SSC-N-543
Sep. 1988

DEGREE OF CONFIDENCE IN PREDICTIONS OF "COLD" MULTIPOLES BASED ON "WARM" FIELD MEASUREMENTS ON SSC DIPOLE MAGNETS *)

B. Berkes, SSC/CDG, Berkeley

SUMMARY **)

Because only a small number of the dipole magnets will be measured both cold and warm, and the rest mainly warm [1], one has to determine this number in order to comply with the requirements for the overall field quality of cold magnets. Based on various degrees of confidence, and regardless of the magnetic field measurement precision, the number of magnets n to be measured in both states was determined. However, the figures for n depend very strongly upon the field requirements for the particular multipole and are generally of the order of several hundred. The results do, in any case, exceed by far the number of 100 magnets, anticipated in [1], even for a lower value of the degree of confidence (90%).

The measurement precision is not correlated to the magnetic field quality distribution, which is only a function of the magnet design and manufacturing tolerances. However,with the help of certain assumptions, one can find a correlation between the precision of the measuring equipment,the number of measurement steps along the magnet (in order to determine the integral multipoles),and the magnitude of the random multipole tolerance. The calculations show that it is possible to comply with the requirements for the multipole tolerances already by achieving a measurement precision (sigma) of the order of 0.5 "units", corresponding to a range error of +-1.50 "units" ([6], Table 12 , p. 248) due to electrical and mechanical uncertainities.

---

*) Talk given at the Magnet System Integration Meeting on Aug.11/12, 1988 at Brookhaven National Laboratory, Upton, NY. This report represents the revised and modified version of the draft (and its annex) attached to the minutes of the said MSIM (Sep. 1, 1988). Therefore,some data in the report will not necessarily be identical to those in the draft.

**) In what follows, it is - if not otherwise stated - implied that one is dealing with normal and skew INTEGRAL geometric multipoles (harmonics); see definitions.

LIST OF SYMBOLS

N	Population (finite) of an entirety of elements (for the case of SSC this refers to the total number of dipole magnets).
n	Sample of elements (magnets), taken out of the total population of N magnets.
$\bar{x}$	Value of an integral geometric multipole of a magnet; the average of a particular multipole over the ENTIRE MAGNETIC LENGTH of the magnet.
μ	Mean value of the distribution of of a multipole of N magnets (systematic multipole tolerance [12] *)).
σ_μ	Standard deviation of the distribution of multipoles of N magnets (random multipole tolerance [12] *)).
$\bar{\bar{x}}$	Average of the population of a sample of n magnets.
$\sigma_{\bar{\bar{x}}}$	Standard deviation of the distribution of n magnets.
$\langle\bar{\bar{x}}\rangle$	Grand average (identical to μ) of the distribution of averages.
$\sigma_{\langle\bar{\bar{x}}\rangle}$	Standard deviation of the grand average or mean (for the case of SSC this refers to the error of the systematic multipole tolerance).
$\delta\sigma_{\bar{\bar{x}}}/\sigma_{\bar{\bar{x}}}$	Relative error of the standard deviation of $\sigma_{\bar{\bar{x}}}$.
$2R$	Difference between the smallest and the largest readings in a sample.

---

*) Neither μ nor σ_μ are necessarily equal to the tolerances according to Table I , even with the best manufacturing care (see also ch. A , 3rd para.). However, for reasons of simplicity, it will be assumed that they comply with the spcified values.

LIST OF DEFINITIONS

ALPHA	Level of significance; the probability with which one is willing to risk rejecting the hypothesis about the population mean even though it is true ([5],p. 209).
CONFIDENCE	The chance (expressed as a percentage) that a certain percentage of all future values generated by a measurement process will lie within bounds determined by previous sets of measurements.
KURTOSIS	The peakedness or the flatness of the graph of a frequency distribution, esp. to the normal concentration of values near the mean as compared with the normal distribution ([4], p. 55).
SKEWNESS	Nonsymmetric distribution about the mean ([4], p. 55).
STANDARD DEVIATION	The square root of the average of the squared deviations from the population mean.Because the mean is,in most practical situations, unknown, it has to be replaced by the sample average. Here, the squared deviations from the average are smaller than from the mean ([5], p. 183), and therefore one has to replace the size of a sample n with n - 1. In this report, the difference between n and n - 1 will be neglected, due to anticipated n >> 1 .
UNIT	Numerical value: 1.0*10**(-4); in the SSC terminology normally used in relation between the amount of a multipole vs. the dipole strength.
VARIANCE	The square of the standard deviation.

A. INTRODUCTION

Once the dipole magnet design has been finalized and its production started, a certain number of magnets [1] shall be - among other tests - proved for their field quality in both the warm and cold states. By comparing the measurement results in both states, one can determine the necessary NUMBER of magnets to be measured, in order to obtain a required DEGREE OF CONFIDENCE in the cold multipole values of consecutive magnets, which will be measured only warm.

Since the results of all measurements are variable, one can make a reasonable guess about the future results (cold field on basis of warm measurements) only if one can obtain a measure of how much variability, or spread, one can expect to see under normal measurement conditions. However, variability cannot be measured accurately with a SMALL number of data, thus, all precision predictions made until now ([2], [3]) have more a character of a JUDGMENT rather than of a serious calculation based on the laws of statistics.

At this time one has to point out clearly that in the present report it is anticipated that all magnets are manufactured within the specified mechanical and electrical tolerances (otherwise, tooling corrections would have to be made), and therefore one has to deal only with statistical problems of the field quality of the magnets themselves.

The next problem, which has to be solved in this context, is the degree of PRECISION of the PRODUCTION MEASURING EQUIPMENT. During the R & D phase of the SSC dipole magnet program, it is obvious that one is primarily interested in the exact knowledge of the prototype magnet field behaviour. Therefore, the measurement results shall be influenced by the measuring equipment as little as possible, i.e., the precision of the equipment used has to be very high.

Unfortunately, the precision of the production measuring equipment is NOT CORRELATED in any way to the magnets, which have to be measured, or to their number. To fulfill the requirements of the accelerator physicist for acceptable values (systematic as well as random) of the field multipoles, one has to make reasonable assumptions for the measurement tolerances for the integral multipoles. This will in turn lead to the PRECISION REQUIREMENT for the measuring equipment.

B. MEASUREMENT CONDITIONS AND DATA HANDLING

No matter which kind of a sensor (coils, NMR-, or Hall-probes) are used for the detection of the magnetic field, the RAW data from these sensors have to be processed in such a way that the final results will consist solely of the terms due to the magnet geometry. Therefore, the raw data have to be subjected to the following sequential procedure:

1. WARM measurements have to be made at the same value of positive and negative current, in order to be able to remove possible effects of the iron magnetization (see para. 4).

2. COLD measurements must be performed at up-ramp as well as down-ramp current changes, in order to be able to remove, or at least considerably reduce, the contribution of the persistent currents (see para. 5). Preceding this,magnets have to undergo a current-cycling procedure.

3. On the basis of raw data - WARM and COLD - Fourier analyses of the magnetic fields in both magnet states have to be made.

4. Subtracting the corresponding multipole for positive and negative WARM magnet currents, one can to determine the remnant part of the particular multipole and deduct it from the total multipole value ([3] *); see also para. 1) .

5. The influence of the persistent current effects on a particular multipole is reduced (see para. 2) due to a cancelation of the up-ramp and down-ramp multipole dependence upon magnet current. A full compensation of the persistent current influence on the multipole will not be possible over the entire current range - at lower currents due to the lack of symmetry of the hysteresys curve and at higher currents because of steel saturation effects. Therefore, one has to choose a reference field level (as an example: 3 T) for cold - warm comparison purposes.

6. TIME DEPENDENT effects on cold magnets [13] can also easily falsify the final data (geometrical multipole). It will be, therefore, necessary to determine a certain time interval before the cold measurements shall start; the value of that delay shall remain CONSTANT for all cold measurements.

7. After the steps described in para. 5 and 6 were performed, the resulting multipoles will be "pure" geometric ones. However,they will still include the FEED-DOWN multipoles,due to off-centering of the measuring device vs. the geometrical axis of the magnet bore, as well as to the difference between the geometrical and magnetic axes of the magnet itself. It will be necessary to develop a method to determine the magnitude of different feed-down multipoles in order to eliminate their influence on the final multipole results. This method would have to be generally accepted by all physicists and magnet engineers involved in the accelerator design. One of such methods, suggested recently by HERA-physicists, seems to be effective [14].

---

*) One shall observe the definition of the multipole sign in [3] (ch. WARM MEASUREMENTS, 2nd para.).

C. ACCELERATOR REQUIREMENTS

For completeness, the tolerances of systematic and random multipoles are presented in Table I ([1],ch. 4.3.1; [9]).

TABLE I.

Systematic and Random Tolerances of the Multipole Field Components *)
[in "units" of dipole field at 1 cm radius]

Multipole Coeff.	Systematic Tol.	Random Tol.
a1	0.2	0.7
a2	0.1	0.6
a3	0.2	0.7
a4	0.2	0.2
a5	-	0.2
a6	-	0.1
a7	-	0.2
a8	-	0.1
b1	0.2	0.7
b2 #)	1.0	2.0 **)
b3 #)	0.1	0.3
b4 #)	0.2	0.7
b5	0.04	0.1
b6	0.07	0.2
b7	0.1	0.2
b8	0.2	0.1

---

*) It shall be explicitely noted that the multipole tolerances concern the magnets as a WHOLE and are NOT meant for INDIVIDUAL units.

**) After "Binning": 0.4 [10].

#) Corrector elements will be added to the accelerator in order to reduce (correct) the systematic and the random sextupole, octupole, and decapole errors in the SSC dipole magnets [11].In this case the value of "h" (see formula (2)) can be chosen differently, which has been intentionally disregarded in the calculations of the present report.

D. MULTIPOLE TOLERANCE ERRORS AND DEGREE OF CONFIDENCE

It is in the nature of things that the distribution law of the multipoles is not known in advance. However, in order to perform any kind of statistical calculations and predictions, one has to deal with certain assumptions. The main one would be that the multipoles of all magnets will behave according to the GAUSSIAN (normal) distribution *). Though there is no conclusive necessity that the magnet multipoles will follow the Gaussian distribution, most of the statistic formulas were developed for that kind of a distribution, which facilitates the calculations. Once the magnets go into production and are measured, one has to PROVE the assumption of the Gaussian distribution by checking the kurtosis and skewness (test of normality: [4], p. 522; see also definitions).

One shall start from the fact that - speaking in terms of statistics - there is a finite population of the size N = 7680 dipole magnets. From this population one shall take out a sample of n magnets and measure them cold and warm. The distributions of N and n magnets will not be exactly the same (Figs. 1a. - 1c.), and therefore the mean μ and the average $\bar{\bar{x}}$ as well as the standard deviations σ_μ and $\sigma_{\bar{x}}$ will be different. However, if one would repeat this procedure of taking out of the magnet population N many times samples of n magnets, one can derive the relation between the standard deviation of the population of averages $\sigma_{<\bar{\bar{x}}>}$ (error of the systematic multipole tolerance) and the sample standard deviation $\sigma_{\bar{x}}$ (random multipole tolerance).

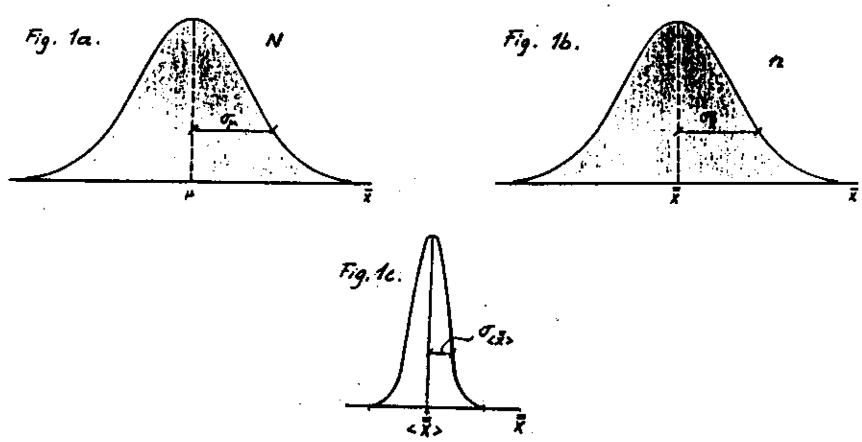

Fig. 1a. Distribution of a large finite population of the size N (total number of magnets) with the mean μ and standard deviation σ_μ.

Fig. 1b. Distribution of a sample of n magnets of measured integral multipole $\bar{x}$, average $\bar{\bar{x}}$, and standard deviation $\sigma_{\bar{x}}$.

Fig. 1c. Distribution (virtual) of the averages $\bar{\bar{x}}$, with the grand average $<\bar{\bar{x}}>$ (equal to μ) and the standard deviation of the sampling distribution of the averages $\sigma_{<\bar{\bar{x}}>}$ (also: standard error of the mean; in the case of the SSC this refers to the error of the systematic multipole).

---

*) Other assumptions can be found in the consecutive text.

Due to the fact that in the statistics there exist distributions for means as well as for standard deviations (both including in their formulas for the degree of confidence the number n of observations (magnets), one has to decide a priori which of these two distributions is of larger importance. From the standpoint of the accelerator it is highly desirable to keep the systematic multipoles below their tolerable values - even for those multipoles, which will be corrected ([15]; see also p. 6, 3rd footnote). Therefore, the basis for further calculations will be the statistics of the mean.

Because neither $<\bar{\bar{x}}>$ nor μ is known, one shall take the average $\bar{\bar{x}}$ of the sample of n magnets as the best estimate for the systematic multipole tolerance μ $(=<\bar{\bar{x}}>)$. Looking for the moment at one kind of measurement only (for instance: warm) and using formula (1) for the confidence limits of an infinite population, one could determine the NUMBER of MAGNETS to be measured in order to fulfill the field requirements for this particular kind of measurement *) . (The values of $z_{\alpha/2}$ for a certain degree of confidence are listed in Table II .)

$$-z_{\alpha/2}\cdot\frac{\sigma_\mu}{\sqrt{n}} < \bar{\bar{x}} - \mu < + z_{\alpha/2}\cdot\frac{\sigma_\mu}{\sqrt{n}} \tag{1}$$

TABLE II.

Dependence of $z_{\alpha/2}$ on the degree of confidence (DOC; [4],p. 460)

α = 0.100	DOC = 90 %	$z_{\alpha/2}$ = 1.645
0.050	95 %	1.960
0.010	99 %	2.576
0.001	99.9 %	3.291

Taking - as an illustrative example - for "h" a maximum value of 30% of the systematic multipole and correcting (1) for the fact that one is dealing with a finite population N rather than with an infinite one (see (2)), the values of of n could be calculated for each multipole. However, one is not interested in the behaviour of n cold or warm

$$z_{\alpha/2}\cdot\frac{\sigma_\mu}{\sqrt{n}}\cdot\sqrt{\frac{N-n}{N-1}} = h \tag{2}$$

magnets ONLY, but rather the PREDICTION of the behaviour of all future N - n cold magnets. These magnets will be measured in the warm state only [1] (NEGLECTING in the following calculations the fact that (N - n)/10 magnets will be ALSO measured in the cold state) and the predictions will be made on basis of the previously determined cold - warm differences of n magnets (see (3)). This, in turn, means that

---

*) In other words, for a given n , formula (1) tells how much the average $\bar{\bar{x}}$ will be "off" the population mean μ by less than a value "h" in $(1-\alpha)*100$ % of the time (α being the "level of significance"). The numerical value of "h" has not yet been determined, but setting it equal to $\sigma_{<\bar{\bar{x}}>}$ would be a reasonable assumption (incidentally, $\sigma_{<\bar{\bar{x}}>}$ has also not yet been determined).

$$(\bar{\bar{x}}_c)_{future} = (\bar{\bar{x}}_w)_{meas.} + (\bar{\bar{x}}_c - \bar{\bar{x}}_w)_{prev.\ determ.} \tag{3}$$

the variances of cold AND warm multipole standard deviations $\sigma_{\bar{\bar{x}}}$ of n magnets must be ADDED in order to get the total variance of the measurements (4) (see also [4], p. 91; [6], p. 69). Therefore, one has to replace $\sigma_\mu/\sqrt{n}$ in (2) by the expression (4) and solve the obtained equation for n with the assumption that both cold and warm variances will be in the first approximation equal (which MAY not necessarily be the case) (5). The data in Table III are obtained accordingly.

$$\frac{\sigma_\mu}{\sqrt{n}} \Longrightarrow \sigma_{\bar{\bar{x}}} \cdot \sqrt{\frac{2}{n}} \tag{4}$$

$$\left(\frac{z_{\alpha/2}}{h} \cdot \sigma_{\bar{\bar{x}}}\right)^2 \cdot \frac{2}{n} \cdot \frac{N-n}{N-1} = 1; \quad n = \frac{2B^2 \cdot \frac{N}{N-1}}{1 + 2B^2 \cdot \frac{1}{N-1}}; \quad B = \frac{z_{\alpha/2}}{h} \cdot \sigma_{\bar{\bar{x}}} \tag{5}$$

The above is exactly true only if the cold and warm samples are "independent" ([4], p. 499; [5], p. 221) i.e. the selection of one sample is in no way affecting the selection of the other. For dependent samples (warm and cold measurements performed on the same sample of n magnets) one has to test by the "null hypothesis" that the differences between the cold and warm data constitute a random sample from a population with the mean equal to the "shift" [2] in the particular multipole between both states ([5], p. 225).

T A B L E III.

Number of magnets n to be masured cold and warm for a required degree of confidence in the cold behaviour of N - n magnets

Multipole	DOC = 90 %	DOC = 95 %	DOC = 99 %
a1	672	920	1,463
a2	1,689	2,195	3,139
a3	672	920	1,463
a4	60	84	145
a5	-	-	-
a6	-	-	-
a7	-	-	-
a8	-	-	-
b1	672	920	1,463
b2	233	327	548
b3	505	699	1,132
b4	672	920	1,463
b5	358	499	823
b6	461	639	1,040
b7	233	327	548
b8	15	21	37

From the data in Table III it is obvious that the number of magnets to be measured is not only quite large but also varies substantially with the requirements for the individual multipole. Looking, for example, at the normal sextupole only (before "Binning"), one would have to measure at least 233, 327 or 548 magnets, in order to comply with the data in Table I .

The values 90%, 95%, and 99% for the degree of confidence in Table III were chosen arbitrarily. In fact, for a project like SSC, where the magnets represent a large portion of the total accelerator cost, even 99% for the degree of confidence appears to be an understatement. However, a requirement of 99.9% for the degree of confidence would mean that practically ALL magnets have to be measured in the warm as well as in the cold state.

As one can see from Table III, the number of measured magnets mentioned in [1] (n = 100; see para. 5.2.11), is NOT SUFFICIENT to determine with the required precision the field quality of cold magnets on the basis of warm measurements. It will, therefore, be necessary to install more magnet test stands at the future SSC-site, in order to measure the number of magnets according Table III within a reasonable time period.

Finally, a short remark concerning the previous procedure: Once n has been determined, the limits (errors) of the random multipole tolerance σ_μ ($\doteq \sigma_{\bar{\bar{x}}}$) can be immediately calculated by the corresponding formula (6). Therefore, a determination of the errors of the random multipole tolerance by the accelerator physicists would appear superfluous.

$$\frac{\sigma_{\bar{\bar{x}}}}{1+\frac{z_{\alpha/2}}{\sqrt{2n}}} < \sigma_\mu < \frac{\sigma_{\bar{\bar{x}}}}{1-\frac{z_{\alpha/2}}{\sqrt{2n}}} \qquad (6)$$

E. PRECISION OF THE PRODUCTION MEASURING EQUIPMENT

Measuring a number of magnets, one has to distinguish between the "product error" $\sigma_{\bar{\bar{x}}}$ (variation of a particular integral multipole from magnet to magnet), which is determined by the design and manufacturing precision of the magnets, and the unavoidable "measurement error" of the equipment $\sigma_{\bar{\bar{e}}}$, used to determine the "product error" [8]. Because the magnet manufacturing procedure is completely independent of the the field measurement, one cannot establish a priori any correlation between the errors of the above mentioned quantities ([8], p. 246); the corresponding relation between the errors will then be

$$(\sigma_{\bar{\bar{x}}})_m^2 = \sigma_{\bar{\bar{x}}}^2 + \sigma_{\bar{\bar{e}}}^2 \qquad (7)$$

where $(\sigma_{\bar{\bar{x}}})_m$ represents the total standard deviation of measurements on a sample of magnets.

In practice, where the errors due to the measuring equipment vary statistically, one has therefore either to reduce their influenece on the total variance of the measurements or to make use of two or more identical measuring instruments for repeated measurements on the same "product" in order to determine $\sigma_{\bar{\bar{x}}}$ [8]. The second solution is very elaborate and time-consuming; it is therefore preferable to solve the problem with an assumption about the variance of the INTEGRAL measurement error distribution.One of the reasonable assumptions could be that the standard deviation of the measurements $\sigma_{\bar{\bar{\ell}}}$ shall not influence the total standard deviation $(\sigma_{\bar{\bar{x}}})_m$ by more than a certain fraction "f" of the random multipole tolerance. As an example, this fraction could be equal to the error of the random multipole tolerance,determined by (6), which leads to the relation

$$(\sigma_{\bar{\bar{x}}})_m = \left(\sigma_{\bar{\bar{x}}}^2 + \sigma_{\bar{\bar{\ell}}}^2\right)^{1/2} = (1+f)\,\sigma_{\bar{\bar{x}}} \Longrightarrow \sigma_{\bar{\bar{\ell}}} \doteq \sigma_{\bar{\bar{x}}}\cdot\sqrt{2f} \quad (8)$$

On the other hand,because the INTEGRAL multipoles represent the average of the INDIVIDUAL multipoles (which,in turn, are the result of a number of already fitted (Fourier analysed) data), measured at "p" positions along the magnet, one could try to establish a relation between the standard deviation of measurements σ_s at each of these "p" positions and the standard deviation $\sigma_{\bar{\bar{x}}}$ of the integral multipole. For this purpose one shall assume that ALL integral measurement errors are equal to $\sigma_{\bar{\bar{\ell}}}$. Comparing now (8) with the expression for the integral error $\sigma_{\bar{\bar{\ell}}} = \sigma_s/\sqrt{p}$, one can obtain the REQUIRED PRECISION of the measuring equipment for a SPECIFIED number of measuring positions as

$$\sigma_{\bar{\bar{\ell}}} = \frac{\sigma_s}{\sqrt{p}} = \sigma_{\bar{\bar{x}}}\cdot\sqrt{2f} \Longrightarrow \sigma_s = \sigma_{\bar{\bar{x}}}\cdot\sqrt{2.p.f} \quad (9)$$

T A B L E IV. *)

Multipole	$\delta\sigma_{\bar{\bar{x}}}/\sigma_{\bar{\bar{x}}}$ [%]	σ_s [p = 30]	2R	σ_s [p = 200]	2R
a1	4.8	1.18	4.9	3.07	17.1
a2	3.0	0.81	3.4	2.08	11.6
a3	4.8	0.51	2.1	3.07	17.1
a4	17.8	0.65	2.7	1.69	9.4
a5	-	-	-	-	-
a6	-	-	-	-	-
a7	-	-	-	-	-
a8	-	-	-	-	-
b1	4.8	1.18	4.9	3.07	17.1
b2	8.3	4.46	18.6	11.52	64.0
b3	5.5	0.55	2.3	1.41	7.8
b4	4.8	1.18	4.9	3.07	17.1
b5	6.6	0.20	0.8	0.51	2.8
b6	5.8	0.37	1.5	0.96	5.3
b7	8.3	0.45	1.9	1.15	6.4
b8	43.4	0.51	2.1	1.32	7.3

---

*) All σ_s - and 2R-values were calculated for 95% DOC and are in "units".

The above is valid only under the assumption that the production errors along the magnet will show (in the first approximation) the same effect in the cold as well as in the warm state. In this case they can be considered as systematic errors and will cancel in the expression for the cold - warm difference, so that one has to deal with statistical errors only.Furthermore, one has to assume that the measuring instrument shows NO systematic error or that this error is negligible; in case the systematic error is not negligible, one would have to determine it and subtract from the total measurement error of ALL measured data.

Table IV shows the data for $\delta\sigma_{\bar{\bar{x}}} / \sigma_{\bar{\bar{x}}}$, as well as those for the standard deviation of the measurement at each position σ_s with p = 30 (BNL "mole"-type of equipment) and p = 200 (an array of any kind of sensors, located in one plane and pulled through the magnet) for all multipoles.One can see from the data that the required precision of the measuring equipment can be relaxed by increasing the number of measurement steps along the magnet. Also,due to the relation between the standard deviation and the range of a sample distribution ([6], Table 12 , p. 248), one can determine the range of the measurement errors; the corresponding values can be found in Table IV .

The errors of the measuring equipment are of electrical as well as of mechanical origin.Assuming equal contributions from both effects to the total error, one eventually ends up with an allowable error (range) of the electrical part of the measuring equipment of some +-0.20 "units" for p = 30 vs. approx. +-0.70 "units" for p = 200 ; these most critical values were calculated from the data for the normal geometric 12-pole (Table IV) .

F. CONCLUSIONS

Pursuing the idea of measuring the magnetic field in the COLD state ONLY on every 10th dipole magnet, one would have to face the fact that the behaviour of ALL other magnets, measured solely in the WARM state, can be PREDICTED only with a certain DEGREE OF CONFIDENCE. If this confidence has to be reasonably high, as appears to be necessary for the SSC dipoles, one would have to measure the magnetic field at the very beginning of the magnet production both in the COLD and WARM state on MANY HUNDREDS of magnets.

The NUMBER of magnets to be measured in both states does not depend on the PRECISION of the measuring equipment, but only on the ACCELERATOR REQUIREMENTS and the DEGREE OF CONFIDENCE. In order to establish a correlation between the number of magnets and measurement precision, one has to make certain assumptions. With the assumption that the standard deviation of the integral measurements shall not be larger than the error of the random multipole tolerance, one can find out that the TOTAL ERROR (RANGE) of a "point" measurement along the magnet can be easily of the order of +-1.50 "units". This is valid even though the integral values of the multipoles have to be known to an order of magnitude higher precision.

ACKNOWLEDGEMENTS

J. Peterson, SSC/CDG, deserves my heartiest thanks for many valuable discussions and suggestions during the preparation of this report. The review of the manuscript and critical comments by T. Ferbel, R. Schermer and R. Coombes, all SSC/CDG, as well as K. Halbach, LBL, are also very much appreciated. I am particularly indebted to K. Metropolis for her help with my English.

Berkeley, CA / Brookhaven, NY
August 11/12, 1988
(1. Revision: September 12/15, 1988)
(2. Revision: November 01/05, 1988)

R E F E R E N C E S

[1] SSC/CDG : Conceptual Design of the Superconducting Super Collider, SSC-SR-2020, Berkeley (March 1986)

[2] Berkes, B. : SSC Dipole Magnets / Review of "Cold" vs. "Warm" Magnetic Field Measurement Data Comparison, SSC-N-529 (June 1988)

[3] Wanderer, P. : Comparison of Warm and Cold Multipoles in DSS Magnets, SSC-N-516 (June 1988)

[4] Duncan, A.J. : Quality Control and Industrial Statistics, 3rd edition, Richard D. Irwin, Homewood (1986)

[5] Freund, J.E. : Statistics, Prentice-Hall,Inc., Englewood Cliffs (1970)

[6] Crow, E.L. et al. : Statistics Manual,Dover Publications,Inc., New York (1960)

[7] Avalone, E.A. - Baumeister III, T. (editors) : Marks' Standard Handbook for Mechanical Engineers, 9th ed., McGraw-Hill Book Co., New York (1987), p. 17-20

[8] Grubbs, F.E. : On Estimating Precision of Measuring Instruments and Product Variability, Journ. of the Amer. Statistical Assn., Vol 42, No. 242 (June 1948), p. 243

[9] Chao, A. : Errors & Corrections, SSC-N-440 (Jan. 8, 1988)

[10] Peterson, J. : Private Communication (May 1988)

[11] Peterson, J. : Strength Requirements for a two-lump Corrector Scheme, SSC-N-512, Berkeley (June 1988)

[12] Berkes, B. : Terminology for the Magnetic Field of SSC Dipoles (SSC-Memo, July 1988 *))

[13] Hanft, R. : Private Communication (May/August 1988)

[14] Peterson, J. : Status about HERA-magnets, Report presented at the SSC Magnet Division Meeting (Nov. 2, 1988)

[15] Peterson, J. : Private Communication (Nov. 1988)

---

*) The term "tolerance" used in the above memorandum concerns ONLY the fact that the particular value of a multipole, as measured and/or determined by the engineer in charge, is accepted by the accelerator physicist as a marginal value. It DOES NOT mean that the accelerator can be operated can be operated with the measured/accepted value without introducing supplementary field corrections.