October 31, 1988

Finite Element Analaysis of NC-9 Dipole
Note #3
Dependence of Coil Azimuthal Stress on Material Properties, Planar Behavior Assumption, and Midplane Displacements

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Introduction

The accuracy of a finite element stress analysis is directly related to the accuracy with which material properties and loads are known. In the absence of reliable information, and because there will probably be a significant variation even after the standardization of tooling and construction technique, it is possible to use the finite element model to determine the effect of variation of these quantities on the magnitude and distribution of the coil stresses. If the collar dimensions and collar material properties are considered invariant, then:

$$S(x,y) = f(d,E,G,Nu,I)$$

Where:

S(x,y) = stress tensor in coil at spatial coordinates x,y
d = uncompressed geometry of coil
E = Young's modulus of coil material
G = Shear modulus of coil material
Nu = Poisson's ratio of coil material
I = Imponderables

The material properties E,G, and Nu vary microscopically and anisotropically in the real coil, but a practical finite element model must smear the properties over a relatively large element and vary them at most orthotropically. Displacements based on the uncompressed coil geometry, however, can be input in great detail.

The "imponderables" include effects such as the non-linearity and load path dependent (non-conservative) material response, and the exact planar behavior assumption (probably somewhere between plane stress and plane strain). This work considers the planar behavior by imposing the two extremes.

Simplifying assumptions may allow future work to consider some of the non-linearities.

The Finite Element Models

Eight models were created, each representing the variation of a single parameter. The models were loaded with specified displacements on the midplanes of the inner and outer coil, which is the condition encountered by collaring. Two assumptions are common to all models:

1. Adjacent conductors do not slide relative to each other
2. Material properties of the coil do not vary with stress

Model characteristics are listed in the tables with the model results.

Young's modulus, EX

Note: This property can vary orthotropically in the model (EX,EY,EZ), but if isotropic properties are assumed, only the EX value is input.

Four different values of EX were studied:

1. $1.5(10^6)$ psi uniform radially across coil
2 $0.75(10^6)$ psi uniform radially across coil
3. $0.75(10^6)$ psi at outer radius, varying linearly to $1.5(10^6)$ at the inner radius
4. $1.125(10^6)$ psi uniform radially across coil

In case 3, the radial variation was done stepwise across the three elements available in the radial direction in each coil, with $1.5(10^6)$ psi on the inner 1/3, $1.125(10^6)$ psi on the middle 1/3, and $0.75(10^6)$ psi on the outer 1/3 of each coil.

Shear modulus, GXY

Note: This property can vary orthotropically in the model (GXY,GYZ,GXZ), but if isotropic properties are assumed, only the GXY value is input.

This property will assume a default value of EX/2(1+NU) if unspecified. The default value was used in all but one case, in which a value of 1500 psi was specified. This represents a material with negligible shear stiffness.

Poisson's ratio, Nu

This was either 0.3 (the default of the ANSYS program) or 0.001.

Midplane displacement

Three cases were studied:

1. A unform displacement of 0.018 in on the midplane of the inner coil and 0.016 in on the midplane of the outer coil.
2. A linearly varying displacement of 0.017 at the inner radius of the inner coil to 0.019 in. at the outer radius of the inner coil, and 0.015 in at the inner radius of the outer coil to 0.017 in at the outer radius of the outer coil. The average midplane displacement of each coil is equal to that in case 1.
3. A linearly varying displacement of 0.019 at the inner radius of the inner coil to 0.017 at the outer radius of the inner coil, and 0.017 in at the inner radius of the outer coil to 0.015 at the outer radius of the outer coil. The average midplane displacement of each coil is equal to that of case 1.

Planar Behaviour

This was either plane stress or plane strain.

Results

The relative effects of Young's modulus, shear modulus, and Poisson's ratio are shown in Table I and Figs. 1-4 for four locations on the coils. Loading in each case is by uniform midplane displacement. The figures show that the average stress and the bending stress components are reduced at the inner coil midplane by 25% and 45%, respectively, as Young's modulus is reduced from $1.5(10E^6)$ to $0.75(10E^6)$ psi. Reduction of Poisson's ratio results in nearly the same bending and only a small (10%) reduction in membrane stress. At all four locations the reduced shear modulus resulted in the smallest bending stress. At the pole tip of the inner coil, the reduced shear modulus produces a reversal of bending stress. It is not clear if this is a real effect, or an artifact of the mesh.

Table II and Figs 5-8 show the effects of planar assumption for constant midplane displacement and material properties. The plane stress model is less stiff, as expected, and results in reductions of inner coil membrane and bending stresses by 9% and 1.4%, respectively, from the plane strain results.

The effects of variable midplane displacement are shown in Table III and Figs. 9-12 for constant material properties and planar behaviour assumption. A variation in displacement from inner to outer radius of 0.002 in. at the inner

coil midplane results in a variation of maximum stress of plus or minus 30%. The effects at the pole tips are much less pronounced.

Table IV and Figs. 13-16 show the effects of a linear variation of the coils' Youngs modulus from $0.75(10^6)$ psi at the inner third of each coil to $1.5(10^6)$ at the outer third. These results are compared to a model in which the Young's modulus was a constant $1.125(10^6)$ across the coil. The models produce essentially identical membrane stress, demonstrating similar average azimuthal stiffnesses. The bending stress at the inner coil midplane, however, is 55% higher for the variable modulus model. The bending stress at the inner coil pole tip increases by a factor of 1.6 with the variable modulus, while at the outer coil pole tip the bending stress actually decreases by 15%.

Conclusion

The results of this work indicate that the most important parameters in the calculation of coil stresses are Young's modulus, shear modulus, and midplane displacements. Consultation and review of other analyses has shown that plane stress is the proper planar behavour assumption, and the variation of stresses with Poissons ratio is insignificant within any realistic range of values.

Table I.
Effects of Material Property Variation on Coil Stresses for Constant, Uniform Midplane Displacement

	Model Number				
	1	2	3	4	5
Yngs Modulus (10^6 psi)	1.5	.75	1.5	1.5	1.5-.75
Shr. Modulus (10^6 psi)	.58	.29	.0015	.58	.58-.29
Poisson's Ratio	0.3	0.3	0.3	0.001	0.3
Coil Pln. Strs/Strn?	strain	strain	strain	strain	strain
Inner Coil Midplane					
Membrane (psi)	-8900	-6600	-8550	-7950	-7950
Bending (psi)	-5000	-3450	-3100	-4950	-6900
Tot. Stress, IR.	-13900	-10250	-11650	-12900	-14850
Tot. Stress, OR.	-3900	-3150	-5450	-3000	-1050
Inner Coil Pole Tip					
Membrane (psi)	-8900	-6600	-8500	-7850	-7850
Bending (psi)	-1600	-450	1000	-850	-2800
Tot. Stress, IR.	-10500	-7050	-7500	-8700	-10650
Tot. Stress, OR.	-7300	-6150	-9500	-7000	-5050
Outer Coil Midplane					
Membrane (psi)	-6850	-5350	-6400	-6050	-6100
Bending (psi)	-3300	-2600	600	-3250	-4700
Tot. Stress, IR.	-10150	-7950	-5800	-9300	-10800
Tot. Stress, OR.	-3550	-2750	-7000	-2800	-1400
Outer Coil Pole Tip					
Membrane (psi)	-6400	-5050	-6100	-5650	-5700
Bending (psi)	5750	3150	1700	5000	3050
Tot. Stress, IR.	-750	-1900	-4400	-650	-2650
Tot. Stress, OR.	-12150	-8200	-7800	-10650	-8750

Table II.
Effects of Planar Behaviour Assumption on Coil Stresses for Constant, Uniform Midplane Displacement

	Model Number	
	2	6
Yngs Modulus (10^6 psi)	.75	.75
Shr. Modulus (10^6 psi)	.29	.29
Poisson's Ratio	0.3	0.3
Coil Pln. Strs/Strn?	strain	stress
Inner Coil Midplane		
Membrane (psi)	-6600	-6000
Bending (psi)	-3450	-3400
Tot. Stress, IR.	-10250	-9400
Tot. Stress, OR.	-3150	-2600
Inner Coil Pole Tip		
Membrane (psi)	-6600	-5950
Bending (psi)	-450	-300
Tot. Stress, IR.	-7050	-6250
Tot. Stress, OR.	-6150	-5650
Outer Coil Midplane		
Membrane (psi)	-5350	-4900
Bending (psi)	-2600	-2450
Tot. Stress, IR.	-7950	-7350
Tot. Stress, OR.	-2750	-2450
Outer Coil Pole Tip		
Membrane (psi)	-5050	-4650
Bending (psi)	3150	2800
Tot. Stress, IR.	-1900	-1850
Tot. Stress, OR.	-8200	-7450

Table III.
Effects of Midplane Displacements on Coil Stresses for Constant Material Properties

	Model Number		
	1	7	8
Yngs Modulus (10^6 psi)	1.5	1.5	1.5
Shr. Modulus (10^6 psi)	.58	.58	.58
Poisson's Ratio	0.3	0.3	0.3
Coil Pln. Strs/Strn?	strain	strain	strain
Midplane Displacement, Inner Coil,in.(IR-OR)	0.018 uniform	0.017-0.019	0.019-0.017
Midplane Displacement, Outer Coil,in.(IR-OR)	0.016 uniform	0.015-0.017	0.017-0.015
Inner Coil Midplane			
Membrane (psi)	-8900	-8950	-9150
Bending (psi)	-5000	-1700	-8100
Tot. Stress, IR.	-13900	-10650	-17250
Tot. Stress, OR.	-3900	-7250	-1050
Inner Coil Pole Tip			
Membrane (psi)	-8900	-9150	-8900
Bending (psi)	-1600	-1350	-1750
Tot. Stress, IR.	-10500	-10500	-10650
Tot. Stress, OR.	-7300	-7800	-7150
Outer Coil Midplane			
Membrane (psi)	-6850	-6450	-6900
Bending (psi)	-3300	950	-6800
Tot. Stress, IR.	-10150	-5500	-13700
Tot. Stress, OR.	-3550	-7400	-100
Outer Coil Pole Tip			
Membrane (psi)	-6400	-6300	-6250
Bending (psi)	5750	4750	3950
Tot. Stress, IR.	-650	-1550	-2300
Tot. Stress, OR.	-12150	-11050	-10200

Table IV.
Effect of Variable Coil Modulus on Coil Stresses for Constant, Uniform, Midplane Displacement

	Model Number	
	1	5
Yngs Modulus (10^6 psi)	1.125	0.75-1.5
Shr. Modulus (10^6 psi)	.58	.58-.29
Poisson's Ratio	0.3	0.3
Coil Pln. Strs/Strn?	strain	strain
Inner Coil Midplane		
Membrane (psi)	-8050	-7950
Bending (psi)	-4450	-6900
Tot. Stress, IR.	-12500	-14850
Tot. Stress, OR.	-3600	-1050
Inner Coil Pole Tip		
Membrane (psi)	-8000	-7850
Bending (psi)	-1050	-2800
Tot. Stress, IR.	-9050	-10650
Tot. Stress, OR.	-6050	-5050
Outer Coil Midplane		
Membrane (psi)	-6150	-6100
Bending (psi)	-3200	-4700
Tot. Stress, IR.	-9350	-10800
Tot. Stress, OR.	-2950	-1400
Outer Coil Pole Tip		
Membrane (psi)	-5800	-5700
Bending (psi)	3550	3050
Tot. Stress, IR.	-2250	-2650
Tot. Stress, OR.	-9350	-8750

Fig. 1

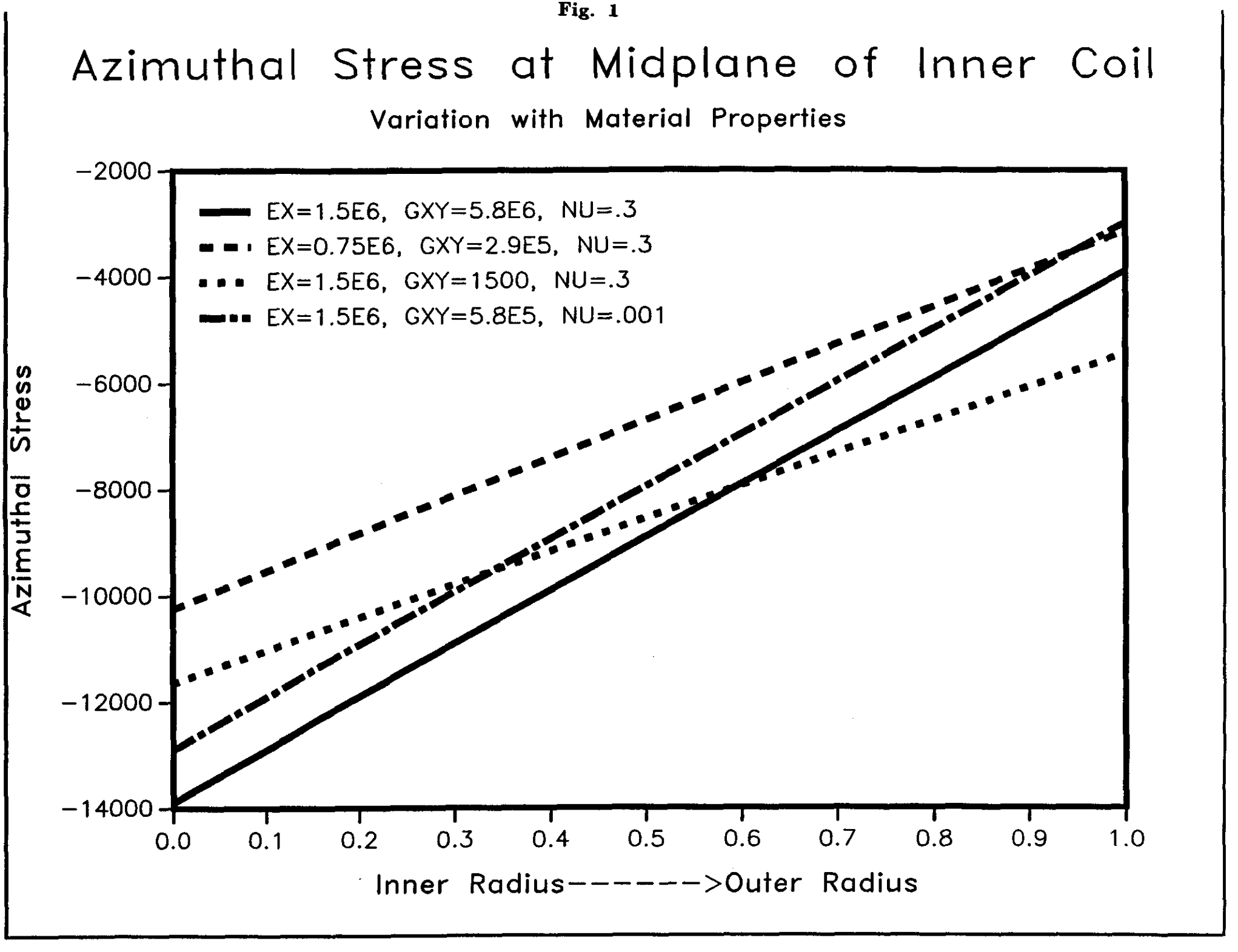

Azimuthal Stress at Midplane of Inner Coil
Variation with Material Properties
EX=1.5E6, GXY=5.8E6, NU=.3
EX=0.75E6, GXY=2.9E5, NU=.3
EX=1.5E6, GXY=1500, NU=.3
EX=1.5E6, GXY=5.8E5, NU=.001
Azimuthal Stress
-2000
-4000
-6000
-8000
-10000
-12000
-14000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius

Fig. 2

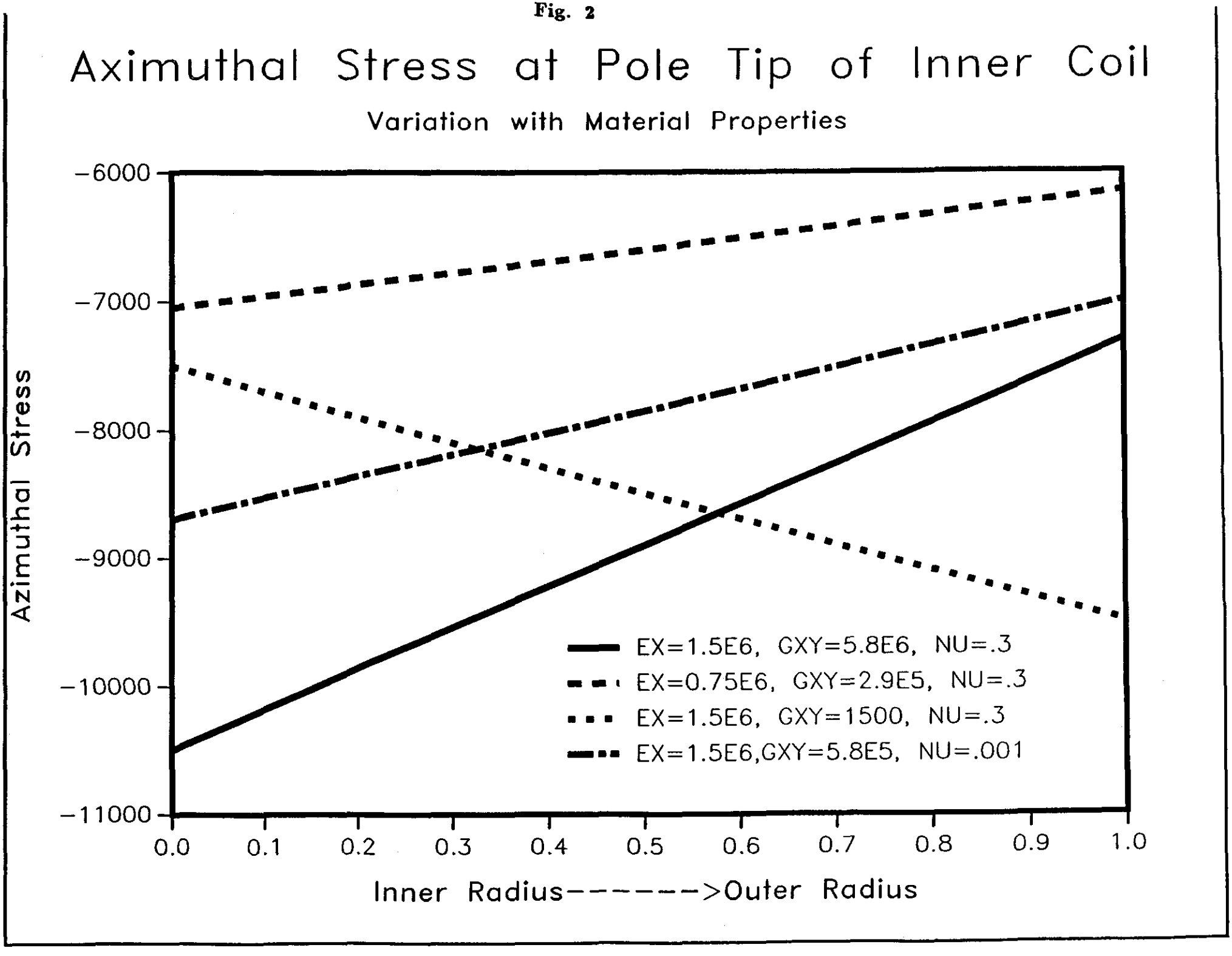

Aximuthal Stress at Pole Tip of Inner Coil
Variation with Material Properties
Azimuthal Stress
-6000
-7000
-8000
-9000
-10000
-11000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius
EX=1.5E6, GXY=5.8E6, NU=.3
EX=0.75E6, GXY=2.9E5, NU=.3
EX=1.5E6, GXY=1500, NU=.3
EX=1.5E6,GXY=5.8E5, NU=.001

Fig. 3

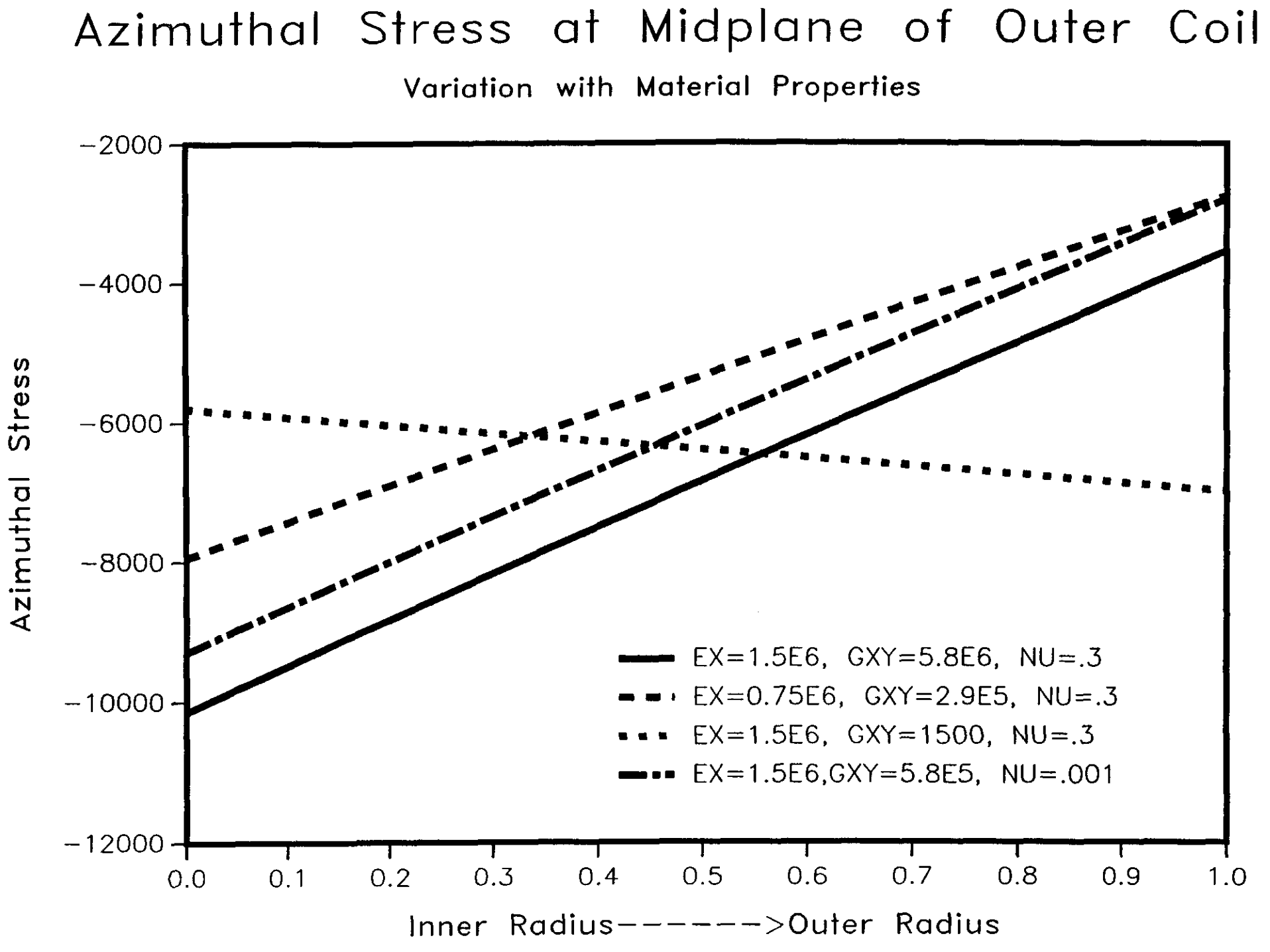

Azimuthal Stress at Midplane of Outer Coil
Variation with Material Properties
-2000
-4000
-6000
-8000
-10000
-12000
Azimuthal Stress
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius
EX=1.5E6, GXY=5.8E6, NU=.3
EX=0.75E6, GXY=2.9E5, NU=.3
EX=1.5E6, GXY=1500, NU=.3
EX=1.5E6,GXY=5.8E5, NU=.001

Fig. 4

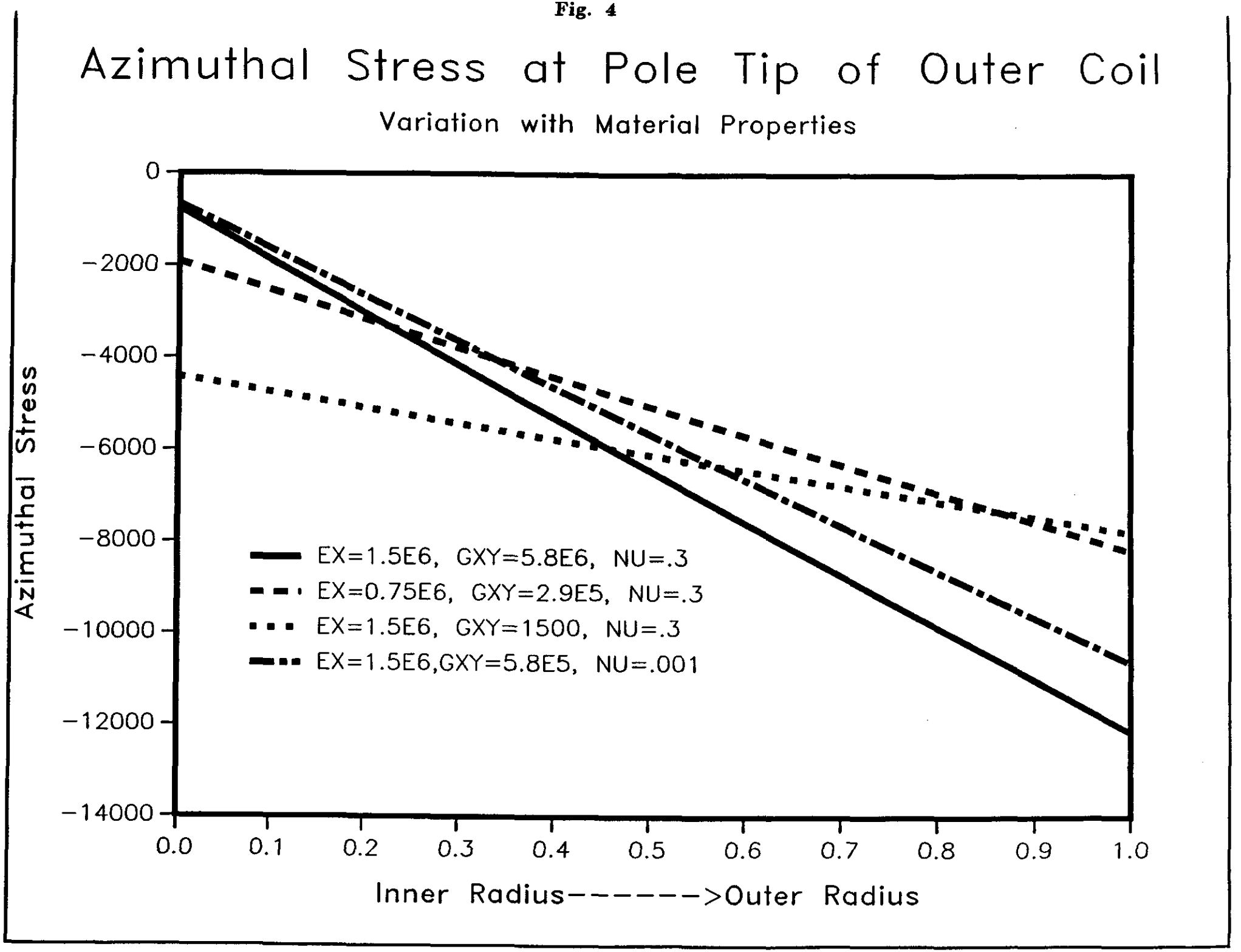

Azimuthal Stress at Pole Tip of Outer Coil
Variation with Material Properties
Azimuthal Stress
0
-2000
-4000
-6000
-8000
-10000
-12000
-14000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------>Outer Radius
EX=1.5E6, GXY=5.8E6, NU=.3
EX=0.75E6, GXY=2.9E5, NU=.3
EX=1.5E6, GXY=1500, NU=.3
EX=1.5E6,GXY=5.8E5, NU=.001

Fig. 5

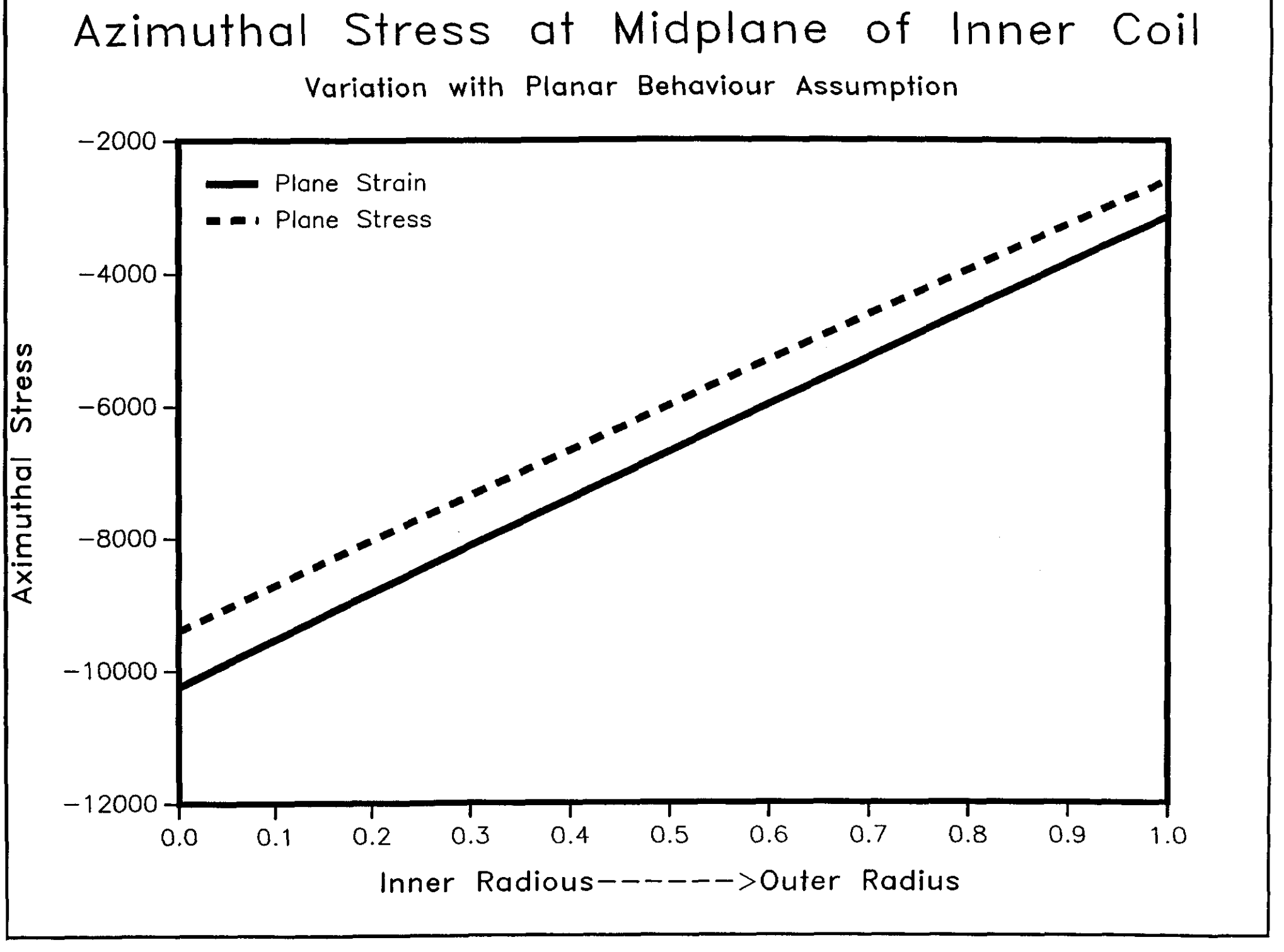

Azimuthal Stress at Midplane of Inner Coil
Variation with Planar Behaviour Assumption
Plane Strain
Plane Stress
-2000
-4000
-6000
-8000
-10000
-12000
Aximuthal Stress
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radious------->Outer Radius

Fig. 6

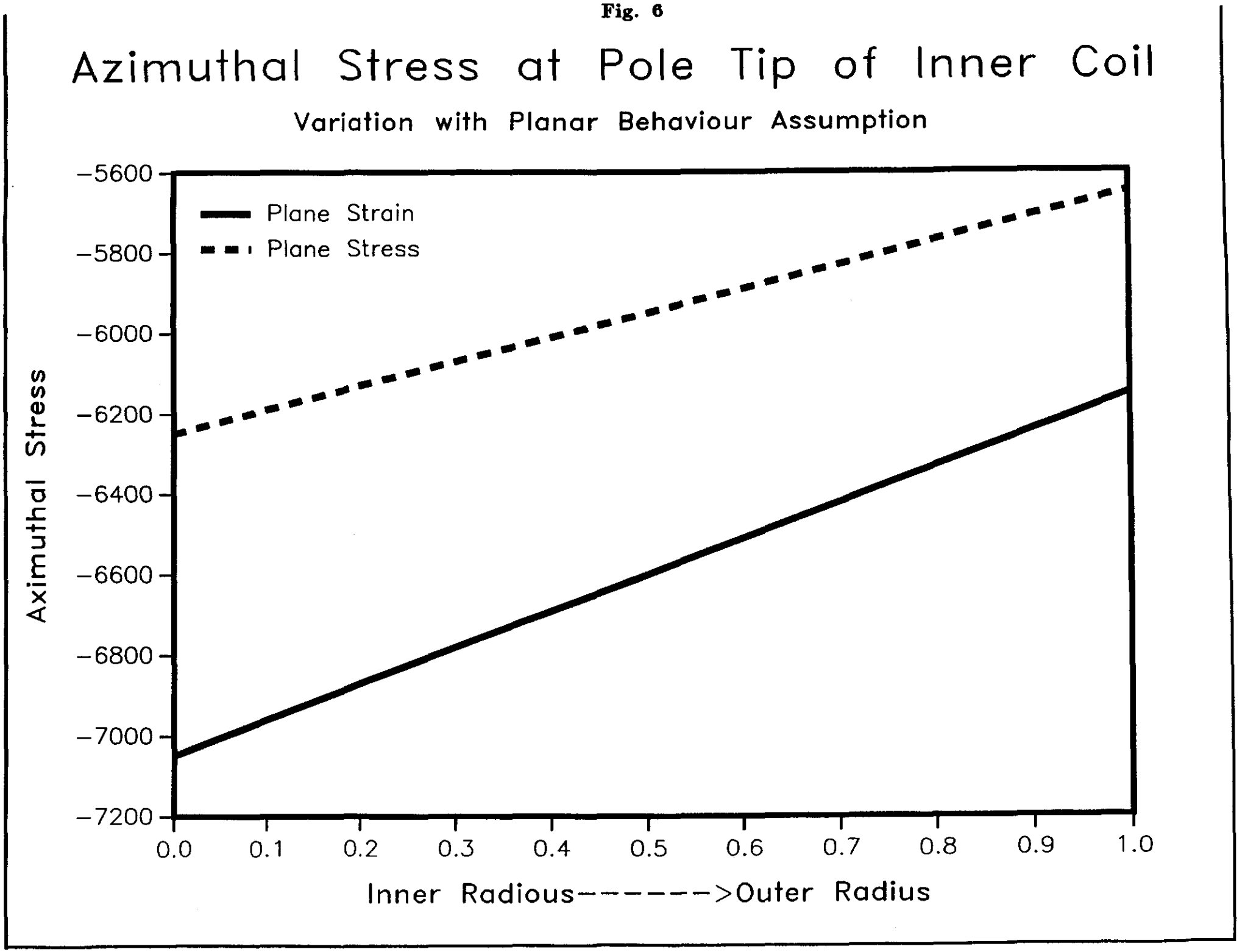

Azimuthal Stress at Pole Tip of Inner Coil
Variation with Planar Behaviour Assumption
Plane Strain
Plane Stress
-5600
-5800
-6000
-6200
-6400
-6600
-6800
-7000
-7200
Aximuthal Stress
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radious------->Outer Radius

Fig. 7

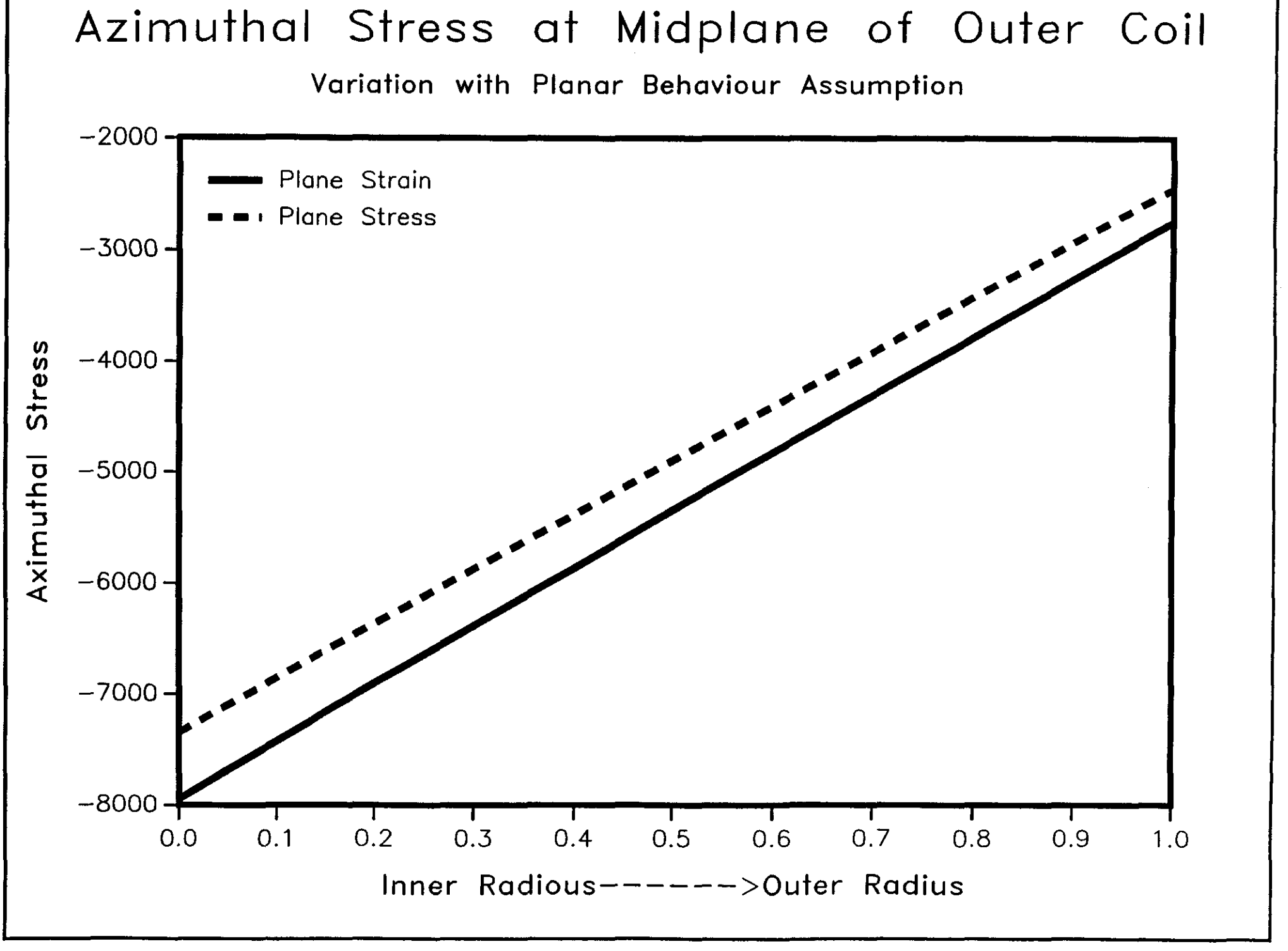

Azimuthal Stress at Midplane of Outer Coil
Variation with Planar Behaviour Assumption
Plane Strain
Plane Stress
Aximuthal Stress
-2000
-3000
-4000
-5000
-6000
-7000
-8000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radious------->Outer Radius

Fig. 8

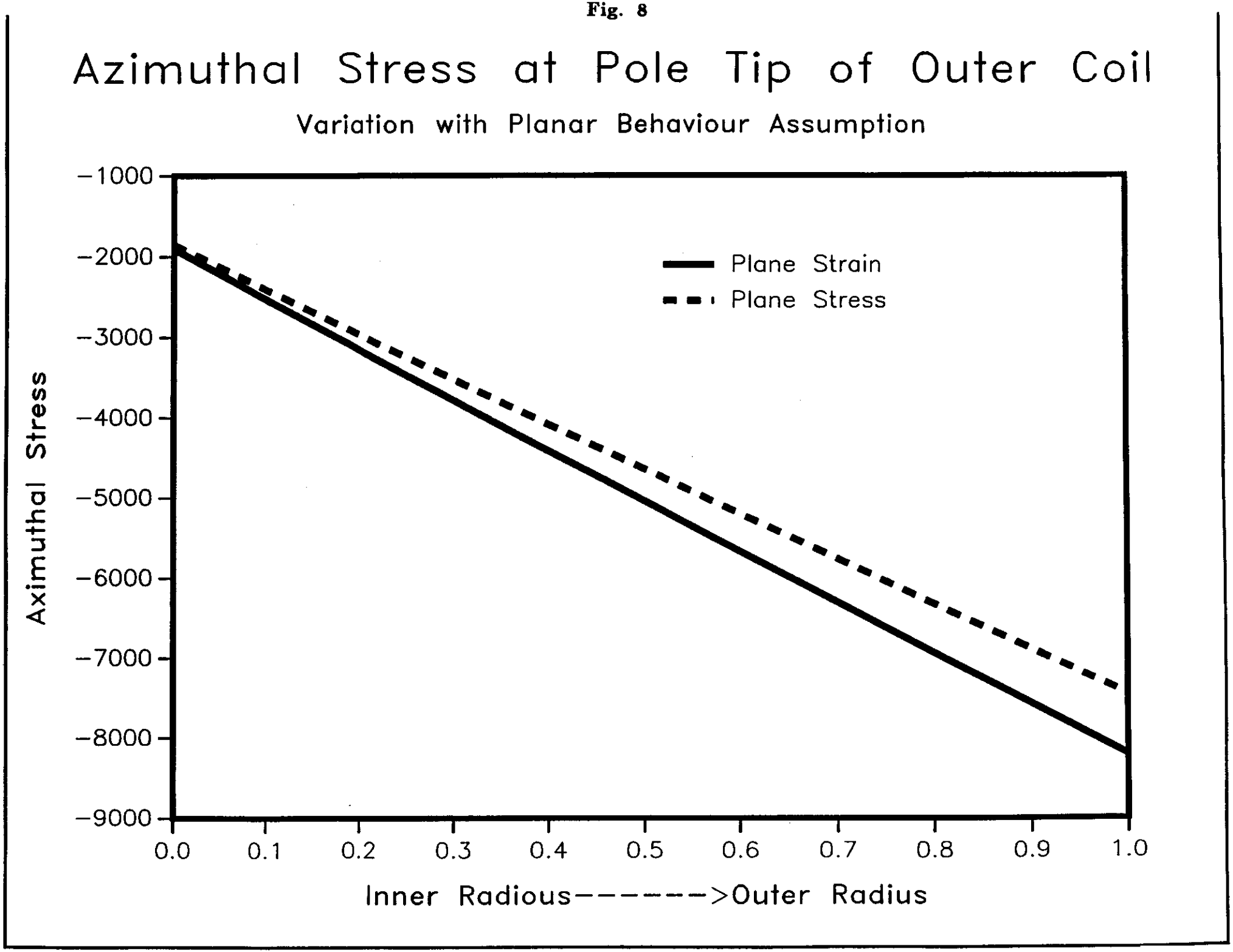

Azimuthal Stress at Pole Tip of Outer Coil
Variation with Planar Behaviour Assumption
Plane Strain
Plane Stress
Aximuthal Stress
-1000
-2000
-3000
-4000
-5000
-6000
-7000
-8000
-9000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radious------->Outer Radius

Fig. 9

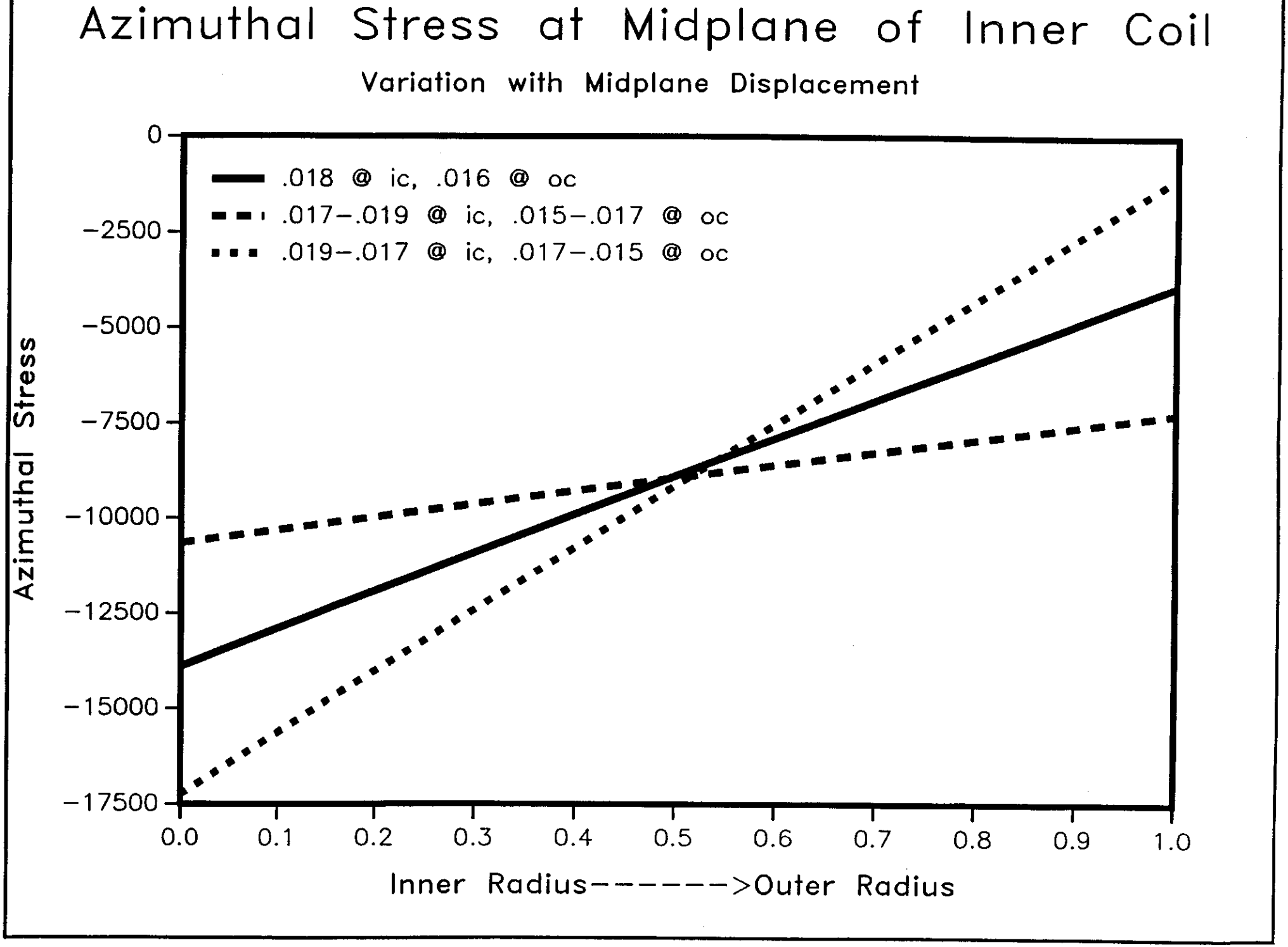

Azimuthal Stress at Midplane of Inner Coil
Variation with Midplane Displacement
.018 @ ic, .016 @ oc
.017–.019 @ ic, .015–.017 @ oc
.019–.017 @ ic, .017–.015 @ oc
Azimuthal Stress
0
−2500
−5000
−7500
−10000
−12500
−15000
−17500
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------>Outer Radius

Fig. 10

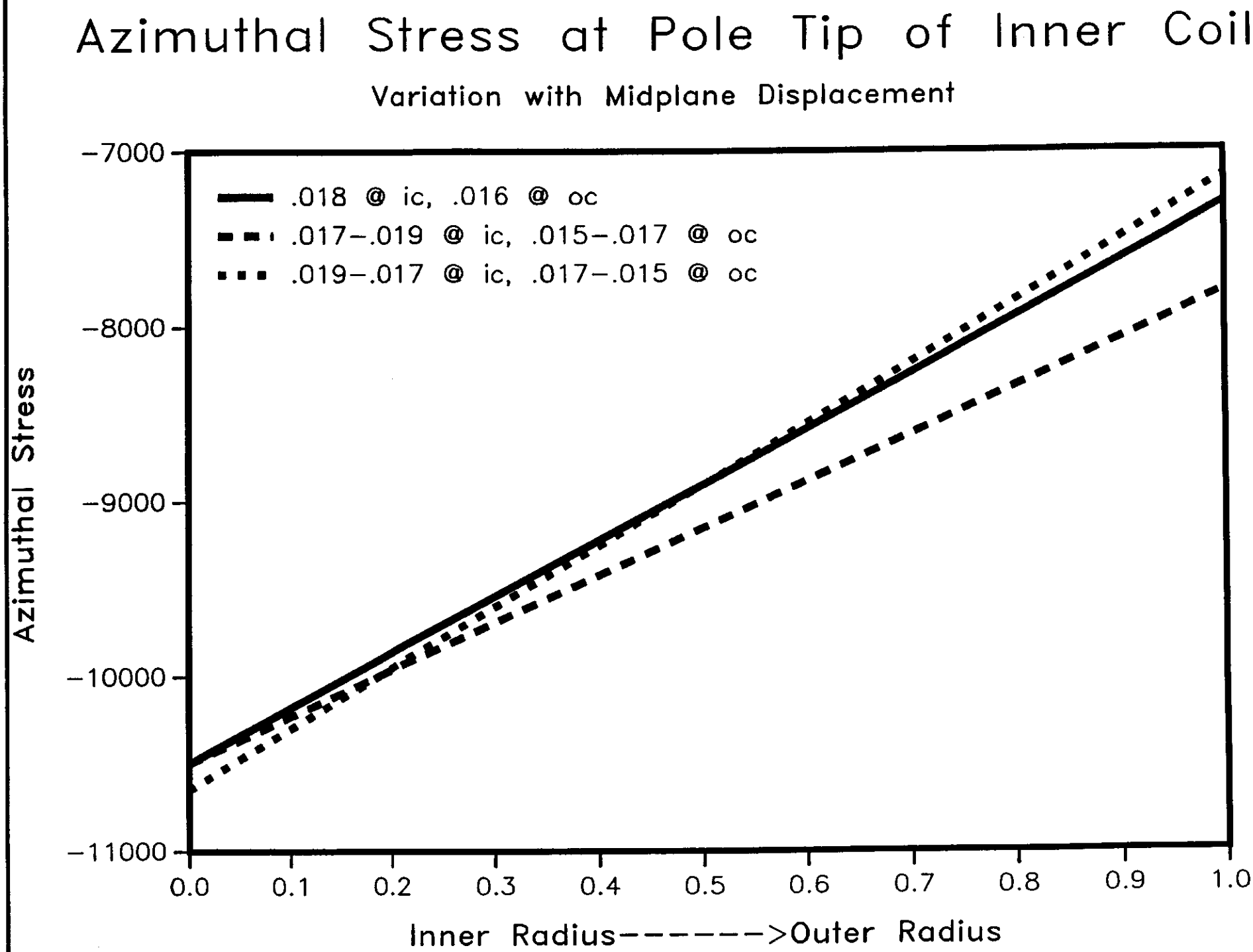

Azimuthal Stress at Pole Tip of Inner Coil
Variation with Midplane Displacement
.018 @ ic, .016 @ oc
.017–.019 @ ic, .015–.017 @ oc
.019–.017 @ ic, .017–.015 @ oc
Azimuthal Stress
−7000
−8000
−9000
−10000
−11000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------>Outer Radius

Fig. 11

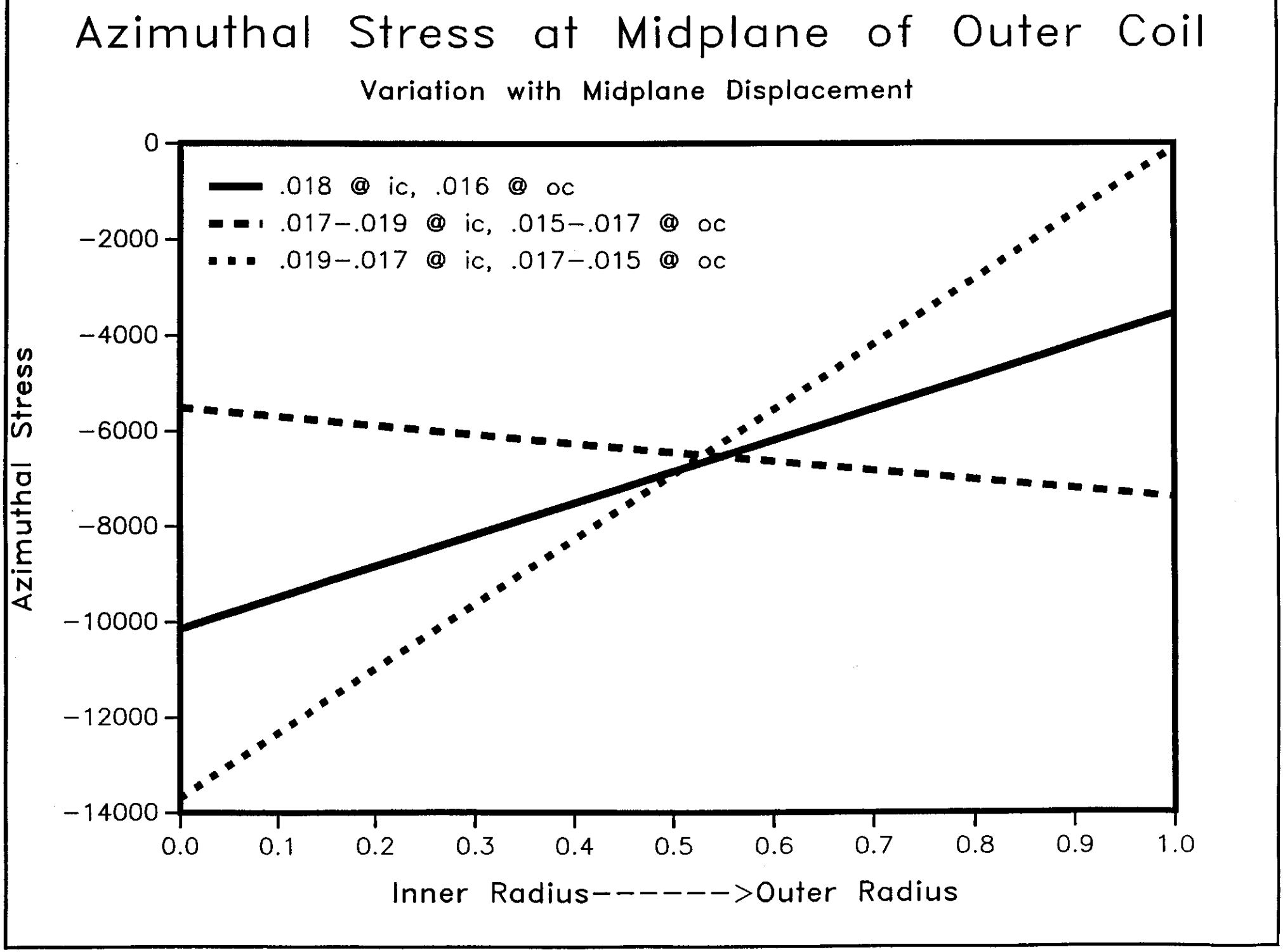

Azimuthal Stress at Midplane of Outer Coil
Variation with Midplane Displacement
.018 @ ic, .016 @ oc
.017–.019 @ ic, .015–.017 @ oc
.019–.017 @ ic, .017–.015 @ oc
Azimuthal Stress
0
−2000
−4000
−6000
−8000
−10000
−12000
−14000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------>Outer Radius

Fig. 12

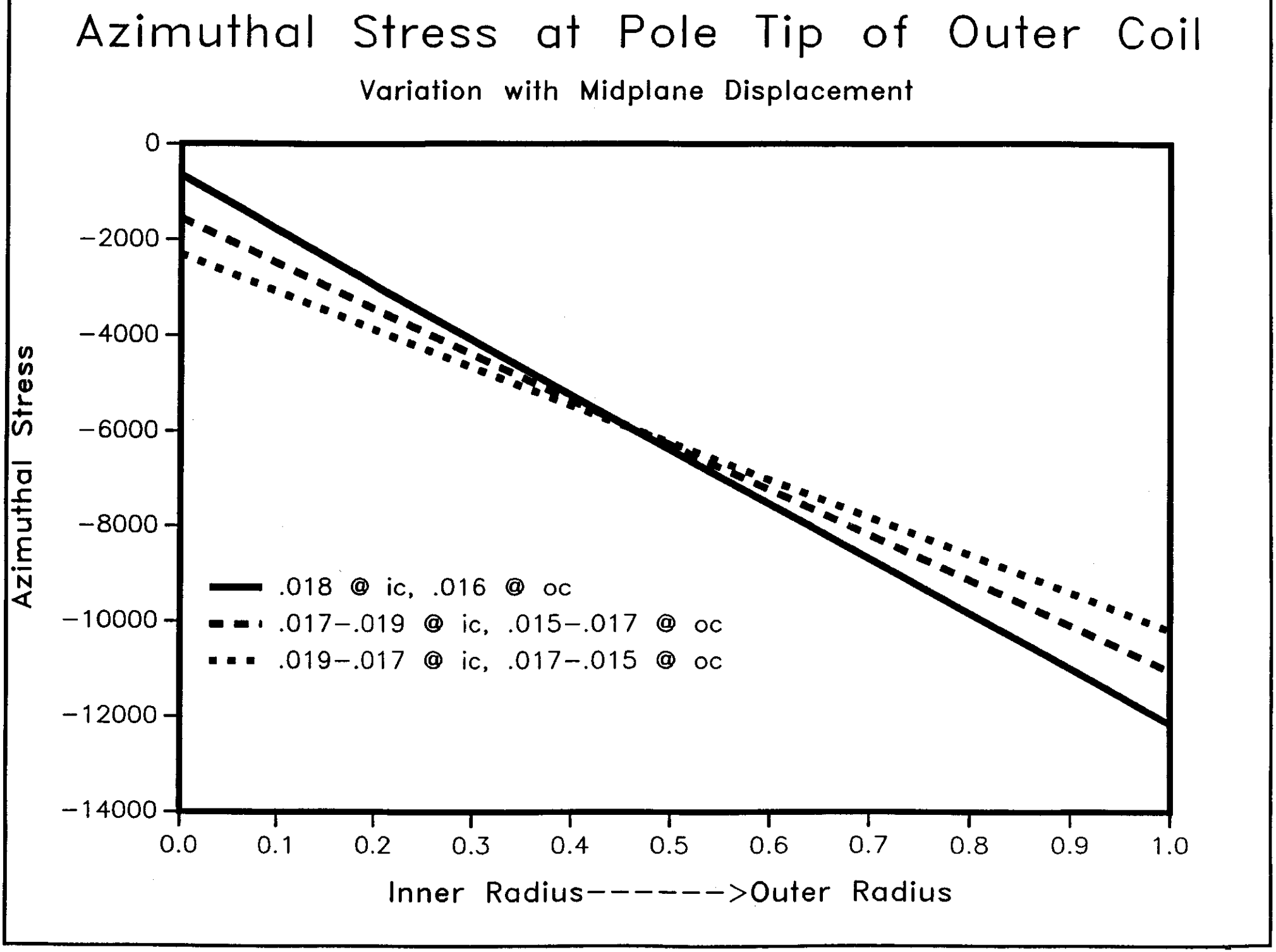

Azimuthal Stress at Pole Tip of Outer Coil
Variation with Midplane Displacement
Azimuthal Stress
0
-2000
-4000
-6000
-8000
-10000
-12000
-14000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius
.018 @ ic, .016 @ oc
.017-.019 @ ic, .015-.017 @ oc
.019-.017 @ ic, .017-.015 @ oc

Fig. 13

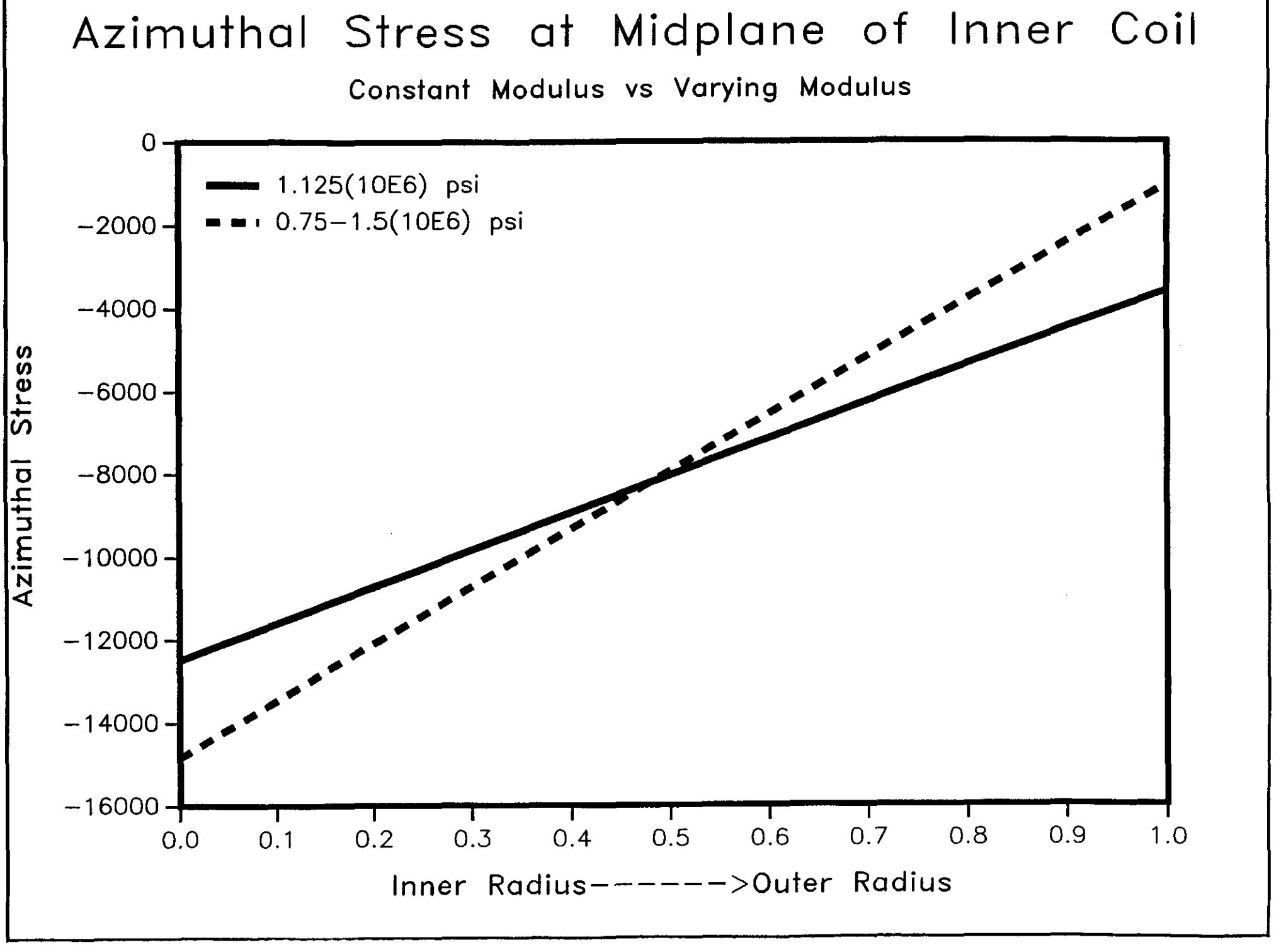

Azimuthal Stress at Midplane of Inner Coil
Constant Modulus vs Varying Modulus
1.125(10E6) psi
0.75–1.5(10E6) psi
Azimuthal Stress
0
–2000
–4000
–6000
–8000
–10000
–12000
–14000
–16000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------>Outer Radius

Fig. 14

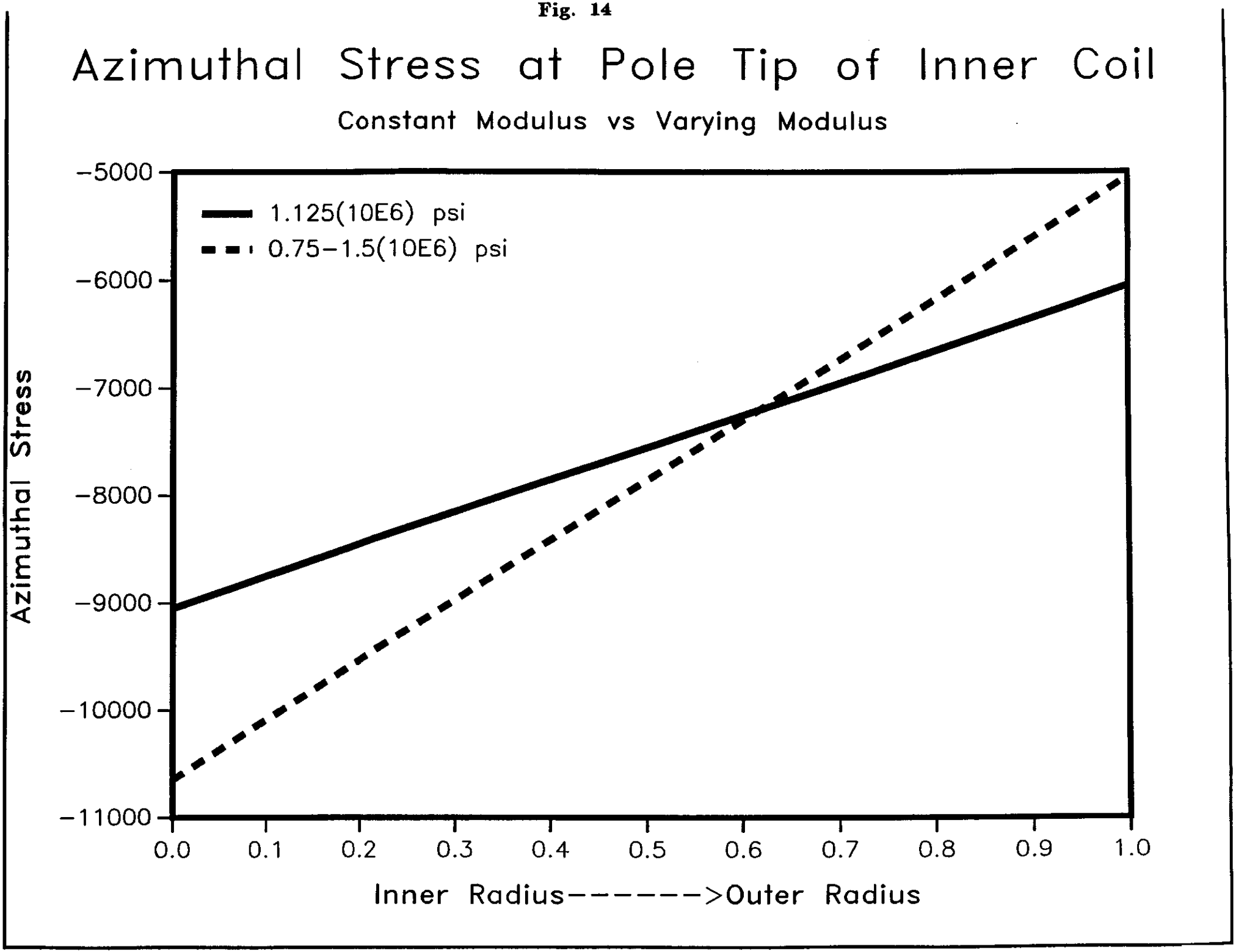

Azimuthal Stress at Pole Tip of Inner Coil
Constant Modulus vs Varying Modulus
1.125(10E6) psi
0.75–1.5(10E6) psi
Azimuthal Stress
-5000
-6000
-7000
-8000
-9000
-10000
-11000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius

Fig. 15

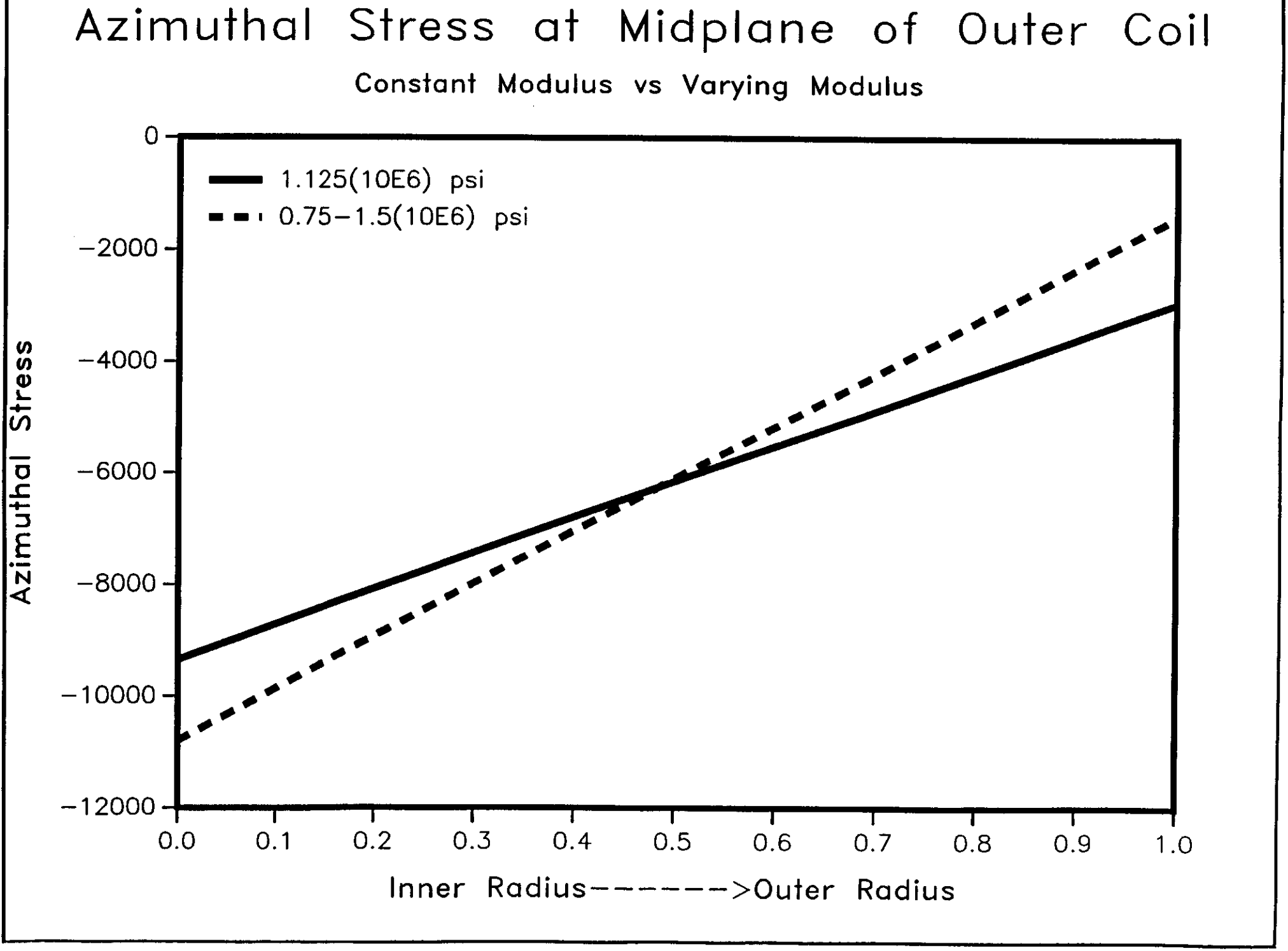
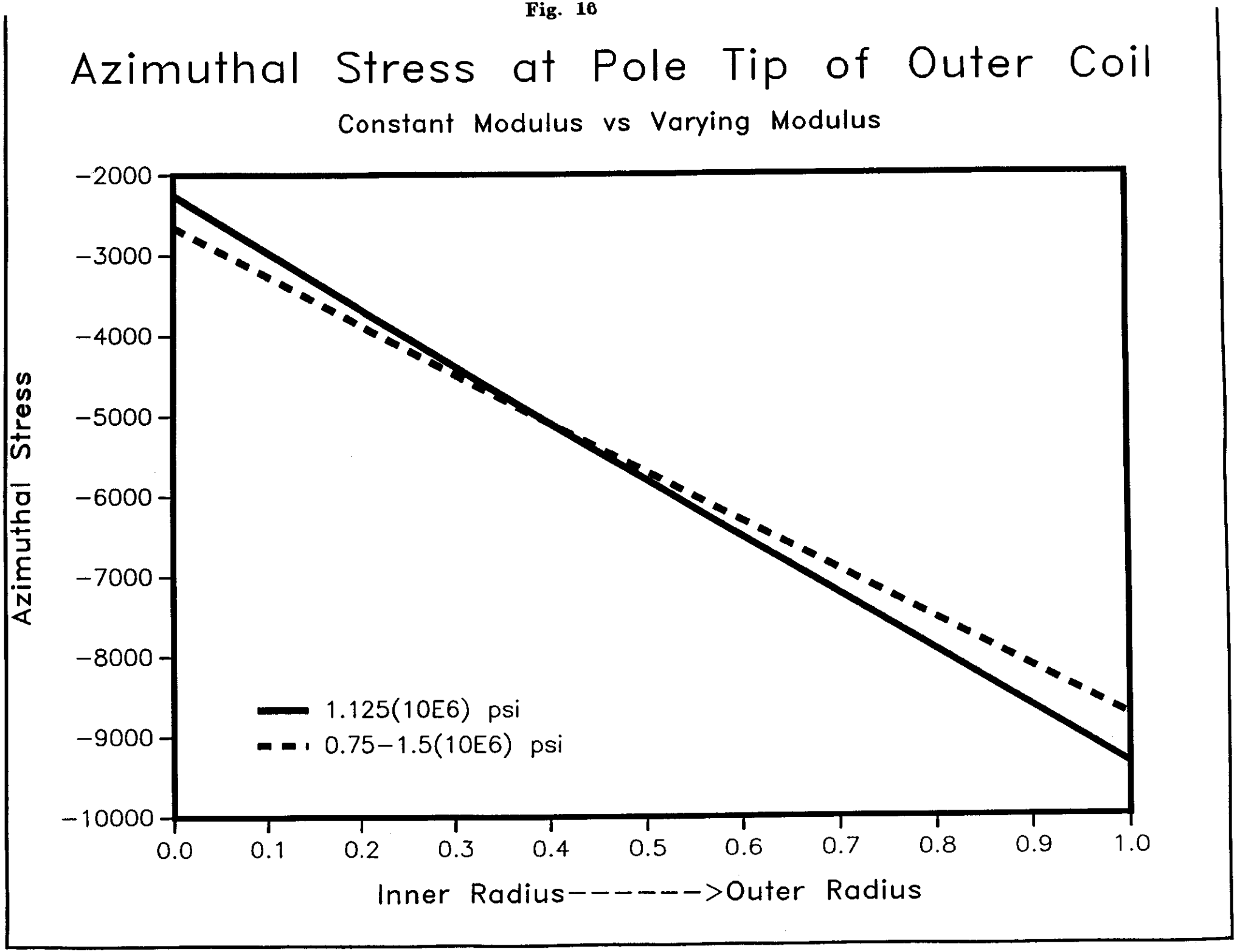

Azimuthal Stress at Midplane of Outer Coil
Constant Modulus vs Varying Modulus
1.125(10E6) psi
0.75–1.5(10E6) psi
Azimuthal Stress
0
–2000
–4000
–6000
–8000
–10000
–12000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius

Fig. 16


Azimuthal Stress at Pole Tip of Outer Coil
Constant Modulus vs Varying Modulus
Azimuthal Stress
-2000
-3000
-4000
-5000
-6000
-7000
-8000
-9000
-10000
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inner Radius------->Outer Radius
1.125(10E6) psi
0.75-1.5(10E6) psi