SSC-N-512

STRENGTH REQUIREMENTS FOR A TWO-LUMP CORRECTOR SCHEME

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1. Introduction

The strength requirements are estimated for a two-lump (Neuffer[1]) corrector scheme capable of correcting the systematic and the random normal sextupole, octupole, and decapole (b_2, b_3, and b_4) errors in the SSC dipole magnets and also for correcting the natural chromaticity. (The correction of the a_1 and b_1 (quadrupole) errors are not included in this discussion. None of the other higher multipoles are expected to require correction.)

2. The Two-Lump Corrector Scheme

The two-lump corrector scheme uses one corrector (C_{QF} or C_{QD}) in the spool next to each arc quadrupole and one (C_M) at or near the midpoint of each half cell.

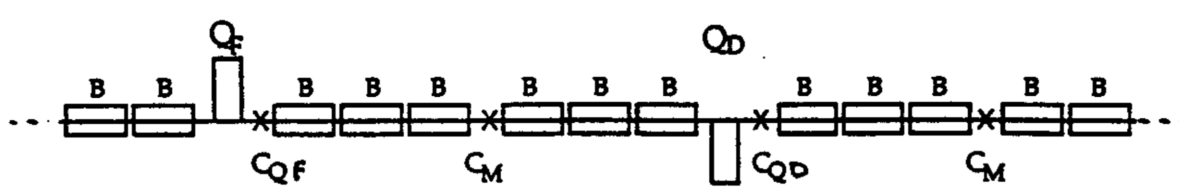

Fig. 1. The scheme with two lumped correctors per half cell.

3. Corrector Strengths

3a. Systematic. The correctors must compensate for the multipole errors due to persistent superconducting currents, for geometric errors in the coil construction, and for saturation effects. The largest absolute values of the required strengths all occur at 20 TeV. The required correction is given by the dipole specifications listed in SSC-N-183:

$$b_2 = 1.0 \text{ unit}$$
$$b_3 = 0.1 \text{ unit}$$
$$b_4 = 0.2 \text{ unit}$$

(Contingencies for dipoles not meeting these specifications will be considered in a later section.)

Using the "Simpson's Rule" ratio of 2:4 for the quadrupole to mid-cell corrector strengths, B_o = 6.6 Tesla, 6 dipoles per half cell, and the dipole length L_o of 16.54 m, we find the following corrector strengths at 1 cm for the systematic multipoles:

$$(CL)_{QF} = (CL)_{QD} = 2\, b_n\, B_o\, L_o$$
$$= 2 \times 1 \times 10^{-4} \times 6.6\text{ T} \times 16.54\text{ m} = .0218\text{ T-m}, \ (n=2)$$
$$= 2 \times 0.1 \times 10^{-4} \times 6.6\text{ T} \times 16.54\text{ m} = .00218\text{ T-m}, \ (n=3)$$
$$= 2 \times 0.2 \times 10^{-4} \times 6.6\text{ T} \times 16.54\text{ m} = .00437\text{ T-m}, \ (n=4)$$

$$(CL)_m = 4\, b_n\, B_o\, L_o = .0437\text{ T-m}, \ (n=2)$$
$$= .00437\text{ T-m}, \ (n=3)$$
$$= .00873\text{ T-m}, \ (n=4)$$

3b. Random. The correctors are used to compensate the variations in the multipole errors in the dipole magnets only if a binning arrangement is employed. For the specified random errors (SSC-N-183) the present view is that only the random normal sextupole (b_2) requires correction:

RMS Variation Specifications

n	a_n	b_n
2	0.6	2.0 units
3	0.7	0.3
4	0.2	0.7
5	0.2	0.1
6	0.1	0.2
7	0.2	0.2
8	0.1	0.1

The algorithm for determining corrector strengths needed to correct a known distribution of random errors (SSC-N-383) is conceptually applied to a three-lump system, as indicated below:

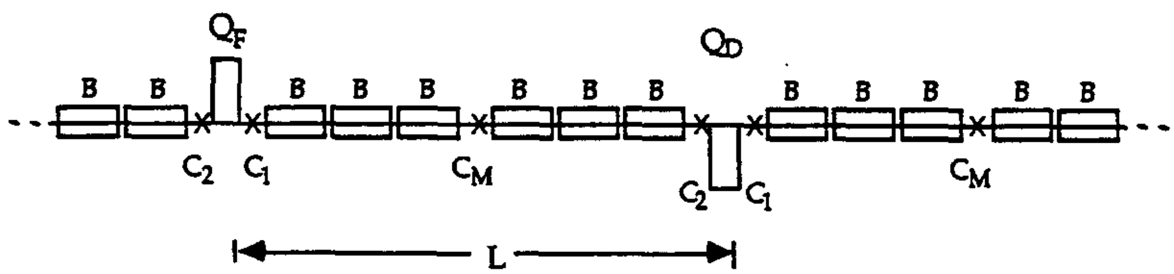

Fig. 2. The "three-lumped-correctors-per-half-cell" correction scheme. B = bend, Q = quad, C_M = mid-cell corrector, C_1 and C_2 = correctors before and after each quad.

Then the two correctors at each quadrupole (C_{C1} and C_{C2}) are combined into one corrector (Q_{CD} or Q_{CF}) located in the spool on the right side of each quadrupole, thus transforming the three-lump system into a two-lump. Note: that the strength of C_{C1} is based on the errors in the six dipoles of one half cell while that of C_{C2} is based on those in the adjacent half cell. Thus they are statistically independent.

For a given set of n-pole errors $\alpha_1, ..., \alpha_6$ in the six dipoles of a half cell, the optimum n-pole corrector strengths are given by:

$$(CL)_1 = -\frac{L_o}{108}(83\,\alpha_1 + 41\,\alpha_2 + 11\,\alpha_3 - 7\,\alpha_4 - 13\,\alpha_5 - 7\,\alpha_6)$$

$$(CL)_2 = -\frac{L_o}{108}(-7\,\alpha_1 - 13\,\alpha_2 - 7\,\alpha_3 + 11\,\alpha_4 + 41\,\alpha_5 + 83\,\alpha_6)$$

$$(CL)_m = -\frac{2\,L_o}{27}(4\,\alpha_1 + 10\,\alpha_2 + 13\,\alpha_3 + 13\,\alpha_4 + 10\,\alpha_5 + 4\,\alpha_6)$$

The average strengths for correcting random variations then are obtained by adding these contributions in quadrature, resulting in

$$(CL)_{1,avg} = (CL)_{2,avg} = 0.876\ \sigma_n L_o$$
$$(CL)_{m,avg} = 1.768\,\sigma_n L_o$$

Since the $(CL)_1$ and $(CL)_2$ at each quadrupole are statistically independent, the corresponding average strength $(CL)_{QF}$ or $(CL)_{QD}$ of the two-lump system is

$$\text{avg}\,(CL)_{QF\text{ or }QD} = \sqrt{2}\ (CL)_{Q1,avg} = 1.239\,\sigma_n L_o$$

Note that the total corrector strength per half cell for correcting random errors with a two-lump system

$$\text{avg}\ (CL)_{tot} = (CL)_{Q,avg} + (CL)_{m,avg} = 3.007\,\sigma_n L_o\ ,$$

whereas, if correctors are applied to the individual dipoles, the required half-cell integral strength would be $6.0\,\sigma_n L_o$ — i.e., twice as strong.

The maximum corrector strengths for correcting a random error distribution, truncated at 2σ, then are:

$$
\begin{aligned}
(CL)_{QF} = (CL)_{QD} &= 2 \times 1.239\,\sigma_n L_o \\
&= 2 \times 1.239 \times 2 \times 10^{-4} \times 6.6\ \text{T} \times 16.54\ \text{m} \\
&= 0.0542\ \text{T-m}\ \ (\text{at 1 cm, } n = 2) \\
(CL)_m &= 2 \times 1.768\,\sigma_m L_o \\
&= 2 \times 1.768 \times 2 \times 10^{-4} \times 6.6\ \text{T} \times 16.54\ \text{mm} \\
&= 0.0773\ \text{T-m}\ \ (\text{at 1 cm, } n = 2)
\end{aligned}
$$

3c. Binning Complications. If the two lumped correctors are used only to correct the random sextupole errors, seven staggered circuits are sufficient to accomplish binning, assuming that the mid-cell correctors can have 1.43 times the strength of the quad correctors and can use the same excitation current. However, if in addition these correctors must have another function, such as correcting the systematic sextupole and/or the chromaticity with relative strengths differing in ratio from 1.43:1, then either more circuits or separate windings must be used.

The case of systematic sextupole correction is not a problem, because the quadratic sextupole correction is not sensitive to the ratio of mid-cell to quad-position corrector strengths. Any ratio 1:1 and 2:1 works well. Thus 7 circuits can take care of the systematic as well as the random sextupole correction, as long as each of the seven circuits can be individually varied to accommodate the variation with B and t of the systematic and random sextupole strengths in the dipoles.

The chromaticity correction is not easily compatible with the random correction, since it uses two sextupole-corrector strengths (of opposite sign) and does not use the mid-cell corrector at all, and furthermore, the chromaticity correction strength depends on the I. R. optics. Thus, it seems simpler to use separate chromaticity sextupole windings in the quad-correctors and to power them with two separate circuits. Since the ratio of the two chromaticity corrector strengths is constant (-2.003:1 for the standard injection and collision optics), independent of the magnitude of the chromaticity correction, it would in principle be possible to power the two chromaticity correctors with just one circuit. But to allow for lattice imperfections and for tune changes, two chromaticity circuits should be provided.

3d. Chromaticity. To correct the natural chromaticity only the sextupole correctors adjacent to the quadrupoles will be used. The required sextupole strengths needed in the worst case (collision optics at 20 TeV) is obtained from the SYNCH program (SSC-146)

$$KSF \equiv (B''\,L)_{SF}/B\rho = 0.009992\ m^{-2}$$

$$KSD \equiv (B''\,L)_{SD}/B\rho = -0.02002\ m^{-2}$$

Thus

$$(CL)_{QF} = KSF\,\frac{r_o^2}{2}\,B\rho$$

$$= .009992\,(.01\ m)^2\ 6.671 \times 10^4\ T\text{-}m/2$$

$$= 0.0333\ T\text{-}m \quad \text{(at 1 cm, sext.)}$$

$$(CL)_{QD} = -\,0.02002\,(.01\ m)^2\ 6.67 \times 10^4\ T\text{-}m/2$$

$$= -\,0.0668\ T\text{-}m \quad \text{(at 1 cm, sext.)}$$

3e. Contingencies. The systematic multiples in the SSC dipoles are the least well known of the three contributions to the corrector strengths. They depend on the design of the coil geometry and do not necessarily scale from the Tevatron dipoles. Accordingly a contingency of 100% is assigned to the systematics contribution.

The random strengths are better known, since they depend mostly on manufacturing reproducibility. Scaling from the Tevatron data is likely to be valid and conservative. Therefore a 50% contingency is assigned to the random contributions.

The chromaticity correction should be known quite well from the lattice calculations. However, a 100% contingency is assigned to the chromaticity strengths in order to allow for operating six standard low-beta interaction regions plus a 20% margin for special low-beta optics.

4. Summary of Two-Lump Corrector Strengths at 1 cm, Including Contingencies

<table>
<tr><th></th><th></th><th colspan="3">Correction Strength</th><th></th></tr>
<tr><th>Corrector</th><th>Multipole</th><th>Systematic</th><th>Random</th><th>Chromaticity</th><th></th></tr>
<tr><td>C_{QF}</td><td>b_2</td><td>0.054</td><td>0.081</td><td>0.066</td><td>T-m</td></tr>
<tr><td></td><td></td><td colspan="2">0.135</td><td></td><td></td></tr>
<tr><td></td><td>b_3</td><td>0.0044</td><td>--</td><td>--</td><td>T-m</td></tr>
<tr><td></td><td>b_4</td><td>0.087</td><td>--</td><td>--</td><td>T-m</td></tr>
<tr><td>C_{QD}</td><td>b_2</td><td>0.054</td><td>0.081</td><td>-0.134</td><td>T-m</td></tr>
<tr><td></td><td></td><td colspan="2">0.135</td><td></td><td></td></tr>
<tr><td></td><td>b_3</td><td>0.0044</td><td>--</td><td>--</td><td>T-m</td></tr>
<tr><td></td><td>b_4</td><td>0.0087</td><td>--</td><td>--</td><td>T-m</td></tr>
<tr><td>C_M</td><td>b_2</td><td>0.077</td><td>0.116</td><td>--</td><td>T-m</td></tr>
<tr><td></td><td></td><td colspan="2">0.193</td><td></td><td></td></tr>
<tr><td></td><td>b_3</td><td>0.0087</td><td>--</td><td>--</td><td>T-m</td></tr>
<tr><td></td><td>b_4</td><td>0.0175</td><td>--</td><td>--</td><td>T-m</td></tr>
</table>

Note: In this table the systematic mid-cell and quad corrector strengths are in the ratio of 1.43:1 for b_2 and 2:1 for b_3 and b_4.

If binning of the sextupole correctors is not required, the required at 20 TeV, the random b_2 contributions in the table become zero, and those coils become much smaller.