Feed-Down Correction in Superconducting Magnets

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Introduction

The HERA magnet measuring group corrects for feed-down errors by making use of the current dependence of the multipole strengths.[1] This simple technique should, of course, be applicable to the SSC superconducting magnets as well. The technique is limited in the case where appreciable non-allowed persistent-current multipoles are produced due to asymmetric composition of the magnet coils. A similar technique was discussed by Herrera et al.[2]

Background

"Feed-down" errors are magnetic measuring errors caused by off-centering errors in the measuring equipment. Consider the magnetic field of a dipole described as usual by a multipole expansion about the center of the magnet ($x = 0, y = 0$):

$$B_y + iB_x = B_0 \sum (b_n + ia_n) \left(\frac{x + iy}{r_0}\right)^n \tag{1}$$

where b_n and a_n are the "normal" and "skew" coefficients for the $2(n + 1)$ multipole, B_0 is the dipole field strength, and r_0 is the reference radius (1 cm for SSC magnets). If the center of the measuring equipment is off by Δx horizontally and Δy vertically, additional contributions to each multipole appear from higher multipoles. To first order (using only terms linear in Δx and Δy) the additional contributions to b_{n-1} and a_{n-1} are:

$$\Delta b_{n-1} = n \left(\frac{\Delta x}{r_0} b_n - \frac{\Delta y}{r_0} a_n\right) \tag{2a}$$

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[1]Rainer Meinke, private communication, Oct. 1988.

[2]J. Herrera, H. Kirk, A. Prodell and E. Willen, "Magnetic Field Measurements of Superconducting Magnets for the Colliding Beam Accelerator," 12th International Conference on High Energy Accelerators, August 1983, p. 563.

$$\Delta a_{n-1} = n \left(\frac{\Delta x}{r_0} a_n + \frac{\Delta y}{r_0} b_n \right) \quad (2b)$$

and if the magnet has up-down symmetry ($a_n = 0$), these expressions simplify to:

$$\Delta b_{n-1} = n \left(\frac{\Delta x}{r_0} \right) b_n \quad (3a)$$

$$\Delta a_{n-1} = n \left(\frac{\Delta y}{r_0} \right) b_n \quad (3b)$$

Feed-Down Correction

The method of finding the off-center error in the measuring equipment used by the HERA group is to exploit the current dependence of the persistent-current multipoles. Consider the feed down from the normal sextupole coefficient to the normal and skew quadrupole coefficients. Each measured coefficient can, in principle, have contributions from: (a) geometric effects, which do not depend on the dipole excitation level (we are excluding coil deformation and yoke saturation effects, which should be negligible at low excitation), (b) persistent-current effects in the superconducting coils, which do depend on magnet excitation level, and (c) feed down from the next higher multipoles a_{n+1} and b_{n+1}.

In well made dipole magnets, only allowed normal persistent-current multipoles should be produced—i.e., only b_0, b_2, b_4, etc., should be present due to persistent-currents. In this case the measured normal sextupole coefficients can be expressed as:

$$b_2 = b_{2g} + b_{2pc}(\mathrm{I}) + 3 \left(\frac{\Delta x}{r_0} \right) b_3 - 3 \left(\frac{\Delta y}{r_0} \right) a_3 \quad (4a)$$

$$a_2 = a_{2g} + 3 \left(\frac{\Delta y}{r_0} \right) b_3 + 3 \left(\frac{\Delta x}{r_0} \right) a_3 \quad (4b)$$

where b_{2g} and a_{2g} are the geometric sextupole contributions and $b_{2pc}(\mathrm{I})$ is the normal persistent-current contribution, which is a function of current. The octupole feed-down contributions are assumed to be geometric only. The two measured quadrupole coefficients then are:

$$b_1 = b_{1g} + 2\left(\frac{\Delta x}{r_0}\right) b_2 - 2\left(\frac{\Delta y}{r_0}\right) a_2$$

$$a_1 = a_{1g} + 2\left(\frac{\Delta y}{r_0}\right) b_2 + 2\left(\frac{\Delta x}{r_0}\right) a_2$$

or

$$b_1 \cong b_{1g} + 2\left(\frac{\Delta x}{r_0}\right) b_{2g} - 2\left(\frac{\Delta y}{r_0}\right) a_{2g} + 2\left(\frac{\Delta x}{r_0}\right) b_{2pc}(\mathrm{I}) \tag{5a}$$

$$a_1 \cong a_{1g} + 2\left(\frac{\Delta y}{r_0}\right) b_{2g} + 2\left(\frac{\Delta x}{r_0}\right) a_{2g} + 2\left(\frac{\Delta y}{r_0}\right) b_{2pc}(\mathrm{I}) \tag{5b}$$

where we have dropped the octupole terms because they are quadratic in Δx and Δy. (The octupole terms could have been retained since they are purely geometric in this case.)

Using the current dependence of b_1, a_1, and b_2 now eliminates the geometric terms:

$$\frac{\Delta b_1}{\Delta \mathrm{I}} = 2\left(\frac{\Delta x}{r_0}\right)\frac{\Delta b_2}{\Delta \mathrm{I}} \tag{6a}$$

$$\frac{\Delta a_1}{\Delta \mathrm{I}} = 2\left(\frac{\Delta y}{r_0}\right)\frac{\Delta b_2}{\Delta \mathrm{I}} \tag{6b}$$

Thus we can solve for the off-center errors Δx and Δy in terms of the slopes of the measured b_1, a_1, and b_2 excitation curves.

A check on this scheme is to solve for Δx and Δy from the decapole/octupole feed down ($b_4 \rightarrow b_3, a_3$) as well as from the sextupole/quadrupole feed down and to compare the results. For the decapole/octupole feed down we similarly obtain:

$$\frac{\Delta b_3}{\Delta \mathrm{I}} = 4\left(\frac{\Delta x}{r_0}\right)\frac{\Delta b_4}{\Delta \mathrm{I}} \tag{7a}$$

$$\frac{\Delta a_3}{\Delta \mathrm{I}} = 4\left(\frac{\Delta y}{r_0}\right)\frac{\Delta b_4}{\Delta \mathrm{I}} \tag{7b}$$

As a test of this scheme the measured multipoles of the BNL 1.8-meter model dipole magnet DSS–010 were used (extracted from up-ramp, center 30" table)[3]:

I	b_1	a_1	b_2	b_3	a_3	b_4
400A	−.72	1.64	−14.81	.31	−.73	2.31
600A	−.47	1.14	−8.87	.23	−.59	1.49
Δ	+.25	−.50	+5.94	−.08	+.14	−.82
$\Rightarrow \frac{\Delta x}{r_0}$	+.021			+.024		
$\frac{\Delta y}{r_0}$		−.042			−.043	

Thus the two sets of $\Delta x, \Delta y$ agree remarkably well, showing both that the method does indeed work and that the magnet DSS–010 was quite symmetric in its arrangement of superconducting material.

This method of evaluating the centering offset is not so simple if there are appreciable non-allowed persistent-current multipoles, such as $b_{1pc}, a_{1pc}, b_{3pc}, a_{3pc}$. Calculations by Mike Green[4] indicate that a 10 percent up-down asymmetry in critical current or filament diameter, for example, produces "non-allowed" persistent-current multipoles strengths of $b_1 = 0$, $a_1 = 1.0$ unit, $a_2 = 0$, $b_3 = 0$, $a_3 = 0.04$, for the nominal case of 5 micron filaments, J_c 2420A per mm^2 (4.2K, 5T), and "allowed" multipole strengths of $b_2 = -6.5$, $b_4 = 0.6$ unit at the SSC injection field 0.33 tesla.

If such non-allowed persistent-current multipoles are present at a significant level, their existence would be indicated either by: (a) discrepancies in the $\Delta x, \Delta y$ obtained from sextupole/quadrupole analysis versus those obtained by decapole/octupole analysis or (b) the resultant $\Delta x, \Delta y$ being "current dependent," since real persistent-current a_1 coefficients (due to magnet asymmetry) have a current dependence that is somewhat different from that of the persistent-current b_2, as illustrated in Figure 1.

[3]BNL Internal Report TMG-376, Apr. 1988.
[4]M. A. Green, SSC-N-377, Aug. 1987.

A further check on the feed-down correction can, in principle, be made using the current dependence of the measured multipoles caused by saturation effects in the iron yoke, as was mentioned by Peter Wanderer.[5] Yoke saturation can cause significant changes mainly in the sextupole component, so that the same feed-down-correction technique can be applied here as well as in the persistent-current regime.

[5]Peter Wanderer, private communication, Nov. 1988.

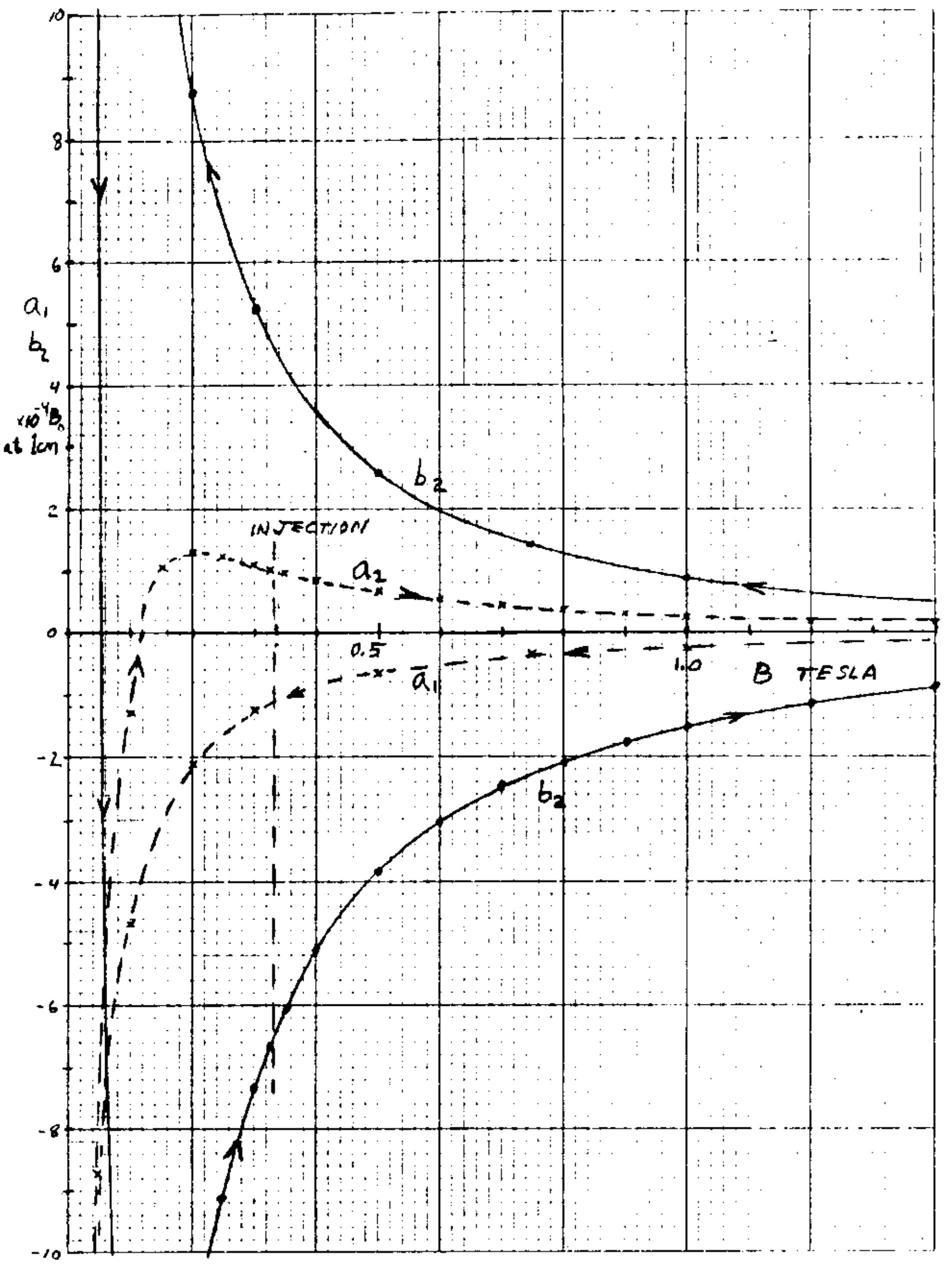

Fig. 1. The computed variation of the persistent-current skew quadrupole coefficient a_1 and the normal sextupole coefficient b_2 during a down ramp and the following up ramp of a magnet excitation cycle between 0 and 5.5 tesla for a magnet with an up-down asymmetry of 10% in filament diameter (5.0 ± 0.25 microns). Calculations by M.A. Green[4] for J_c of 2.42 kA/mm^2 at 5T and 4.2K in an NC-7 (LBL) dipole.