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Minimal Energy Deposition in S.S.C. Dipoles

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I present here the results of a brief calculation I have done at Saclay[1] to determine the minimal energy necessary to initiate a quench. The thermal equation with constant material properties has been solved analytically using the elegant method presented by L. SCHWARTZ[2].

I consider a bare, straight and infinite conductor, behaving adiabatically. I assume that the energy W per unit volume is deposited at time t = 0 on a length (2p) of conductor. A sufficient condition for the establishment of a propagating zone mode is that the two dimensionless parameters a and λ, defined by :

$$a = \frac{W}{2C(T-T_0)} \qquad \text{and} \qquad \lambda = 2\,\frac{p^2 P_{j0}}{DW}$$

satisfy one or the other of the two following sets of inequalities :

i) $\lambda \leq K_m$ and $a > 1$

ii) $0 < \lambda < K_m$ and $a > \text{Max}\,[1, q(\lambda)]$

where :

- K_m is the maximum value of the function K(x) defined by :

$$K(x) = \frac{1}{\sqrt{2\pi}}\,\frac{\exp(-1/2x)}{x^{3/2} g(1/\sqrt{x})}$$

The curve K versus x is represented on figure 1,

• q(x) is the function defined by :

$$q(x) = \frac{1}{g\left[1/\sqrt{K^{-1}(x)}\right] + \int_0^{K^{-1}(x)} ds\ g(1/\sqrt{s})}$$

The curve q versus x is represented on figure 2,

• g(x) is the French ERF function defined by :

$$g(x) = \sqrt{\frac{2}{\pi}} \int_0^x du\ \exp(-u^2/2)$$

• $K^{-1}(x)$ is the inverse function of K(x) defined from $]0, K_m[$ to $]x_m, +\infty[$,

• C is the specific heat per unit volume of the conductor evaluated at T_t ($J/m^3/K$),

• D is the thermal diffusivity of the conductor evaluated at T_t (m^2/s),

• P_j is the average density per unit volume of the power dissipated in the conductor at the normal resistive state (J/m^3),

• T_c is the critical temperature at the given field and zero current (K),

• T_{ci} is the critical temperature of the cable at the given field and the given current,

• $T_t = (T_c + T_{ci}) / 2$,

• T_0 is the operating temperature.

It yields that for a thermal disturbance of a given length ($2p_0$) the minimal energy deposition W_0 is the solution of the implicit equation :

$$\left(\frac{P_{jo}}{4D[C(T_t-T_0)]^3}\right)^{1/2} W_0\, p_0 = W(\lambda)$$

where W(x) is the function defined by :

$$W(x) = \begin{cases} \left\{ \mathrm{Max}\left[1;\ q(x)\right] \right\}^{3/2}\ x^{1/2} & \text{if } x \in\ ]0;\ K_m[\\ x^{1/2} & \text{if } x \in [K_m;\ +\infty[\end{cases}$$

The curve W versus x is represented on figure 3.

The mimum of the curve of figure 3 defines the absolute minimal energy deposition, which is given by :

$$\lambda_{min} = 0.29 \qquad \text{and} \qquad a_{min} = 1.08$$

I have written a FORTRAN program which takes care of all the calculations. The inputs of the program are : the location of the coil (inner or outer), the operating temperature, the operating current, the copper/superconductor ratio, the copper RRR, the critical current at 4.2 K and 5 T, and the half-length of the disturbance. The output is the minimal energy deposition W_0.
This file is : LBL117:[DEVRED.STAB]MINI.EXE.

To illustrate the use of this program, I have reported on figure 4 the curves W_0 versus p_0 for the inner coil cable of magnet DD0010 at 4.4 K and for two values of current : 5000 and 6000 A. On figures 5 and 6, I show, for these two currents, the influence of the copper/superconducting ratio, all other parameters being constant (included the short sample critical current density J_{ss}). It appears that for the lower current, which corresponds to the range of the training quenches, the effect is less than 15 %.

REFERENCES :

[1]A. DEVRED, "Etude des conditions d'amorcage des régimes de propagation de la zone normale," C.E.N. Saclay - D.Ph.P.E/S.T.I.P.E, internal report CRYOMAG/85-17 R&D (1985).

[2]L. SCHWARTZ, "Méthodes mathematiques pour les sciences physiques," Herman (Paris).

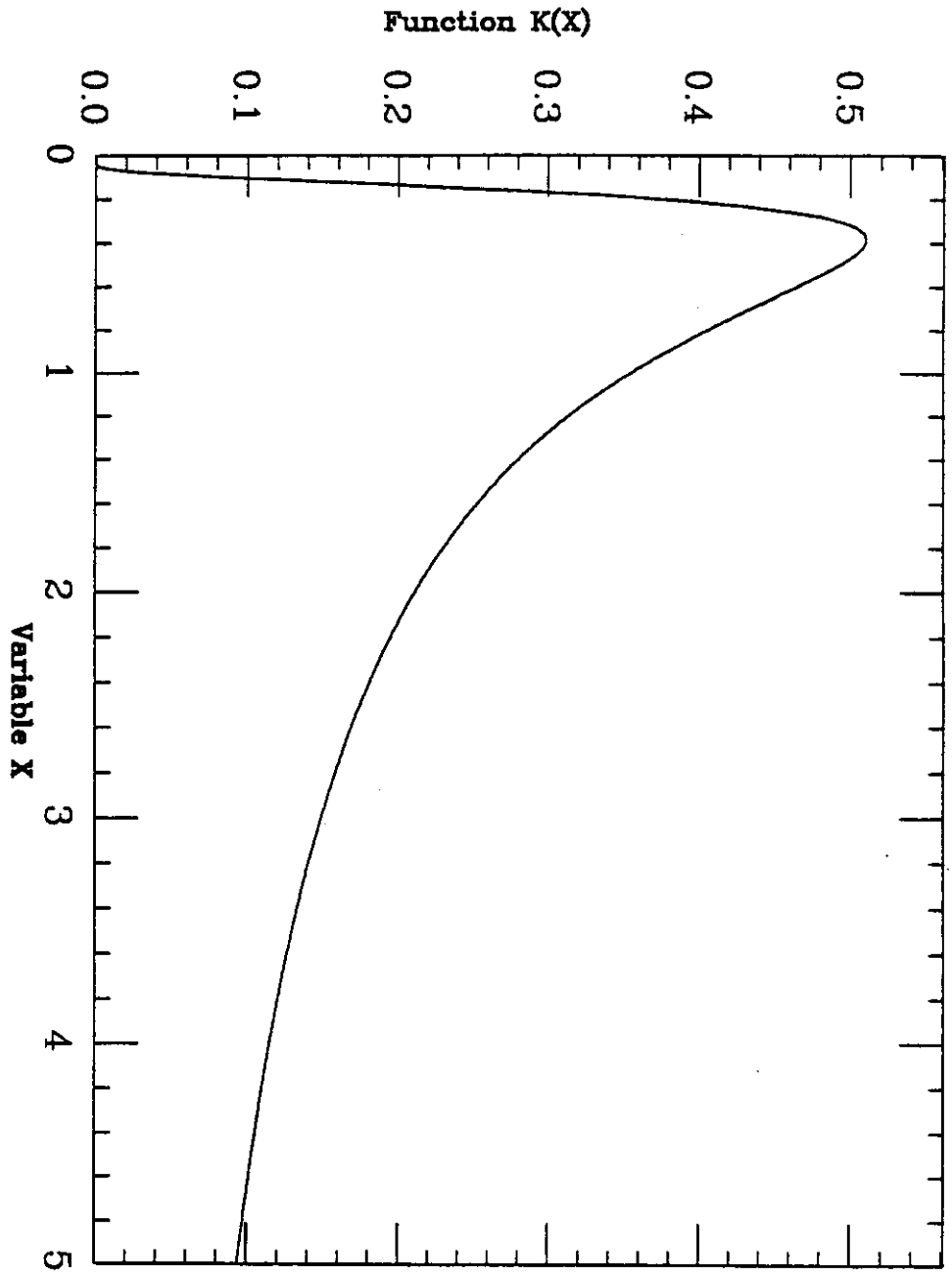

FIGURE 1

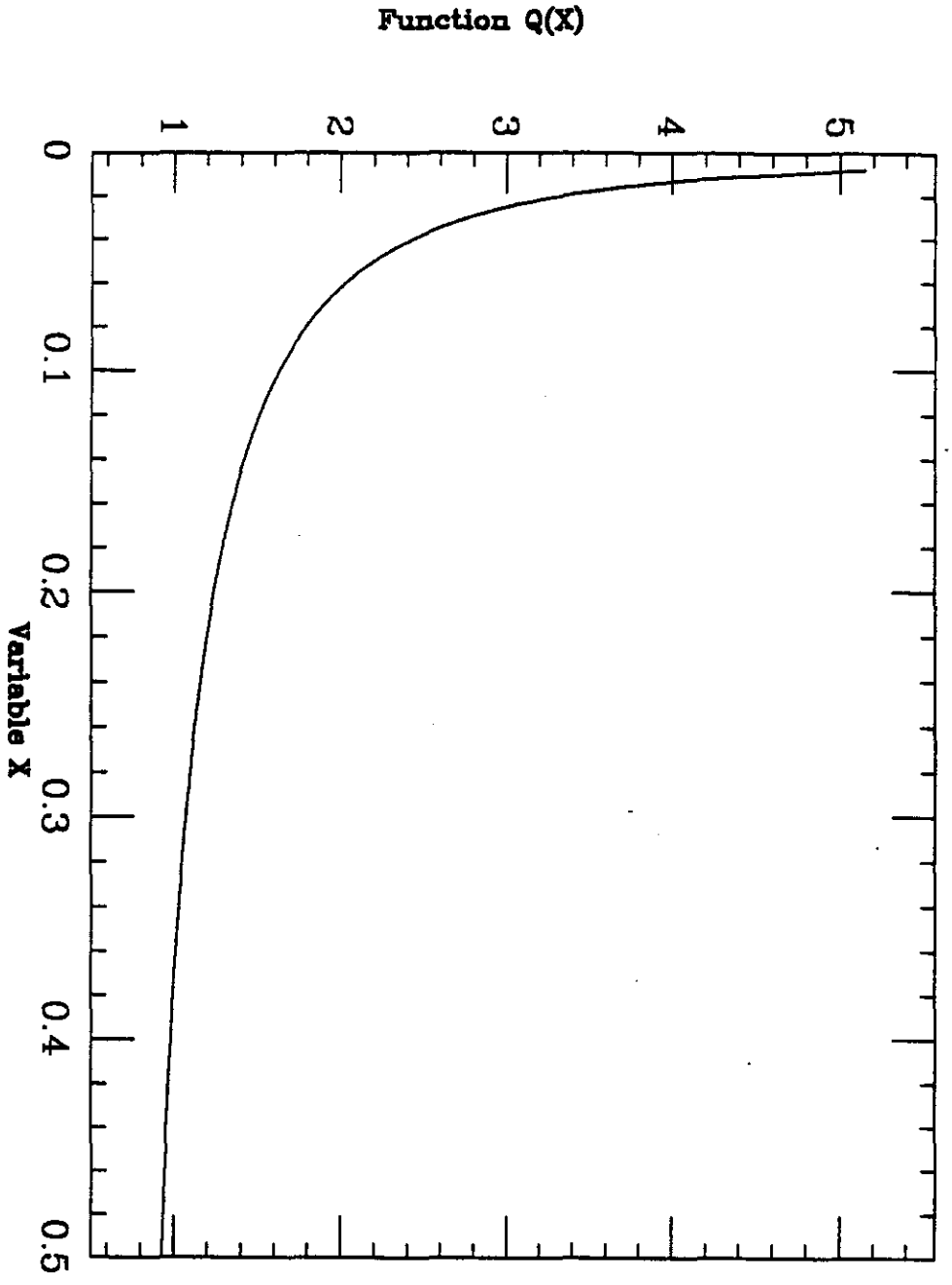

FIGURE 2

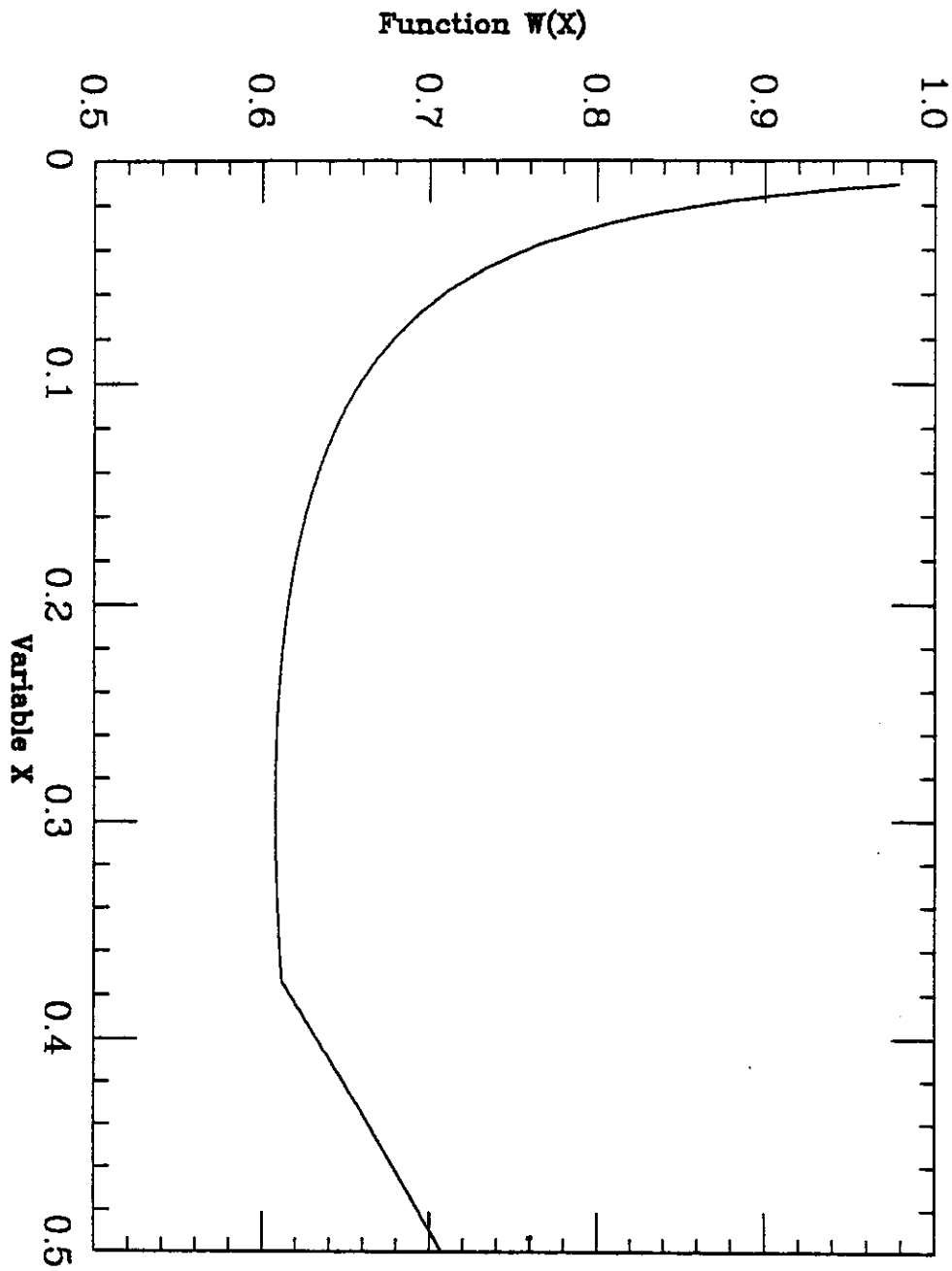

FIGURE 3

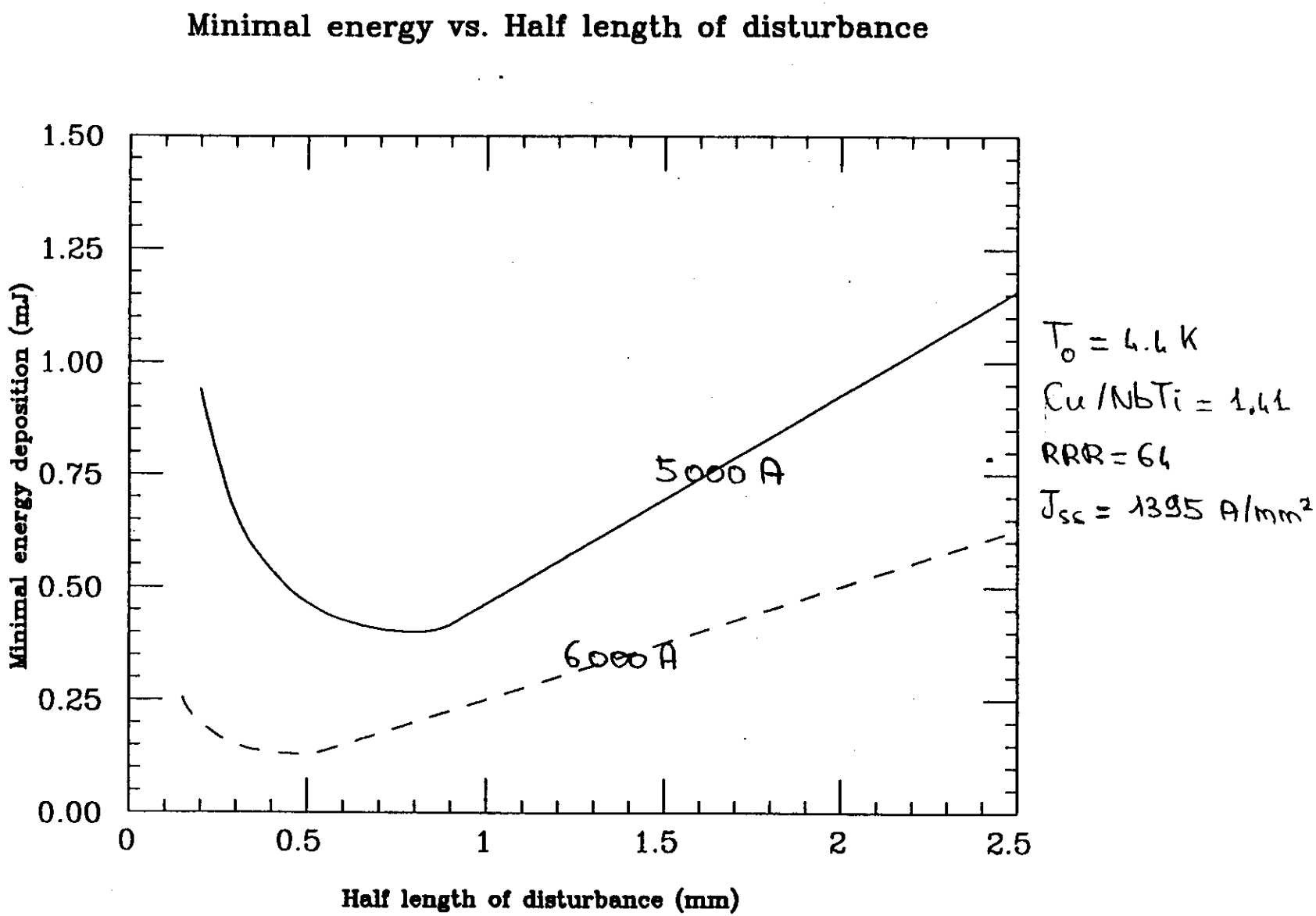
Minimal energy vs. Half length of disturbance
Minimal energy deposition (mJ)
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
0.5
1
1.5
2
2.5
Half length of disturbance (mm)
5000 A
6000 A
T_0 = 4.4 K
Cu/NbTi = 1.41
RRR = 64
J_{sc} = 1395 A/mm²

FIGURE 4

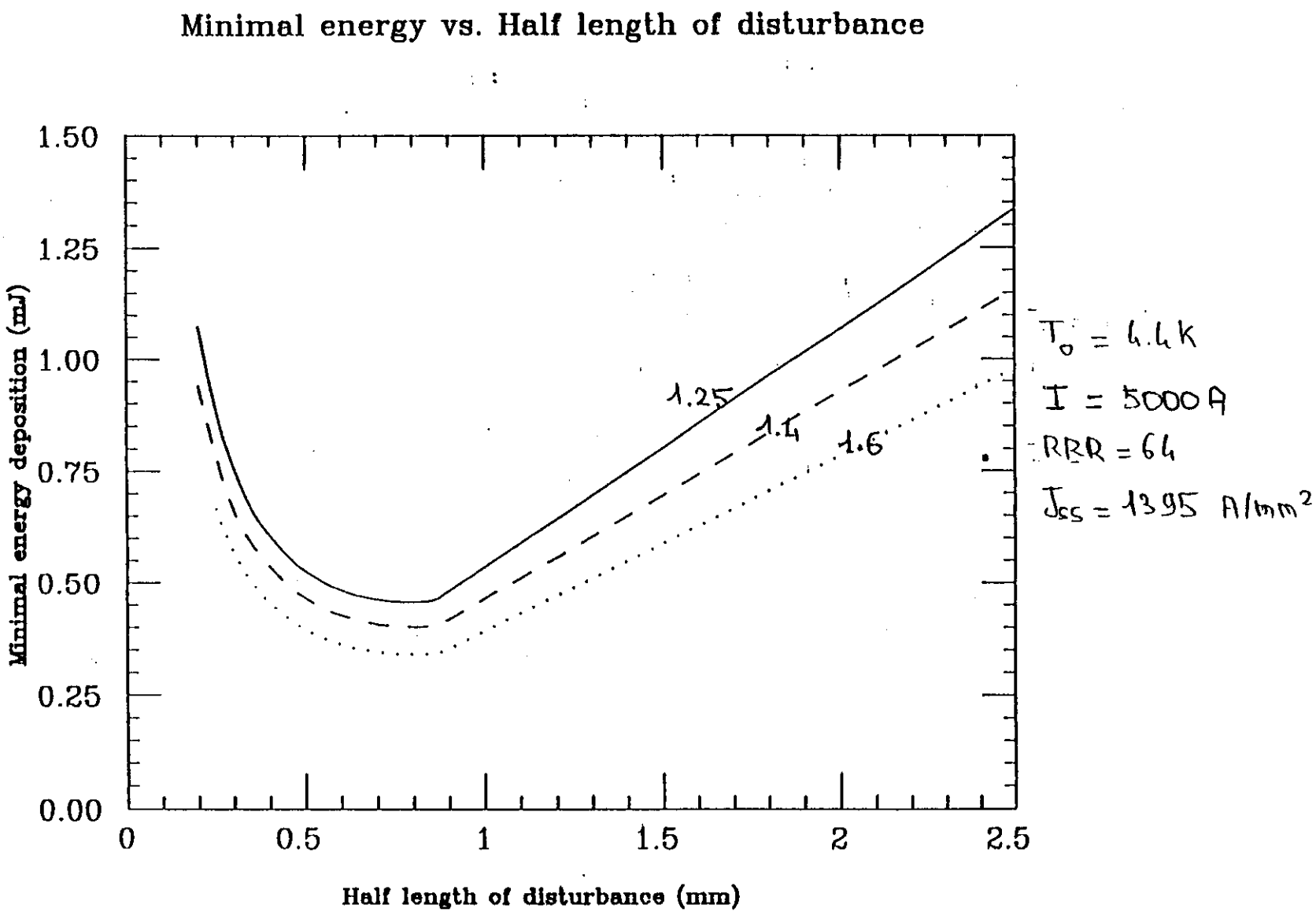
Minimal energy vs. Half length of disturbance
Minimal energy deposition (mJ)
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
0.5
1
1.5
2
2.5
Half length of disturbance (mm)
1.25
1.4
1.6
T_o = 4.4 K
I = 5000 A
RRR = 64
J_{ss} = 1395 A/mm²

FIGURE 5

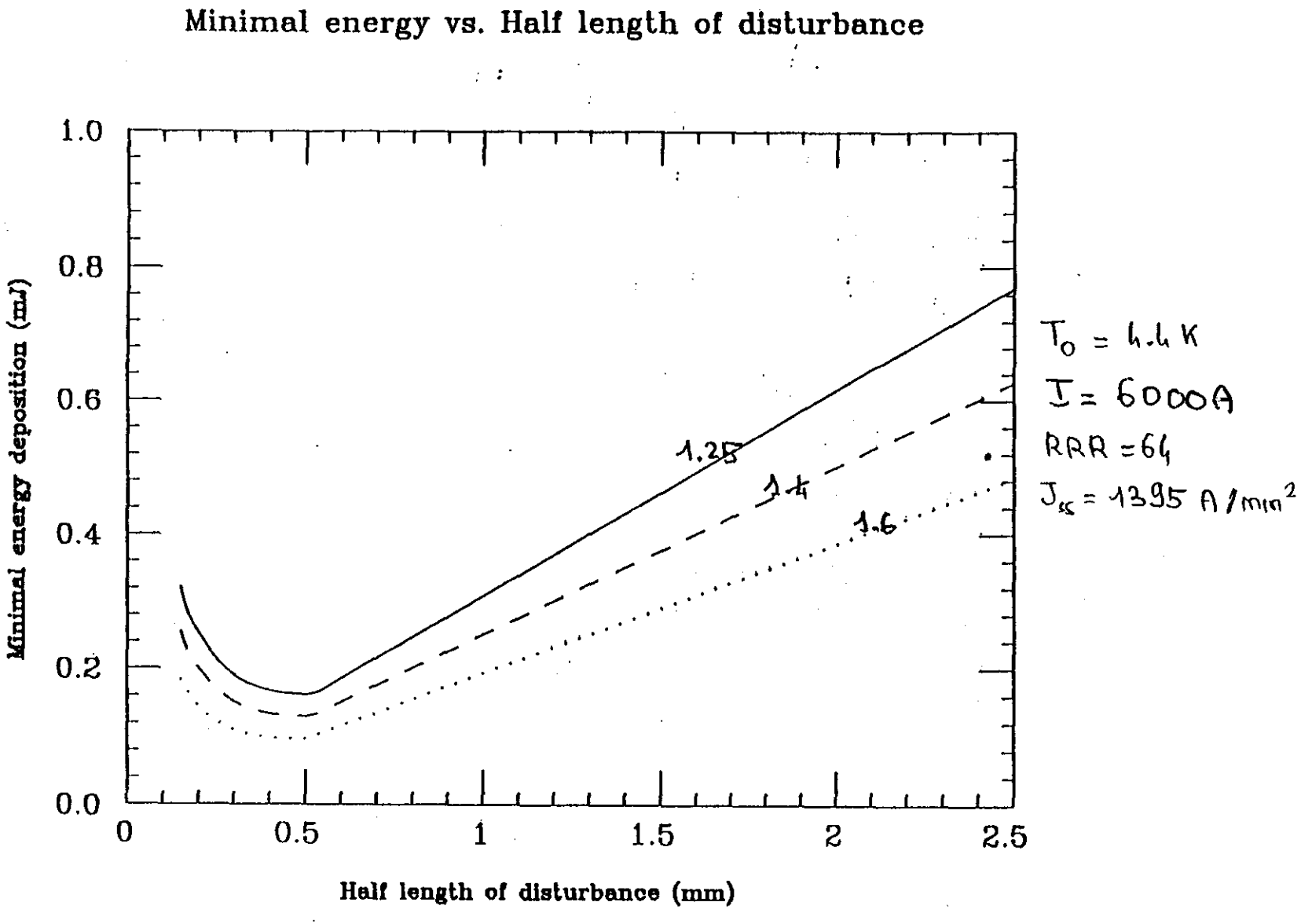
Minimal energy vs. Half length of disturbance
Minimal energy deposition (mJ)
1.0
0.8
0.6
0.4
0.2
0.0
0
0.5
1
1.5
2
2.5
Half length of disturbance (mm)
1.25
1.4
1.6
T_0 = 4.4 K
I = 6000A
RRR = 64
J_{ss} = 1395 A/mm^2

FIGURE 6