SSC-N-505

Closed-Orbit Excursions Due To A Systematic Dipole Tilt

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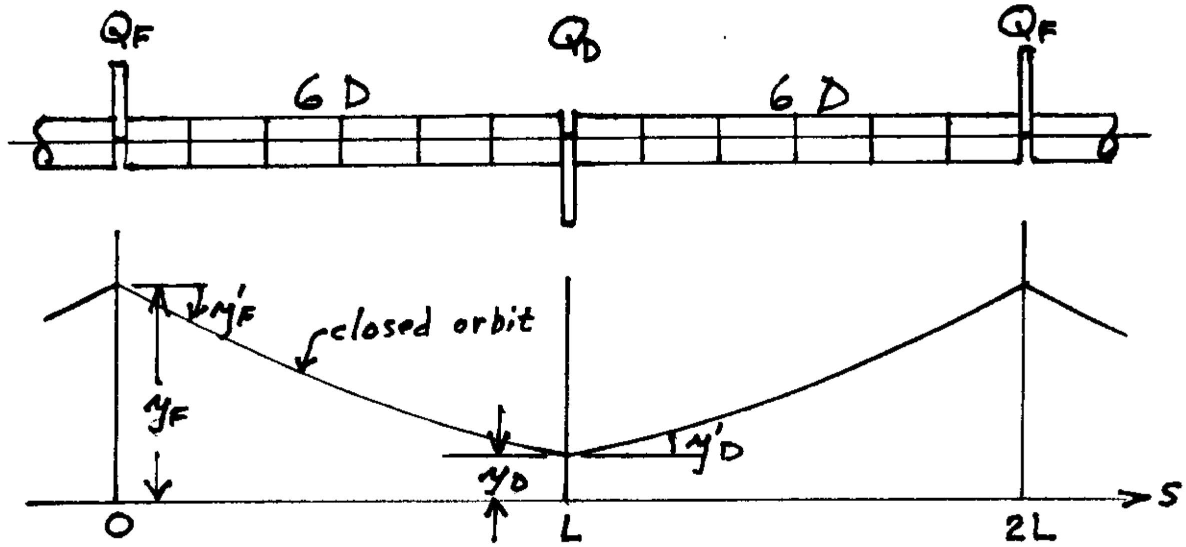
April 21, 1988

A systematic tilt of the dipole magnets produces a vertical closed-orbit excursion. To estimate the magnitude of such a closed-orbit excursion, it is convenient to model the SSC as made up simply of 319 90-degree arc cells (3828 dipoles) with no imperfections other than the tilted dipoles and to use thin-lens quads and dipoles that fill the space between quads.



The orbit that closes on itself has a pattern that repeats in each cell. Since by symmetry $y' = 0$ at the center of each quad, we see that the angles in and out of each quad are of opposite sign and equal to

$$y'_F = -y_F/2f$$
$$y'_D = +y_D/2f \qquad (1)$$

where f is the quad focal length ($1/f = B'_Q L_Q / B\rho$). Between Q_F and Q_D the slope $y'(s)$ varies as

$$y'(s) = -\frac{y_F}{2f} + \phi\,\frac{s}{\rho} \tag{2}$$

where ϕ is the systematic angle of tilt of the dipoles. ρ is the magnetic radius of curvature. By integrating we find y_D and y'_D in terms of y_F and ϕ:

$$\begin{aligned} y_D &= y_F\left(1 - \frac{L}{2f}\right) + \frac{\phi L^2}{2\rho} \\ y'_D &= +y_F/2f - \phi L/\rho = y_D/2f \end{aligned} \tag{3}$$

where L is the half-cell length. From these equations we find

$$y_F = \frac{\phi f}{\rho}(4f + L) \tag{4}$$

and

$$y_D = \tfrac{\phi f}{\rho}(4f - L) \quad .$$

To correct for the fact that in the SSC arcs the dipoles occupy only about 87% of the space, we can multiply these expressions for y_F and y_D by 0.87. Evaluating (4) with $L = 114.25$ m, $f = L/2\sin\mu/2 = 80.8$ m, $\rho = 1.009 \times 10^4$ m, and $\phi = 10^{-3}$ radian, e.g., we obtain:

$$\begin{aligned} y_F &= 3.04 \text{ mm} \\ y_D &= 1.45 \text{ mm} \quad . \end{aligned}$$

The rms value of this closed-orbit excursion in such a 90° lattice is readily evaluated to be

$$\langle y \rangle = 1.75\,\phi\,L^2/\rho$$

which for the given parameters is equal to 2.27 mm.

Thus the SSC could tolerate and correct for a systematic tilt angle on the order of 1 milli-radian. However, operationally it would be desirable to reduce it to a much lower value.