of M measurements (determining $\bar{w}_M$ and σ_w') is not correlated with the previous N measurements. However, because of the strong cold-warm correlation, we expect $\sigma_d < \sigma_w'$, and since $M < N$, we see that $\sigma \sim \sigma_w'$, where, of course, the prime refers to the observed width of the distribution for the new sample of M magnets. We can therefore obtain an estimate of the cold mean, the standard deviation, and the appropriate uncertainties in these quantities, and now, as before, require that these parameters be within some range. Because the "next" sample will be smaller than the already available set, we can be somewhat less restrictive in the level of confidence we set for rejection or for panic. Perhaps $n \leq 2.6$ (≤ 0.01 confidence level) might be appropriate at this stage of selection, while a level of $n < 3.3$ (≤ 0.001 confidence) might be appropriate for the accruing sample.

Conclusion

The main conclusions that can be surmised from this note are the following: (i) Deciding on how many magnets are required to assure compliance with tolerance specs is an iterative procedure. Measurements are needed to set the scales, and then to track the production. (ii) If the correlations between cold and warm multipoles are as strong as observed for DSS magnets, then we can expect $\sigma_d << \sigma_w \approx \sigma_c$; this implies that warm measurements are just as appropriate as cold ones for setting any confidence levels, or for tracking and assuring the quality of magnet production related to the multipole structure of the fields. (iii) To establish the correlations between cold and warm multipoles will require an initial measurement of a batch of magnets in both their cold and warm states. Assuming that correlations will be comparable to those observed for the sextupole (not quadrupole!) moments of the six DSS dipoles ($\overline{d} = -0.60 \pm 0.09$ for the normal, and -0.11 ± 0.02 for the skew sextupole terms), we anticipate that a sample of about 100 magnets will provide sufficient statistics for determining the cold-warm corrections for the normal sextupole term to a superb accuracy of about 0.02 units!

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References

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3. A. Chao and M. Tigner, SSC-N-183 (May 27, 1986).