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NEW COEFFICIENTS FOR A $J_c(B,T)$ ANALYTIC FORM

G. Morgan and W. B. Sampson

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New Coefficients for a $J_c(B,T)$ Analytic Form

A previous note[1] gave several analytic forms for fitting $J_c(B,T)$ data. More recently, an unpublished variation of these has been used, and is given by equation 1.

$$J_c(B,T) = P_1\left(1. + \frac{P_2u+P_3u^2+P_4u^3}{1+P_5v}\right)\left(\frac{1+P_6v}{1+P_7v+P_8v^2}\right) \qquad (1)$$

In equation 1, $u = T - 4.222$ kelvin and $v = B - 5.0$ tesla. This makes P_1 the value of J_c at the reference temperature and field, 4.222 K and 5.0 T, to be supplied by the user. Data for high-homogeneity NbTi supplied by W.B. Sampson were used to evaluate P_2 thru P_6, using a two-independent-variable, non-linear least-squares program, DUBNAL. These data were valid only in the range 5 to 8 T, and were supplemented by a previous parameterization which was used to obtain P_7 and P_8 and extended the range of validity to 3 to 8 T. More recently, magnetization data supplied by A. Ghosh for typical SSC NbTi were used to extend the range to zero tesla, but this extension was valid only near 4.2 K.

Recently, it was learned that the $J_c(B,T) = 0.$ curve given by this parameterization did not match very well an expression[2], given by eqn (2), known to be good for recent Nb, 46.5 wt. % Ti.

$$T_c = 9.2\left(1. - \frac{B}{14.5}\right)^{0.59} \qquad (2)$$

The difference between the parameterization and eqn (2) was almost -0.3 degree at 5 T. It is surmised that the data for the original parameterization may have been for Nb, 50 wt. % Ti.

New data, covering the range 4.0 to 8.0 T and 2.5 to 8.0 K , again supplied by W.B.S., are shown in Figure 1 and Table 1, comprising 44 points.

Table 1

Relative J_c as a function of B and T

B, T → / Temp, K ↓	4.	5.	6.	7.	8.
2.5	--	1.50	1.26	1.05	0.79
3.0	1.59	1.36	1.14	0.94	0.70
3.5	1.46	1.23	1.02	0.81	0.59
4.0	1.31	1.08	0.87	0.67	0.46
4.5	1.13	0.91	0.71	0.52	0.33
5.0	0.95	0.74	0.55	0.37	0.19
5.5	0.77	0.57	0.39	0.22	0.06
6.0	0.58	0.40	0.23	0.07	--
6.5	0.40	0.23	0.07	--	--
7.0	0.22	0.06	--	--	--
7.5	0.04	--	--	--	--

Note that these data give $J_c = 1.0$ at 4.2 K and 5.0 T. Data from the various manufacturers differ somewhat, and these data are representative of

an average material (from IGC). These numbers are based mainly on BNL measurements. Four points were added, two from eqn (2) at 2.0 and 3.0 T, and two at 4.222 K from Ghosh's magnetization data at 2.0 and 3.0 T, 1.942 and 1.496, resp. Figure 2 is Ghosh's curve. Note that self-field effects markedly lower the low-field curves. The solid curve is used for the present parameterization. An expression which fits this curve is given by equation 3.

$$f = \frac{a}{\sqrt{B+B_0}} - b \qquad (3)$$

In eqn (3), a,b, and B_0 have the values 2.46324, .39588, and .1142, resp. This equation has the value f = 1 at B=3.0.

When eqn (1) is fitted to the 48 points, the parameters have the values given in Table 2, which also includes the older values of the parameters.

Table 2

Parameter	2	3	4	5	6	7	8
Old value	-.35424	-.02335	.00614	-.15335	-.21080	.00655	-.01669
New value	-.31532	-.01528	.00161	-.16309	-.23174	-.02125	-.02042

The rms deviation of the fit, $(\Sigma(y_i-y_{calc})^2/(48-7))^{1/2}$, is .011, and the largest single deviation is .027 at $J_c(3.,4.222) = 1.496$, a point from Ghosh's curve.

Table 3 gives the variation in J_c at 5.6 T and 7.0 T with temperature, computed from the new correlation, and Table 4 gives the variation in J_c with field at 4.222 K.

Table 3

T, K		2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
J_c,	5.6 T	1.355	1.229	1.092	0.946	0.792	0.631	0.465	0.294	0.120
	7.0 T	1.058	0.939	0.812	0.676	0.532	0.382	0.227	0.067	---

Table 4

B, T	0.0	1.0	2.0	3.0	4.0	5.0	5.6	6.0	7.0	8.0
J_c	10.229	2.875	1.926	1.523	1.231	1.000	0.879	0.802	0.613	0.405

The difference in an SSC magnet performance prediction using the new correlation is modest. A report on the C358A coil design [3] gave the estimate $B_0 = 6.831$ T at 6.756 kA, at 4.35 K with Cu/sc = 1.3 in the inner, with $P_1 = 2475$ A/mm^2. The outer had a margin of 2.74 % in J_c at this current. Using the new correlation, the prediction is 6.867 T at 6.788 kA for the inner, with a margin of 3.53 % in the outer.

Figure 3 is a listing of a Fortran program implementing the new correlation.

References

(1) G. Morgan, "Analytic Forms for Critical Current Data", Magnet Div. Note 110-4 (SSC-MD-84), Dec. 11, 1984

(2) J.S. Lubell, "Empirical Scaling Formulas for Critical Current and Critical Field for Commercial NbTi", IEEE Trans. on Magnetics, Vol MAG-19, No. 3, p. 754 (May 1983)

(3) G. Morgan, "C358A - A New SSC Dipole Coil and End Design", Magnet Div. Note 210-1 (SSC-MD-155), March 13, 1987

GC From W.B. Sampson 5/23/88

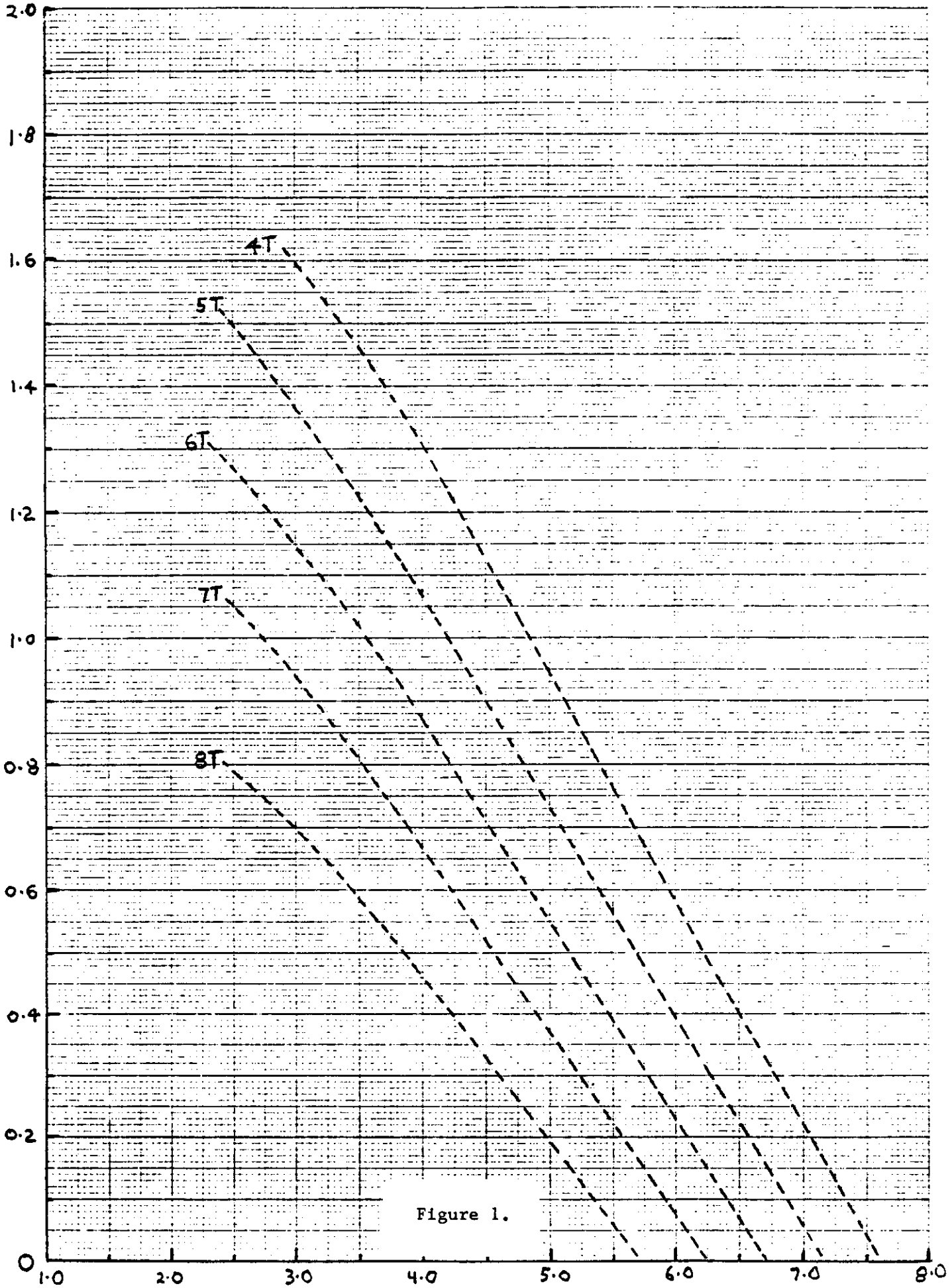

Figure 1.

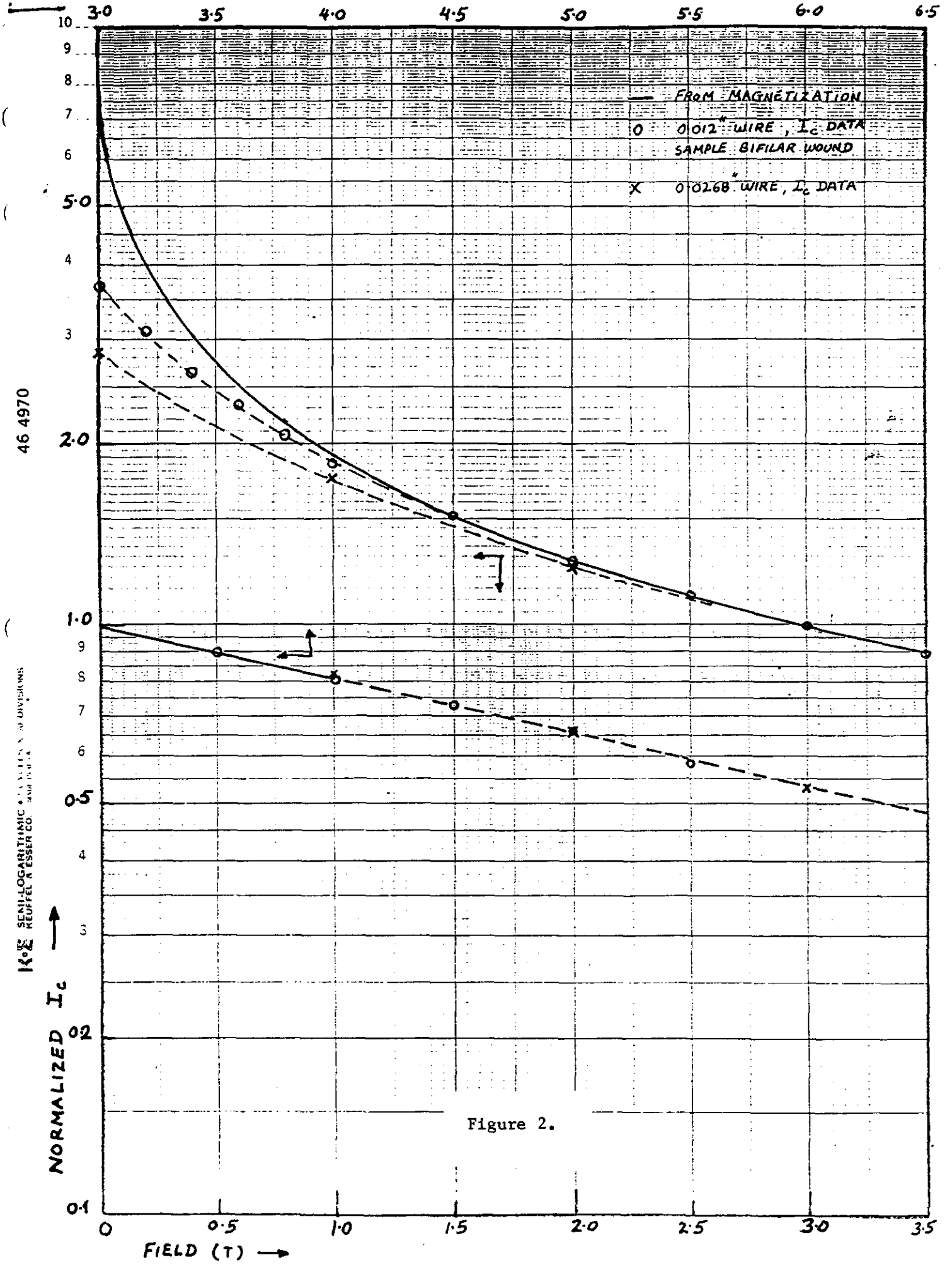
FROM MAGNETIZATION
0 0.012" WIRE, I_c DATA
SAMPLE BIFILAR WOUND
X 0.0268" WIRE, I_c DATA
NORMALIZED I_c
FIELD (T)
46 4970
K&E SEMI-LOGARITHMIC
KEUFFEL & ESSER CO.

Figure 2.

```
      PROGRAM PIC88
 THIS PROGRAM COMPUTES IC FROM B,T,SC AREA AND JC AT 5T & 4.222K
100 PRINT 1
  1 FORMAT(' GIVE T,B,ASC,STDJC'/)
      READ*,T,B,ASC,STDJC
      IF(T.EQ.0.)STOP
      AIC=ASC*STDJC*AJ(B,T)
      PRINT 2,AIC
  2 FORMAT(' IC='F10.3)
      GO TO 100
      END
      FUNCTION AJ(B,T)
 THIS FUNCTION RETURNS RELATIVE JC OF IGC NBTI BASED ON DATA
 SUPPLIED BY W.B.S. 5/23/88, WITH ADDED POINTS FOR 4.222 K AND
 FOR ZERO JC AT 2 AND 3 TESLA, PER GHOSH MAGNETIZATION DATA
 5/28/87, AND EXTENDED TO FIELDS BELOW 2 T, VALID NEAR 4.2 K
      LOGICAL L
      DIMENSION P(8)
      DATA P/1.,-.315319,-.015280,.001611,-.163089,-.231741,
     A-.021249,-.020418/
      IF(B.LT.2.)THEN
      L=.TRUE.
      X=-3.
      ELSE
      L=.FALSE.
      X=B-5.
      ENDIF
      Y=T-4.222
      AJ=(1.+Y*(Y*(Y*P(4)+P(3))+P(2))/(1.+P(5)*X))*
     A(1.+X*P(6))/(1.+X*(X*P(8)+P(7)))
      IF(L)AJ=AJ*(1.89734/SQRT(B+0.1142)-.30493)
      RETURN
      END
```

Figure 3.

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