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A Finite Element Study of Floor Loading by a Model Detector

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ABSTRACT

The SSC model detector depresses a (model) steel reinforced concrete experimental hall floor by distances ranging from a few cm to less than a cm, according to finite element calculations. Because of the insensitivity of these results to the floor thickness and elastic moduli of the rock strata beneath the floor, they can be used to estimate the effects of loading by actual detector systems.

A. Introduction

In terms of sheer mass, the SSC TeV scale particle detectors will dwarf the current detectors of particles with hundreds of GeV energy: the low-β hermetic hadron /EM/ muon detectors contemplated may load the floors of experimental halls with masses of 30,000 to 50,000 tonnes (1 tonne = 10^3 kg). The amplitude of sinking due to the elastic loading is of interest as design parameter of the detectors and experimental halls, since the beam follows a rather tightly defined path through the lattice and center of detector. The uniquely heavy detectors are a considerable extrapolation from operating experience of the mighty CERN $Sp\bar{p}S$ detectors UA-1 and 2, Fermilab's CDF, and detectors currently under construction like SLD for SLC at SLAC and H1 and Zeus for HERA at DESY.

In order to illustrate the situation, consider the following model problem of a cubical detector with a mass of 48,000 tonnes. The density of iron is roughly 8 tonnes/m^3 so the volume of the detector would be 6000 m^3 if it were made from solid iron: the sides of the cube are 18.2 m. The basal area of such a detector is 330.2 m^2, so it exerts a pressure of P = 4.8×10^8 N/(330.2 m^2) = 1.45×10^6 Pa (about 14 atmospheres). Suppose the detector rests on a layer of shale that is L = 1.00 km thick and has a Young's modulus E = 4.8 GPa; the compression of the shale layer will be: $\Delta L = L\,(P/E) = (145/4.8) \times 10^{7-9}$, ΔL = 30.2 cm = deflection of the detector base. This assumes that the layer of rock below the shale is essentially rigid, and illustrates the order of magnitude of compression to be expected in the actual detector halls. Young's modulus for poorly cemented sedimentary rocks is in fact of the order of 5 GPa, and the depth of sedimentary cover over the crystalline basement in the continental interior ranges from values of about 1.0 km in Nebraska to 2 or 3 times that in continental sedimentary basins to around 6.7 km along the Gulf Coast of Texas, with depths of a few hundred meters near mountains to zero sedimentary cover on their crests (e.g., the Sierra Nevada). For proposed SSC sites on the Best Qualified List (BQL), the depth of sedimentary cover over crystalline basement ranges from 0–2 km.

The pressure exerted by the weight of the central calorimeter is given by $P_1 = F_1/A_1 =$ (9.81 N/kg) (30.536) (E6) kg/(32 m × 17 m), P_1 = 0.551 MPa (a Pascal = 1 N/m^2); the pressure at the base of the muon toroids is P_2 = (9.81 N/kg) (8.395) (E6) kg/(2 m ×16 m), P_2 = 2.57 MPa. Because of the relatively small basal area of the muon toroids, $P_2 > P_1$. For comparison, 1 MPa is about 10 atmosphere.

The values of Young's modulus E for common rocks are around 5–20 GPa. For reference, E (concrete) = 20.7 GPa and E (steel) = 200 GPa. Typical reinforcing for concentrated weights consists of a steel mattress with triangular bracing, which is then filled with concrete as the floor is poured. We treat such an integrally reinforced structure as having double the rigidity of ordinary concrete, E (floor) = 40 GPa. Because steel is at once more ductile and rigid than concrete, the effective Young's modulus for ordinary reinforced concrete under compression is usually taken as that of the concrete alone. Our treatment is therefore an adjustment of customary practice to account for the load bearing capacity of the steel reinforcing without specifying the precise geometry of the mattress.

B. Model Detector Characteristics

The Task Force that gave the report on *Collision Hall Limitations* (SSC-SR-1028) invented the model detector with the detailed properties given in that report. Here, we assume that pieces with the masses of the 17-m wide by 32-m long central calorimeter portion of 30,536 tonnes and the 16-m wide by 2-m long muon toroids each with masses of 8395 tonnes have their loads uniformly distributed to steel plates forming the bases. The detector is centered in a type A (25 m wide by 80 m long) collision hall, with distances of 4 m from the sidewalls respectively and 18 m from either end as shown in Fig. 1. The finite element mesh points are also shown in this figure. Floor loading due to the shielded focusing triplets and the forward muon toroids is neglected at this stage, as are any inhomogeneities in loading. Because of the three-dimensional nature of the loading and the effects of the reinforced concrete floor (two layer system), the nature of the resulting floor deflection due to elastic loading is difficult to specify in terms of analytic expressions. Therefore the elastic response of the system to the loading due to detector weight is modeled using the finite element code ANSYS, in the manner to be described below.

The density of concrete is about the same as that of the sedimentary rocks, approximately 2.5 tonne/m^3. The density of steel is about 7.8 tonne/m^3. If the mass of the concrete in the reinforced floor is the same as that of the steel, $M_c = M_{Fe}$, then

$$\rho \text{ (reinforced concrete)} = \frac{M_c + M_{Fe}}{V_c + V_{Fe}} = \frac{M_c}{V_c} \left(\frac{1 + M_{Fe}/M_c}{1 + V_{Fe}/V_c}\right)$$

use $M_c = M_{Fe}$

$$\rho \text{ (reinforced concrete)} = \rho_c \left(\frac{2}{1 + \rho_c/\rho_{Fe}}\right) = 2.5 \left(\frac{2}{1.32}\right)$$

$$\rho \text{ (reinforced concrete)} = 3.79 \text{ tonnes/m}^3.$$

The pressure due to the weight of the slab

$$P = \frac{g \rho V}{A} = g \rho h = 37.1 \text{ kPa} ;$$

the pressure due to the weight of the detectors are of the order of MPa, so body forces may be (and are) neglected in what follows.

C. ANSYS Finite Element Calculations

1) Outline of the method

For many deformation problems, the geometry, loading, or material properties make it very difficult (if not impossible) to obtain analytical mathematical solutions. In problems such as this, finite element analysis can be used to break a complicated problem into a system of simultaneous algebraic equations. A finite element code, such as ANSYS, uses three general phases in the analysis procedure: preprocessing, solution, and postprocessing. The preprocessor is used to define the material properties, geometry, and the loading of the model. The solution phase consists of generating the element matrix and solving for the displacements, stresses, etc. The postprocessor is then used for scanning, display and printing of the data generated by the solution phase.

2) Static Elastic Equations

The general matrix equations used in finite element analysis is

$$\{F\} = [K]\,\{d\}$$

where

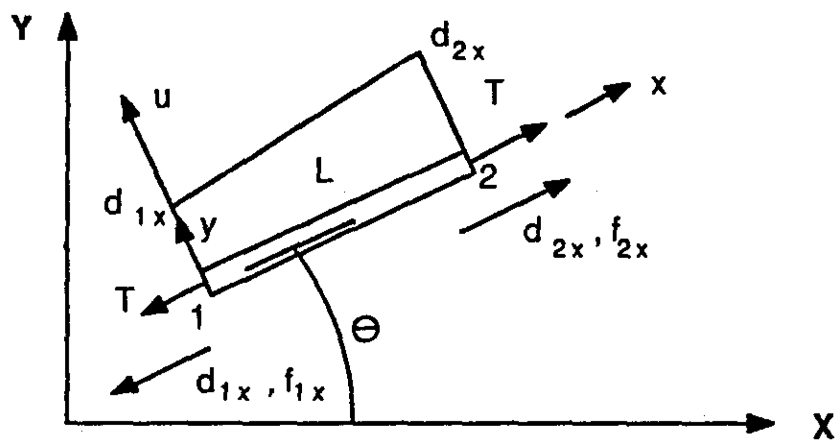
$\{F\}$ = vector of global nodal forces

$[K]$ = total, or global, stiffness matrix

$\{d\}$ = vector of known and unknown nodal displacements

For clarity, throughout the remainder of this discussion { } will refer to a vector, and [ ] will refer to a matrix.

In order to simplify the explanation of how the above equation is derived, consider the example of the beam element in tension as shown in the figure below.



For this beam element the displacement function, u, can be expressed as

$$u = \left(\frac{d_{2x} - d_{1x}}{L}\right) x + d_{1x}$$

or, in matrix notation

$$u = [N_1 \; N_2] \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$

where N_1 and N_2 are the shape functions

$$N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

The strain/displacement relationship is then

$$\varepsilon_x = \frac{d_{2x} - d_{1x}}{L}$$

and the stress/strain relationship is

$$\sigma_x = E\,\varepsilon_x$$

Using basic mechanics

$$T = A\,\sigma_x$$

where T is the applied tension and A is the cross-sectional area of the beam.

The stiffness matrix can now be derived as follows

$$T = \frac{AE}{L}(d_{2x} - d_{1x})$$

Using the sign convention shown in the figure

$$f_{1x} = -T \qquad f_{2x} = T$$

and substituting, these equations become

$$f_{1x} = -\frac{AE}{L}(d_{2x} - d_{1x})$$

$$f_{2x} = \frac{AE}{L}(d_{2x} - d_{1x})$$

or, in matrix form

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$

To put this equation into the more standard form for the element

$$\{f\} = [k]\ \{d\}$$

This equation is generated for each element in the model. These elemental equations must then be converted from the nodal (x,y) coordinate system into the global (X,Y) coordinate system. Once this has been done the equations can be assembled into the global matrix equation

$$\{F\} = [K]\ \{d\}$$

which can be solved for the displacements directly, and the stresses and strain indirectly.

The equations for the three-dimensional hexahedral elements used in this study follow the same general principle as in the derivation above. However, the formulation is sufficiently complicated to be beyond the scope of the simple explanation and introduction to the finite element method presented here.

D. Details of the Method

The model used is composed of three layers: rock substrate, reinforced concrete floor and steel mat, and rebar walls. Since the size of the mesh (i.e., number of elements) did not affect the results, the biggest mesh size possible was used as determined by the loading conditions. By making the mesh large, the number of elements is kept to a minimum, thereby minimizing the amount of computer time required. The mesh of the concrete/mat layer is shown in Figure 1.

The mesh of the rock substrate is basically the same as that of the concrete/mat layer. However, there is one exception: due to the depth of the rock substrate it was necessary to increase the number of elements from one to four in the vertical (Y) direction. The base of the rock substrate is constrained both vertically and horizontally. These constraints simulate the extremely rigid crystalline basement under the rock layer.

The sidewall reinforcing steel (rebar) acts like a spring supported in place by the concrete, itself acting as a partial support for the concrete/mat layer. It was modelled using spring elements attached at the edge nodes. The spring constant, k, was calculated using one inch steel rods (E = 200 GPa), spaced with 15 rods per 2 meters. The height of the rebar was taken to be 50 m, corresponding to the height of the experimental wall. The resulting equation for k is then

$$F = k \Delta L$$

where

$$k = E A/L$$

$$= \frac{(200 \times 10^9 \text{ Pa}) [(15) \pi (1.27)^2 \ (10^{-4} \text{ m}^2)]}{50 \text{ m}}$$

$$= 30.4 \times 10^6 \text{ N/m}$$

The top of the springs are fixed both horizontally and vertically.

The model was loaded using pressure applied at the top of the concrete/mat layer. The pressure were defined by taking the weight of the detector and dividing it by the area directly underneath the detector. The pressures under the toroids are then $P_T = (8.395 \times 10^3$ kg) (9.81 m/s^2)/(2 × 16 m^2) = 2.57 MPa and the pressure under the central detector is P_l = 30,536 × 10^3 kg) (9.81 m/s^2)/(32 × 17 m^2) = 0.55 MPa. These pressures were then applied uniformly over their respective areas (see Fig. 1).

E. Results

The results of the finite element calculations with the boundary condition corresponding to spring elements on the floor edges are displayed in Tables 1–3 and plotted in Figs. 3–5 respectively. As shown in Fig. 3, the maximum deflections are insensitive to increasing thickness of the reinforced concrete slab floor for thickness of more than a meter. This agrees with results from a test case in which the floor elements were rigidly constrained in the vertical direction (y = 0 required for all edge elements).

The dependence of the loading on the underlying rock substrate is shown in Figs. 4 and 5. While the relatively stiff reinforced floor slab deforms relatively little under loading, the underlying rock carries the load in much the same fashion functionally as if the slab were absent. With the slab absent

$$y_{max} = \left(\frac{P}{E}\right) D(\text{rock}) \tag{E.1}$$

is the simple form of the stress/strain proportionality.

The inverse proportionality of y_{max} with E is shown in Fig. 4 and proportionality of y_{max} with depth of rock is shown in Fig. 5. However, the slope of the graph of Fig. 5, for example, is not P (center)/D (rock) but a factor of two less; the smaller deflection is due to the floor slab and sidewall reinforcing taking up part of the loading. This simple model of an effective slope for the deflection dependence on rock substrate allows us to analyze different types of loadings. It is certainly an effective means of interpolation between cases.

We have also studied the effect of the size of the experimental hall floor on the amount of deflection. For the case of zero concrete thickness (support solely from the rock) a symmetrically placed detector in an 80 m long (Type A) experimental hall results in a maximum deflection of the floor of 2.799 cm; the corresponding deflection for the model detector centered in a 50 m long (Type B) experimental hall is 4.326 cm. If the hall size is restricted to the 17 m × 32 m size of the central portion of the detector without the muon toroids, the deflection is 4.791 cm, the same as found from Eq. (E.1). Broadly speaking, the support due to the area not subjected to compressional loading comes from the tension in the unloaded slab surface. These considerations provide a limiting case check on the finite element calculations.

We have also investigated the case of asymmetric detector placement by changing the sidewall spacing from 4 m on each side to 1 m on one side and 7 m on the other. This might be a tempting option in terms of detector staging and assembly. When this is done, the line of maximum deflection shifts from the symmetric longitudinal line below the beam line to the line along the edge of the hall closest to the detector. The maximum deflection for the case of 2 m thickness of concrete for the symmetric case is 2.137 cm; the maximum deflection in the asymmetric case along the edge of the hall is 2.953 cm, giving a slope of 0.86E-3 transverse to the beam line.

F. Conclusions

ANSYS three-dimensional finite element calculations of experimental hall loading have been performed. They yield results in accord with Hooke's law (E.1) in the one-dimensional limit. The other major results of this study are as follows:

1) The dependence of the floor deflections on properties of the rock substrate can be represented by:

$$y_{max} = f\,PD\,(\text{rock})/E(\text{rock}) \tag{F.1}$$

where $0 < f < 1$.

2) The factor f decreases with increasing thickness of concrete, increased support from the reinforcing steel in the sidewalls, and increasing floor area in the halls.

3) For thick enough underlying sediments, floor deflections are of the order of a few centimeters for a wide variety of rock types.

4) Once the region of reasonable structural safety margin has been reached, the maximum deflection is relatively insensitive to increasing depth of concrete floor. As a consequence, treatment of the underlying rock layer is likely to be a more useful tool in increasing foundation stability than increasing floor thickness.

5) Placing the detector to one side of the hall or the other causes the detector to tilt by a small amount, of the order of millimeters per meter.

We do not anticipate that changing the pressure from uniform loading to non-uniform loading will make much difference in the results, because of the lack of special deformations associated with the presence of the muon toroids in comparison with the central mass. In other words, it should be possible to get a reliable estimate of elastic deformations caused by real detectors, by using the constant pressure approximation, especially if the detector is symmetrically placed.

The use of an equivalent Young's modulus E to represent the nonlinear deformation of the reinforced steel mattress floor is perhaps the departure from usual practice most open to criticism. However, the modulus E (reinforced floor) = 2 E (unreinforced concrete) so that is the compression follows the worst case behavior of unreinforced concrete, the deflections would still be little affected because the modulus for unreinforced concrete is still much larger than for most sedimentary rocks. Further studies utilizing multiple layers and possibly some structural shell theory would be necessary to answer this point quantitatively.

Acknowledgement

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References

ANSYS Engineering Analysis System User's Manual — Vol. I & II, Gabriel J. DeSalvo and Robert W. Gorman, Swanson Analysis Systems, Inc., Houston, PA 1987.

Report of the Task Force on Collision Hall Limitations, SSC Central Design Group (April 19878) SSC-SR-1028.

Table 1

Maximum vertical (Y) deflection (cm) vs. thickness of reinforced concrete floor (m) with E (rock) = 11.5 GPa and D (rock) = 1 km.

D_{conc}/ (m)	Y_{max} (cm)
0	2.80
0.5	2.20
1	2.15
2	2.14
3	2.13

Table 2

Maximum vertical (Y) deflections (cm) vs. modulus of rock substrate (GPa) with thickness of concrete layer = 2 m and D (rock) = 1 km.

E_{rock} (GPa)	Y_{max} (cm)
5.75	4.270
11.5	2.137
14.0	1.760
20.7	1.119

Table 3

Maximum vertical (Y) deflections (cm) vs. depth of rock substrate (km) with E (rock) = 11.5 GPa and thickness of concrete layer = 2 m.

E_{rock} (km)	Y_{max} (cm)
0.5	1.079
1.0	2.137
1.5	3.200
2.0	4.260

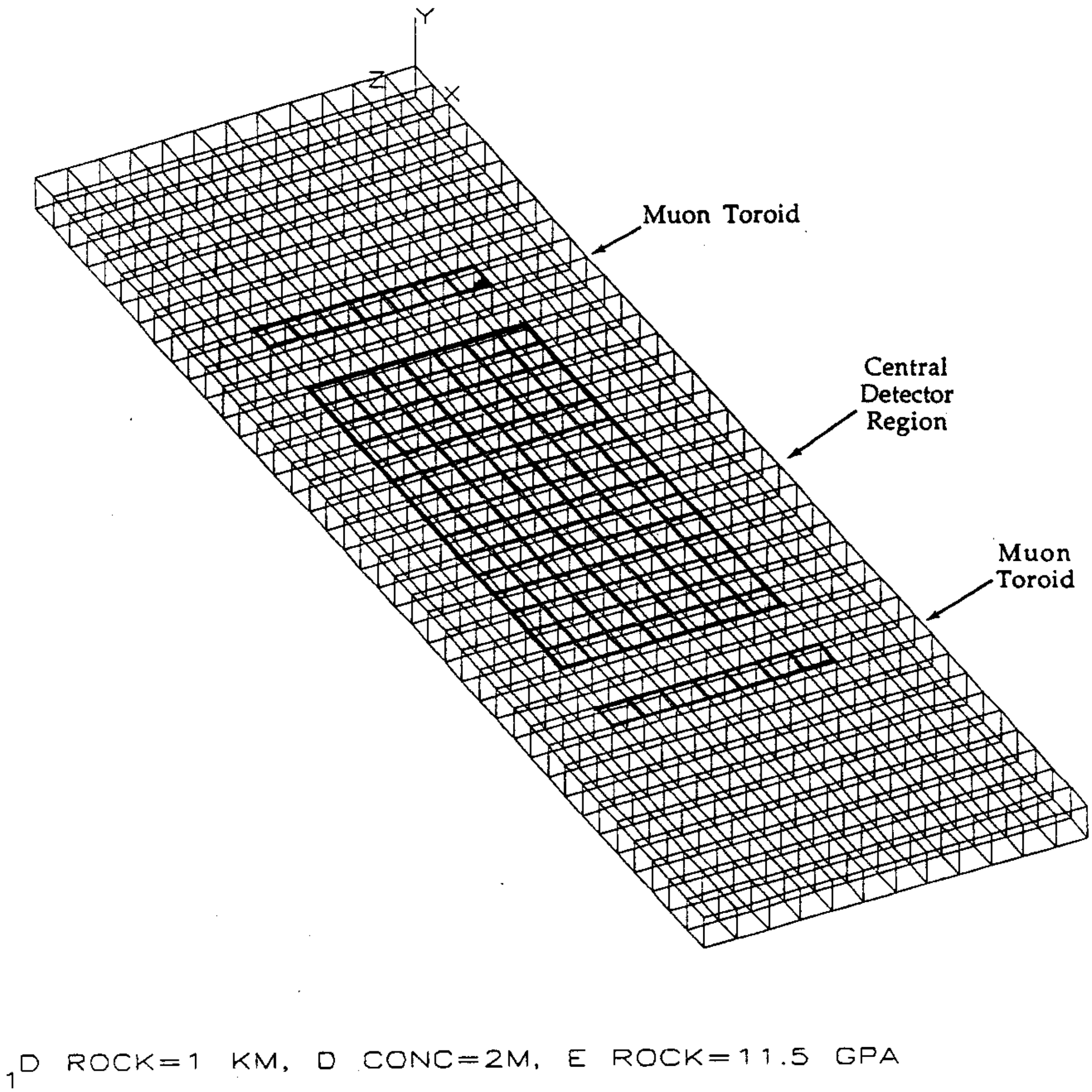

Fig. 1. Mesh and loading of concrete layer.

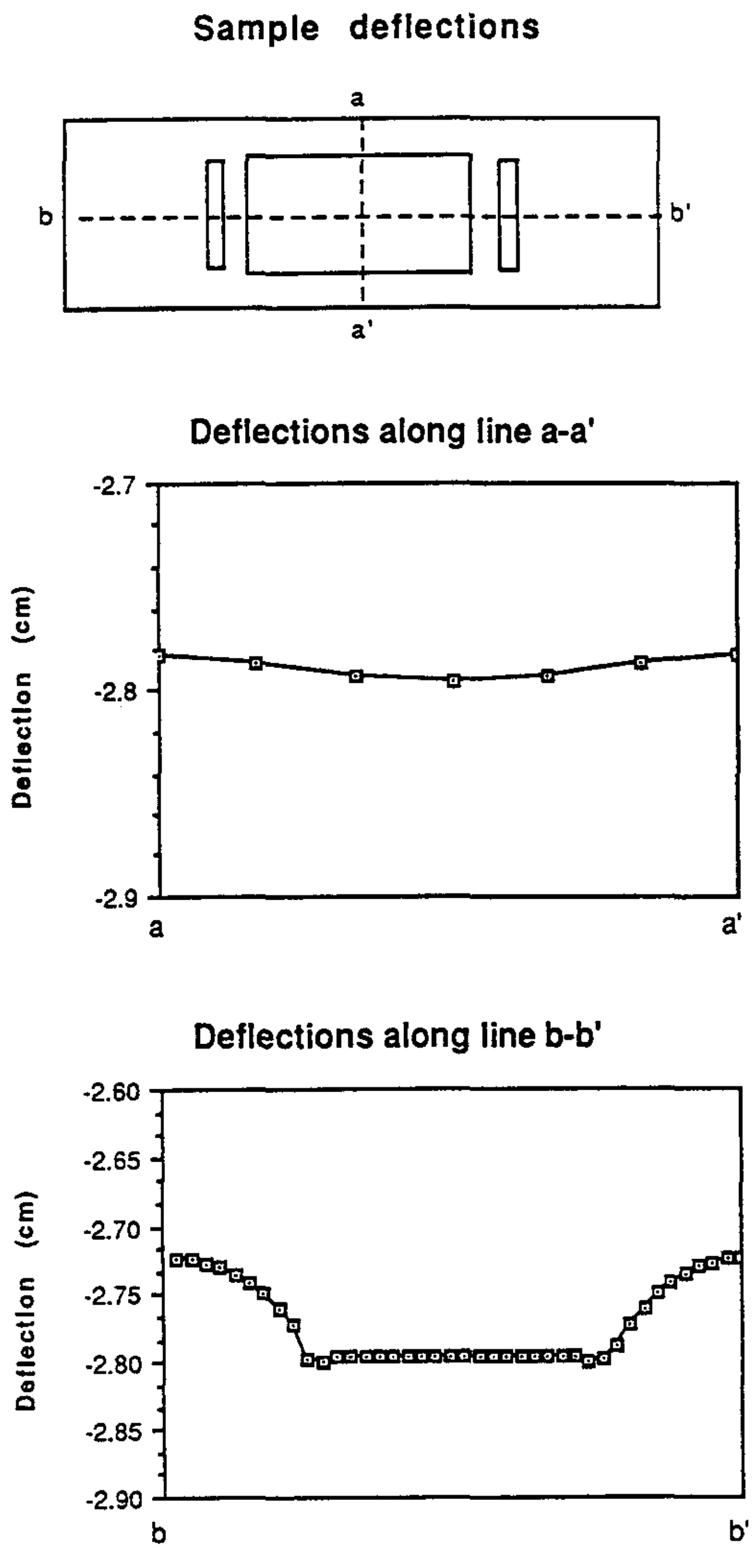

Fig. 2. Sample deformation under load.

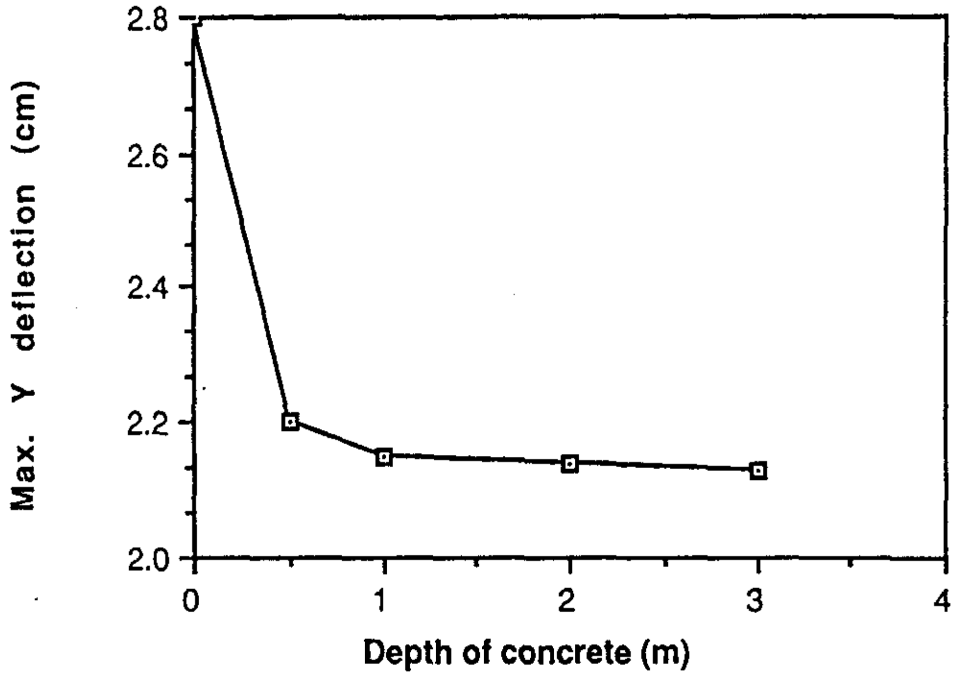

Fig. 3. Maximum vertical (Y) deflection (cm) vs. thickness of reinforced concrete floor (m) with E (rock) = 11.5 GPa and D (rock) = 1 km.

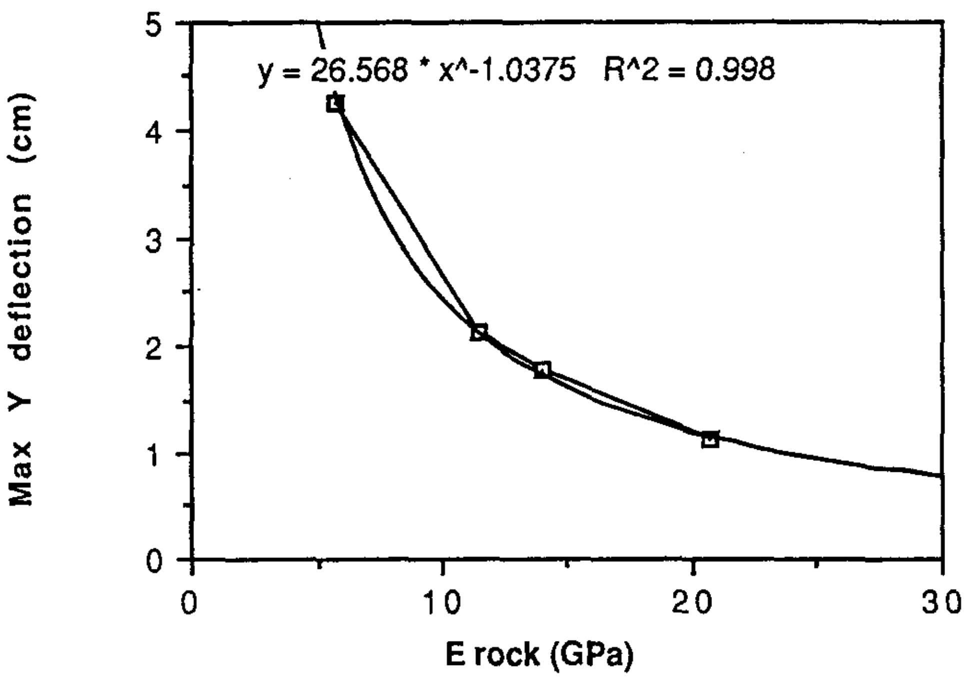

Fig. 4. Maximum vertical (Y) deflections (cm) vs. modulus of rock substrate (GPa) with thickness of concrete layer = 2 m and D (rock) = 1 km.

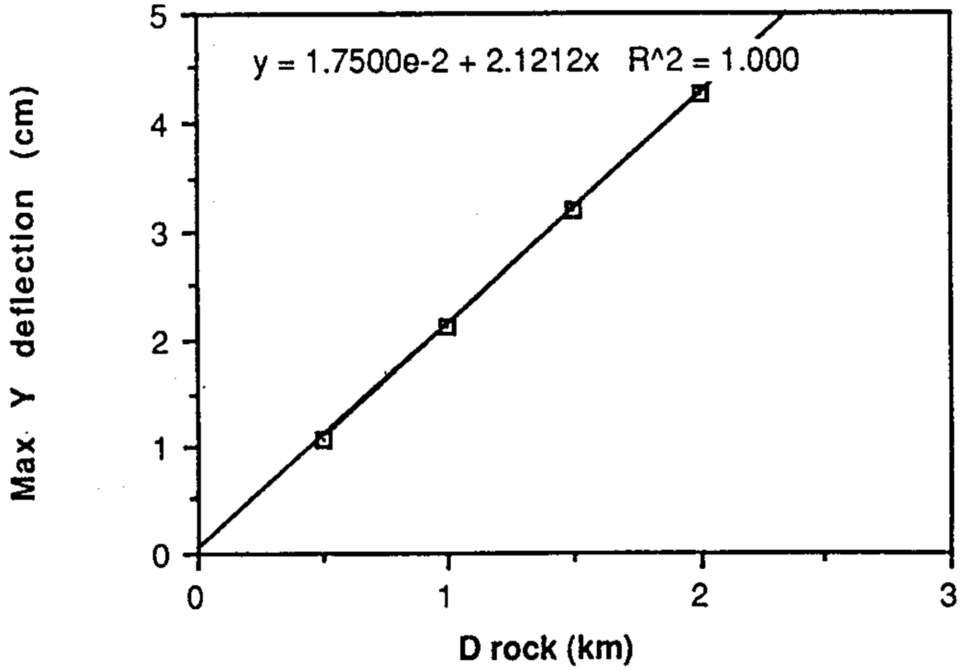

Fig. 5. Maximum vertical (Y) deflections (cm) vs. Depth of rock substrate (km) <u>with</u> E (rock) = 11.5 GPa and thickness of concrete layer = 2 m.