A NOVEL METHOD FOR CORRECTING THE SSC MULTIPOLE PROBLEM*

David Neuffer
At-6, Los Alamos National Laboratory, Los Alamos, NM 87545
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A new method of correcting dynamic nonlinearities due to the multipole content of a large synchrotron such as the Superconducting Super Collider is discussed. The method uses lumped multipole elements placed at the center (C) of the accelerator half-cells as well as elements near the focusing (F) and defocusing (D) quadrupoles at the ends. In a first approximation, the corrector strengths follow Simpson's Rule. Correction of second-order sextupole nonlinearities may also be obtained with the F, C, and D octupoles. Correction of nonlinearities by ~ three order of magnitude is obtained, and a solution to a fundamental problem in large synchrotrons is demonstrated.

A large synchrotron such as the Superconducting Super Collider (SSC)[1] requires adequate linearity for beam stability and reliable operation. Linear motion is required over a working region sufficient to include the beam size and momentum spread with closed orbit deviations and injection errors. This linear

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aperture is severely limited by the SSC multipole content, and the nonlinearities are greatly magnified by the large circumference.

Nonlinearity can be measured by the amplitude and momentum-dependent tune shifts per turn $\Delta\nu_x$, $\Delta\nu_y$. An SSC linear aperture tolerance (SLAT) may be set by requiring $\Delta\nu_x, \Delta\nu_y \leq \pm 0.005$ for orbits with amplitudes A_x, A_y, up to 0.5 cm in the SSC arcs and with momentum offsets $\delta \equiv \frac{\Delta p}{p} \leq \pm 0.001$.[1] Except for short utility and interaction regions (IR), the SSC circumference is composed of ~320 alternating-gradient cells consisting of long dipoles, short focusing (F) and defocusing (D) quadrupoles, and short corrector magnets (see Fig. 1). The nonlinear magnetic fields in the dipoles dominate the nonlinear motion and they may be represented by

$$B_y + iB_x = B_o\{1 + \sum [b_n(s) + ia_n(s)](x+iy)^n\} \quad ,$$

where B_o is the bending field and $b_n(s)$ and $a_n(s)$ are the normal and skew multipole components. The transverse motion may be described by a Hamiltonian,

$$H = \frac{I_x}{\beta_x(s)} + \frac{I_y}{\beta_y(s)} + \Re \sum_n \frac{B_o}{B\rho} \frac{[b_n(s) + ia_n(s)](x+iy)^{n+1}}{n+1} \quad ,$$

where $\beta_x(s)$ and $\beta_y(s)$ are the betatron functions[2] of linear motion. The coordinates x and y of particle motion are represented in action-angle variables (I,ϕ) by $x = \sqrt{2\beta_x I_x}\cos(\phi_x) + \eta\delta, y = \sqrt{2\beta_y I_y}\cos(\phi_y)$; the off-momentum orbit at δ determined by the dispersion $\eta(s)$ is included. The terms $A_x = \sqrt{2\beta_x I_x}$ and $A_y = \sqrt{2\beta_y I_y}$ are the amplitudes. The tune shifts are obtained by averaging the phase advance caused by the field perturbation

$$\Delta\nu_{x,y} = \frac{1}{2\pi}\int \frac{d\phi_{x,y}}{ds} ds = \Re\langle\frac{dH}{dI_{x,y}}\rangle \quad . \tag{1}$$

In first order in the coefficients b_n and a_n only systematic normal multipoles ($\overline{b_n}$) contribute; the resulting expressions for the tune shifts as a function of I_x, I_y, and δ due to sextupole (b_2), octupole (b_3) and decupole (b_4) components are

$$\begin{aligned}
\Delta\nu_x &= \langle b_2\beta_x\eta\delta\rangle + \langle \frac{3}{4}b_3\beta_x^2 I_x - \frac{3}{2}b_3\beta_x\beta_y I_y + \frac{3}{2}b_3\beta_x\eta^2\delta^2\rangle \\
&\qquad + \langle 3b_4\beta_x^2\eta I_x\delta - 6b_4\beta_x\beta_y\eta I_y\delta + 2b_4\beta_x\eta^3\delta^3\rangle \\
\Delta\nu_y &= -\langle b_2\beta_y\eta\delta\rangle + \langle \frac{3}{4}b_3\beta_y^2 I_y - \frac{3}{2}b_3\beta_x\beta_y I_x - \frac{3}{2}b_3\beta_y\eta^2\delta^2\rangle \\
&\qquad + \langle 3b_4\beta_y^2\eta I_y\delta - 6b_4\beta_x\beta_y\eta I_x\delta - 2b_4\beta_y\eta^3\delta^3\rangle \quad . \qquad (2)
\end{aligned}$$

The SLAT may be applied to Eqs. (1) and (2) to obtain tolerance limits[3] on uncorrected $|\bar{b}_n|$; see Table I.

The SSC dipoles are expected to have significant multipole content, particularly in the $\bar{b}_n$ with n even, which are allowed in dipole symmetry. Estimates[5] of the systematic and rms random multipole strengths as extrapolated from the similar Tevatron dipoles [6] or calculated from persistent current and saturation effects[1] have been collected in Table I where they are compared with SLAT values. Serious deficiencies in $\bar{b}_2, \bar{b}_3$, and $\bar{b}_4$ are observed. Initial SSC design included sets of multipole trim coils within every dipole for correction of $\bar{b}_2, \bar{b}_3$, and $\bar{b}_4$, but they greatly complicate the dipoles and may be impractical.

The correction is simplified if it is implemented using short correctors separate from the dipoles. Initial attempts used correctors placed near F and D quads, where chromatic (momentum-dependent) correction sextupoles are placed. Because there are only two first-order b_2 terms, F and D sextupoles can completely correct them; however, second-order sextupole effects are important (see below). For b_3, b_4, and higher multipoles, there are five or more terms and they cannot be completely corrected. The F and D correction is quite inadequate and can reduce the b_3 and b_4 nonlinearities only by a factor of ~2 (Table II).[7,8]

A great improvement is obtained by adding a corrector to the center (C) of each half-cell (Fig. 1). An optimization converges very close to a characteristic solution. For example, a particular corrected b_3 tune-shift term from Eq. (2) may be written as

$$\Delta\nu_x = \frac{3I_x}{4L}\left\{\int_0^L b_3(s)\beta_x^2(s)ds \right. \\ \left. + \left[\frac{S_{3,F}}{B_o}\beta_x^2(0) + \frac{S_{3,C}}{B_o}\beta_x^2(L/2) + \frac{S_{3,D}}{B_o}\beta_x^2(L)\right]\right\} \ . \tag{3}$$

All first-order terms are of similar form. The corrector strengths $S_{n,i}$ are defined by $S_{n,i} \equiv B_{n,i}l_i = -f_{n,i}B_o\bar{b}_nL$, where $B_{n,i}$ and l_i are the corrector lengths and strengths, and L is the half-cell length. The Simpson's Rule[9] solution is $f_\mathrm{F} = f_\mathrm{D} = 1/6$ and $f_\mathrm{C} = 4/6$ per half cell; it corrects all b_3 and b_4 nonlinearities by two orders of magnitude. Optimization about that solution permits another order of magnitude reduction (see Table II).

The F, C, D correction [Eq. (3)] is equivalent to approximating integrals of powers of betatron functions by a sum over discrete points, and Simpson's Rule is a third order integration that is very accurate for smoothly varying functions. Figure 2a shows β_x, β_y, and η, over a full cell; they are smoothly varying over half-cells with a derivative discontinuity at the quadrupoles. All the terms that appear in b_2, b_3, and higher-order tune shifts are smoothly varying on the half-cell level (Figs. 2a and 2b). The F, C, and D correctors provide three-point, Simpson's Rule type cancellation, which is well suited to the correction of all first-order terms.[8] The first-order correction is insensitive to machine perturbations, because it requires only smooth variation on the half-cell level. Second-order effects are also reduced by first-order correction. The method is easily extended to include other multipoles (b_5, b_6, etc.), if necessary. Random multipole effects can be reduced by varying the correctors to follow the local multipole content.[10] The method can be extended to correct quadrupole multipole content, particularly in the long IR quads.

Superconducting dipoles have a very large b_2 content, and second-order terms are important. From perturbation theory,[11–13] second-order sextupole tune-shift terms are of the same form as the first-order octupole terms:

$$\Delta\nu_x = aI_x + bI_y + c\delta^2 \text{ and } \Delta\nu_y = dI_y + bI_x + e\delta^2 \quad . \tag{4}$$

The coefficients a through e scale as b_2^2 and are positive after first-order correction. The expression for a in a simplified lattice is

$$a = \frac{C}{32\pi L}\int_0^{2L} ds\beta_x(s)^{3/2}B_2(s)\int_s^{s+2L} ds'\beta_x(s')^{3/2}B_2(s') \times \left\{3\frac{\cos[\psi_x(s') - \psi_x(s) - \pi\mu_x]}{\sin(\pi\mu_x)} + \frac{\cos 3[\psi_x(s') - \psi_x(s) - \pi\mu_x]}{\sin(3\pi\mu_x)}\right\} \quad ,$$

where C is the ring circumference, $B_2(s)$ is the normalized sextupole strength, $\psi_x(s)$ is the betatron phase, μ_x is the cell tune, and $\Delta\nu_x$ is the full-ring tune shift. These terms are double integrals with phase factors and a discontinuity at $s = s'$ and are not closely fitted to Simpson's Rule integration. The first-order b_2

correction with only F and D elements reduces second-order terms by a factor of $\sim$5. Addition of a C sextupole corrector reduces these terms by another factor of $\sim$5, increasing b_2 tolerance to an acceptable level, but without the Simpson's Rule improvement seen in first-order (Table II).

Second-order b_2 correction can be greatly improved by using the F, C, D octupoles. Unlike ${b_2}^2$ terms, first-order octupole $\Delta\nu$ terms have opposing signs [Eq. (2)], and there are only three correctors for five terms. However, the octupole Hamiltonian has three terms; the $\Delta\nu$ terms that derive from the same Hamiltonian term have similar dependences (Fig. 2b). The correction strategy is to use the C octupoles to correct the *b* and *e* terms in Eq. (4) and the F and D octupoles to correct the others; the ratios of F, C, and D strengths per half-cell are $\sim$ $(1 : -2.7 : 1)$. The correctors can correct completely either amplitude or chromatic $\Delta\nu$ with the remanent terms reduced by $\geq$ 10$\times$. The optimum reduction is by a factor of $\geq$ 30, increasing $\bar{b}_2$ tolerances to $\geq 30 \times 10^{-4}\mathrm{cm}^{-2}$ (see Table II). Other nonlinear effects such as orbit distortion remain small, provided cell resonances are avoided. Because the octupole tune shifts are linear, there is no interference between their b_2^2 and b_3 correction roles. The use of F, C, and D octupoles to correct second-order sextupole nonlinearities adds an extra operational dimension, conceptually similar to the use of F and D sextupoles to correct quadrupole chromaticity. Their use may be extended to control nonlinearities from other elements; for example, all A_i^2 tune shifts can be cancelled to zero, regardless of their sources.

Simple solutions have thus been found for formidable problems of SSC nonlinearities. First-order nonlinearities due to multipoles are correctable by a system of F, C, D elements, with initial strengths determined by the generally valid Simpson's Rule. Second-order sextupole effects can be cancelled with F, C, D octupoles. Tolerances on SSC fields can be increased from the impractical 10^{-6} level to 10^{-3} cm^{-n}. The same corrections can be applied to any synchrotron, with similar improvements.

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TABLE I. Tolerances and estimated strengths of systematic multipole content in the SSC dipoles. All multipole strengths are in units of 10^{-4} cm^{-n}. The tolerances are obtained from the SLAT. Estimated strengths are extrapolated from Tevatron data or calculated from the magnet properties.

Multipole	Tolerance in SSC Lattice (230 m, 90° Cells)	Estimated Random Error (Tevatron)	Systematic Strength (Tevatron)	Persistent Current Multipole Strength	Saturation Multipole Strength
b_2	0.0097	2.0	0.45	-4.7	1.2
b_3	0.017	0.35	-0.14	–	–
b_4	0.031	0.60	-0.33	0.30	-0.05
b_5	0.054	0.06	-0.024	–	–
b_6	0.096	0.08	1.57[a]	0.07	-0.01
b_7	0.17	0.16	0.009	–	–
b_8	0.29	0.02	-2.1[a]	<0.02	0.02

[a] The higher allowed multipoles (b_6, b_8) were not minimized in the Tevatron design; the SSC conductor placement should reduce these within tolerances.[4]

TABLE II. First-Order Correction of b_3, b_4 and Second-Order b_2 Correction. The correction factor is the ratio of uncorrected to corrected $\Delta\nu$ in the SLAT aperture. The tolerance is the maximum corrected b_n permitted under the SLAT.

Correction Condition	Correction Factor	Tolerance (10^{-4}cm^{-n})
b_3 (Octupole) Correction		
No correction	1.0	0.018
F, D chromatic correction[7] ($f_F = 0.28$, $f_D = 0.70$)	1.9	0.033
F, C, D Simpson's Rule (f_F, f_C, f_D) = (1/6, 4/6, 1/6)	93	1.6
F, C, D correction (0.165, 0.66, 0.165)	370	6.7
b_4 (Decupole) Correction		
No correction	1.0	0.029
F, D chromatic correction[7] (f_F, $= 0.24$, $f_D = 0.93$)	1.4	0.04
F, C, D Simpson's Rule (f_F, f_C, f_D) = (1/6, 4/6, 1/6)	31	0.9
F, C, D correction (0.158, 0.663, 0.168)	800	24
Second Order b_2 (Sextupole) Correction		
No correction	1.0	1.2
F, D chromatic b_2 correction	5.1	2.7
F, C, D chromatic b_2 correction, equal weights ($f_C = 0.5$)	24	5.9
F, C, D chromatic correction, Simpson's Rule ($f_C = 0.667$)	23	5.7
F, D first-order b_2 correction ($f_{C,2} = 0$), and F, C, D octupoles	120	13
F, C, D first-order b_2 correction ($f_{C,2} = 0.5$ to 0.67), and F, C, D octupoles	700	32

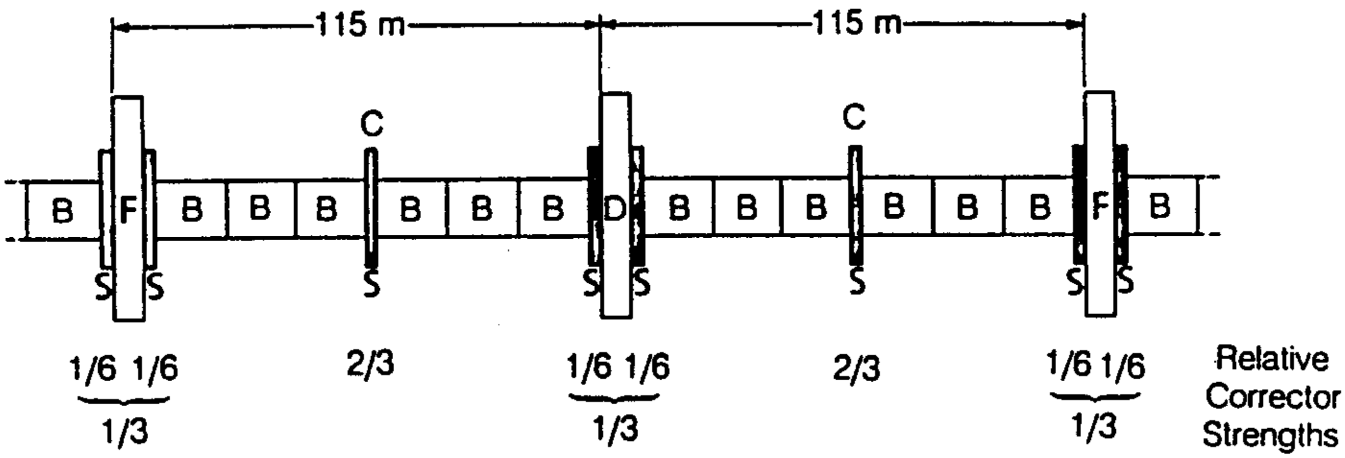

Fig. 1. A symmetrical SSC cell. The element labels are: B - dipoles, F, D - quadrupoles, S - slots for correctors, C - center corrector slot. The correctors on opposite sides of the F and D quads may be combined on either side and exact symmetry is not necessary.

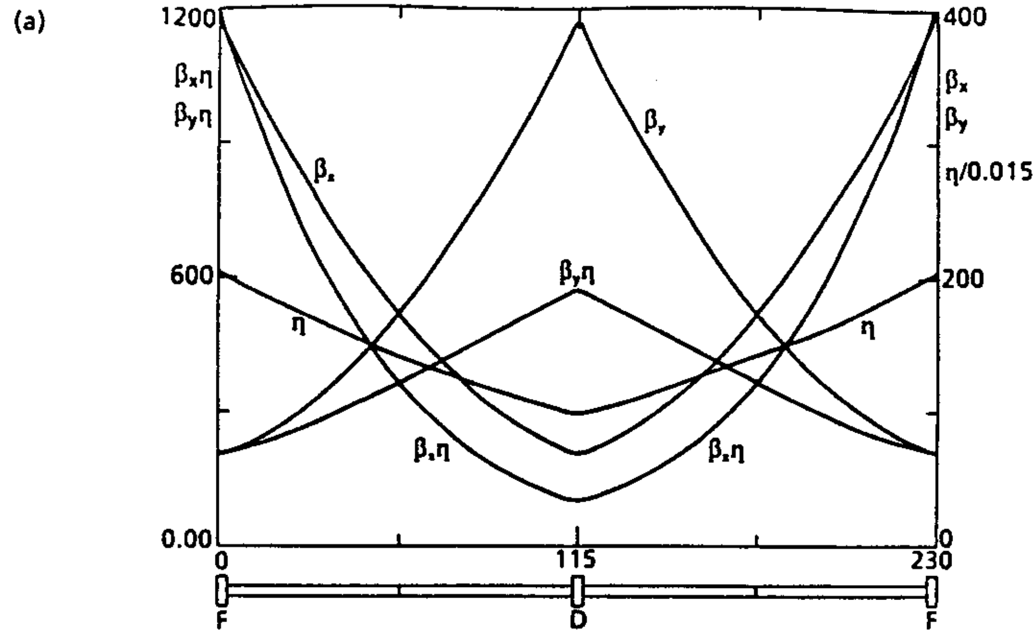

Fig. 2.a) Betatron functions (β_x, β_y, η) for a full SSC cell. The functions that appear in the sextupole tune shifts $(\beta_x\eta, \beta_y\eta)$ are also shown. Note the derivative discontinuity and the reflection symmetry at the D quadrupole.

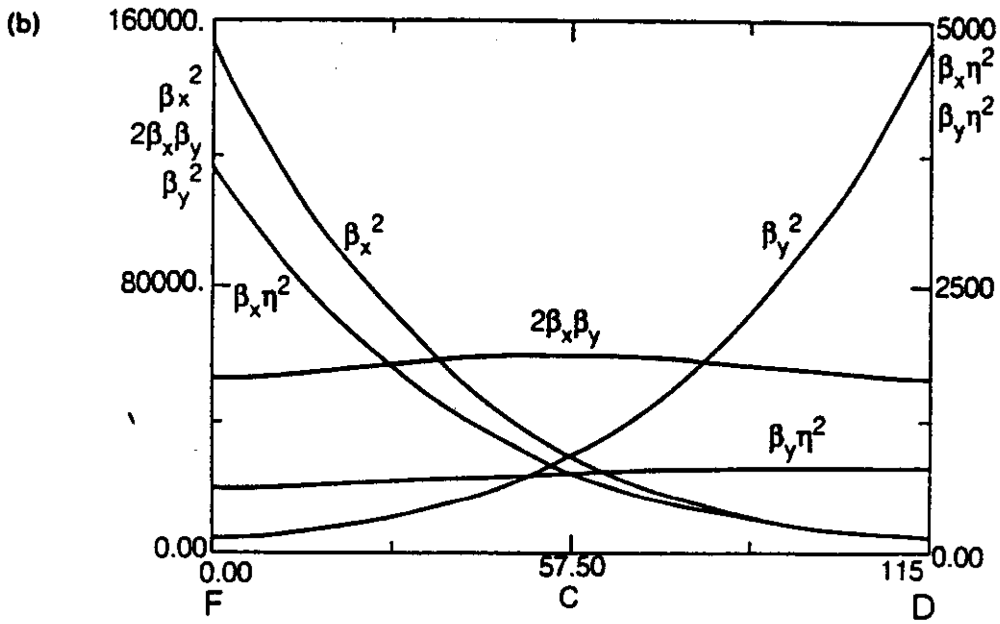

Fig. 2.b) Octupole $\Delta\nu$ functions on a half cell. The $\beta_x{}^2$ and $\beta_x\eta^2$ terms are derived from x^4 in the Hamiltonian; $2\beta_x\beta_y$ and $\beta_y\eta^2$ from $(-x^2y^2)$ and $\beta_y{}^2$ from y^4.