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Numerical Study of Various Lumped Correction Schemes for Random Multipole Errors

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ABSTRACT

The transverse amplitude and momentum dependence due to random multipole errors in the dipoles has been examined for various lumped correction schemes of the SSC lattice. The lumped compensation elements are set using analytical formulas developed by Forest and assume that "fine binning" circuits are available. The results obtained from analysis of particle tracking data using the program TEAPOT have been compared with analytical calculations and found to be in agreement. It is shown that any of the examined lumped correction schemes is capable of compensating for the random errors specified for the SSC, with a scheme due to Neuffer being especially effective.

1. Introduction.

The purpose of this study is to compare three lumped compensation schemes for correction of random magnet errors present in the SSC. We have imposed a constraint that no more than two physical correctors are present per half cell. We have also considered a SSC lattice which has been stripped down to its bare minimum to allow the comparison of the numerical and analytical results. Thus the "arcs only" lattice considered here consists solely of 320 identical cells, which in turn contain only the quadrupoles, dipoles, lumped correction elements and separate chromaticity sextupoles. The deletion of the intersection regions and simplification of the rest of the lattice is made merely to facilitate comparisons. The crucial parameters of the full SSC lattice have been retained. The tune of the lattice were chosen to be

$$Q_x = 81.285$$
$$Q_y = 82.265$$

where the integer tune split was chosen to reduce the effect of systematic coupling.[1] The dipole errors included in the study are the random sextupole, octupole and decapole errors (which dominate the smear in the SSC) including their skew components.

In addition of studying the effectiveness of the correction schemes in the case when all the multipole errors are being compensated for, a more detailed study of the contribution of the individual dipole errors to the smear and tune has been done for the Neuffer scheme only. A few selective tracking runs have been done for the other correction schemes to examine the their consistency. In all cases we have assumed that the lumped correctors are "infinitely binned"[2] , that is the corrector strengths can all be adjusted independently of each other. The degradation due to finite binning can be estimated by calculating the rms field errors remaining after the binning compensation has been performed. The corrector strengths are assigned according to analytical formulas due to Forest.[3] It is found that although all three of the studied schemes satisfy the requirement of random error compensation for the SSC, the lumped corrector scheme proposed by Neuffer[4] is the most effective.

2. Correction Schemes.

We have considered three lumped correction schemes each of which contain two physical lumped correctors per half cell. The first of the three schemes places one of the correctors to one side of the quadrupole, and the other in the center of the half cell. Conceptually, however, this scheme proposed by Neuffer[4] can be regarded to contain three correctors per half cell, as the corrector adjacent to the quadrupole acts as if it is a pair of correctors straddling the quadrupole. This "conceptually three lumped" correction scheme allows the analysis on one half cell to be done without considering the quadrupoles. The strength of the physical corrector is then given by the sum of the two "half" correctors, whose strengths are determined by the dipole errors in the respective half cell. This combination of correctors have been previously found to be effective.[3][4] The two remaining schemes place the correctors symmetrically around the center of the half cell, their positions indicated in the following schematic.

$Q_fC_fBBBC_cBBBC_dQ_d$	Conceptual Three Lumped Scheme
$Q_fC_fBBBC_cBBBQ_d$	Neuffer Scheme or *3*3 Scheme
$Q_fBC_fBBBBC_cBQ_d$	1*4*1 Scheme
$Q_fBBC_fBBC_cBBQ_d$	2*2*2 Scheme

Here Q_f and Q_d signify the focusing and defocusing quadrupoles, B the bending magnets and C_i indicates the locations of the correctors in the half cell. The correction schemes will be referred to in the future by the name next to the configuration. The $*'s$ in the notation of the correction schemes denote the position of the correctors, while the number indicate the number of magnets between them.

The corrector strengths of the three schemes were evaluated by applying the general scheme developed by Forest[3][5] to the individual half cell of six bending magnets and two correctors. We obtain for the three compensation schemes the following corrector strengths.

*3 * 3 Scheme (in "three lumped corrector representation"):

$$\begin{aligned}
C_f L_f &= \frac{L_b}{108}[-83\alpha_1 - 41\alpha_2 - 11\alpha_3 + 7\alpha_4 + 13\alpha_5 + 7\alpha_6] \\
C_c L_c &= -\frac{L_b}{27}[8\alpha_1 + 20\alpha_2 + 26\alpha_3 + 26\alpha_4 + 20\alpha_5 + 8\alpha_6] \\
C_d L_d &= \frac{L_b}{108}[7\alpha_1 + 13\alpha_2 + 7\alpha_3 - 11\alpha_4 - 41\alpha_5 - 83\alpha_6]
\end{aligned}$$

1 * 4 * 1 Scheme:

$$\begin{aligned}
C_f L_f &= \frac{L_b}{8}[-9\alpha_1 - 7\alpha_2 - 5\alpha_3 - 3\alpha_4 - \alpha_5 + \alpha_6] \\
C_c L_c &= \frac{L_b}{8}[\alpha_1 - \alpha_2 - 3\alpha_3 - 5\alpha_4 - 7\alpha_5 - 9\alpha_6]
\end{aligned}$$

2 * 2 * 2 Scheme:

$$\begin{aligned}
C_f L_f &= \frac{L_b}{4}[-7\alpha_1 - 5\alpha_2 - 3\alpha_3 - \alpha_4 + \alpha_5 + 3\alpha_6] \\
C_c L_c &= \frac{L_b}{4}[3\alpha_1 + \alpha_2 - \alpha_3 - 3\alpha_4 - 5\alpha_5 - 7\alpha_6]
\end{aligned}$$

where α_i is the multipole error of the i^{th} dipole magnet. $C_x L_x$ is the integral strength of corrector x. Note that the weights do not depend on the multipole order.

3. Tracking Analysis.

We have included in this study only the random magnetic multipole errors from sextupole to decapole order, whose values are listed in Table 1. The effect of the systematic error on smear and tune is reported elsewhere.[6]

Table 1. RMS Variations of Multipole Errors in SSC Dipoles ($10^{-4}B_0$ at $1cm$).[7]

a_2	0.6 cm^{-1}	b_2	2.0 cm^{-1}
a_3	0.7	b_3	0.3
a_4	0.2	b_4	0.7

To obtain the amplitude and energy dependence of the smear and tune, particles were tracked around the the "arcs only" SSC lattice. The three independent particle variables are x, the horizontal, and y the vertical transverse amplitude and the momentum offset δ. The tracking is performed at all possible combinations of

$$x = \ 0.0\ mm, 5\ mm \text{ at the point where } \beta_x \text{ is maximum}$$

$$y = \ 0.0\ mm, 5\ mm \text{ at the point where } \beta_y \text{ is maximum and}$$

$$\delta = \ 0.0, \pm 0.001,$$

making a total of 12 particles per scheme. The results will be presented in the following format:

$$\begin{pmatrix} S(0,5,+0.001) & S(5,5,+0.001) \\ S(0,5,0.000) & S(5,5,0.000) \\ S(0,5,-0.001) & S(5,5,-0.001) \\ & \\ S(0,0,+0.001) & S(5,0,+0.001) \\ S(0,0,0.000) & S(5,0,0.000) \\ S(0,0,-0.001) & S(5,0,-0.001) \end{pmatrix}$$

where $S = \ S(x, y, \delta)$ is the $\frac{1}{3}$(peak-to-peak) smear. The unit for the smear results is chosen to be a part in hundred such that the acceptable value according to the CDR specification is 10. The tune variations $\Delta Q_x(x, y, \delta)$ and $\Delta Q_y(x, y, \delta)$ after correction for the random errors are in general one to two orders of magnitude smaller than the allowed maximum tune shift variation of ± 0.005. The results will therefore not be presented explicitly.

The tracking was performed using the program TEAPOT.[8] Prior to the actual tracking the random errors are corrected by the above described method with careful consideration that the random errors are identical for each of different schemes. The tunes of the lattice are set and the chromaticities are adjusted to zero by separate chromaticity sextupoles. Each particle is tracked for 256 turns and the tracking results are Fourier analysed to obtain the tune and smear. For the main study, where all multipole errors are compensated, a total of 5 seeds are run for each of the correction schemes in addition to the uncorrected case. In the study of the contribution to the smear by the individual skew multipole errors, only one seed is run. To achieve adequate accuracies for the tracking results, the SSC dipoles are represented by two kicks per dipole. Studies on the lattice with systematic errors have shown that this will provide acceptable accuracy.[6]

4. Results.

The following table lists the results of the amplitude and momentum dependence of the smear for the various schemes. Table 2. give the results of the 5 seed average of the smear for the lattice with random errors before and after correction by the three described compensation schemes. Schemes $2*2*2$ and $1*4*1$ exhibit almost consistently a reduction of the smear by a factor of 5 to 7. The absolute values of the smear imply that these two compensation schemes would be effective in the design of the SSC lattice. The $*3*3$ ("Neuffer") scheme displays a even better reduction factor of the smear. It reduces the smear all around by a factor of 10 to 13, reducing the smear in the worst case to less than one percent.

The relative effectiveness between schemes $2*2*2$ and $1*4*1$ cannot be fully analysed with only 5 random seeds. The fluctuations of the smears from tracking results for the two schemes are simply to large to separate them clearly, although the average over 5 seeds seem to indicate that the $1*4*1$ scheme is better. A more comprehensive study of these two schemes will be necessary, should a choice be made between the $1*4*1$ and the $2*2*2$ compensation scheme. On the other hand, the Neuffer scheme consistently exhibits better results than the other two.

Analytical calculations of the smear have previously been done by Forest[9] for various correction schemes. A direct comparison of the results is marred by slight differences in tunes, the inclusion of decapole errors in the numerical study and the representation of the dipole magnet by two kicks in the numerical study compared with one in the analytical. Nevertheless, the smear reduction factor of the uncorrected case to scheme ($2 * 2 * 2$, $1 * 4 * 1$) to scheme $* 3 * 3$ in the analytical study is approximately 1:6:10, in acceptable agreement with the numerical studies in this report. Quantitatively, the analytical smears at $x = 5\ mm$, $y = 5\ mm$ for the three cases are 9.5, 1.6 and 1.0 compared with 10.2 ± 1.1, $1.33 \mapsto 1.93 \pm 0.2$ and 0.74 ± 0.1. The smaller smear in the numerical Neuffer case may be explained by the difference in the SSC dipole representation between the two methods.

Table 2. 5 Seed Average of Smear

No correction:

		$8.71 \pm .94$			11.1 ± 1.3
	$6.28 \pm .94$			10.2 ± 1.1	
7.80 ± 1.0			10.5 ± 0.7		
		$0.16 \pm .02$			6.73 ± 1.2
	$0.15 \pm .02$			5.52 ± 1.2	
$0.17 \pm .01$			7.40 ± 1.0		

$*3*3$ Scheme:

		$0.59 \pm .09$			$0.82 \pm .15$
	$0.61 \pm .11$			$0.74 \pm .11$	
$0.65 \pm .10$			$0.83 \pm .09$		

		$.011 \pm .002$			$0.55 \pm .53$
	$.009 \pm .002$			$0.42 \pm .53$	
$.010 \pm .002$			$0.49 \pm .44$		

$2*2*2$ Scheme:

		$1.53 \pm .06$			$2.04 \pm .17$
	$1.39 \pm .13$			$1.93 \pm .21$	
$1.24 \pm .23$			$2.01 \pm .14$		

		$.035 \pm .002$			$1.42 \pm .15$
	$.033 \pm .002$			$0.98 \pm .24$	
$.034 \pm .004$			$1.04 \pm .33$		

$1*4*1$ Scheme:

		$1.09 \pm .09$			$1.58 \pm .15$
	$0.98 \pm .18$			$1.33 \pm .24$	
$1.23 \pm .14$			$1.68 \pm .11$		

		$.027 \pm .002$			$1.02 \pm .11$
	$.027 \pm .002$			$0.75 \pm .19$	
$.029 \pm .002$			$1.04 \pm .11$		

Table 3. summarizes the results of the smear for the Neuffer correction scheme when only the specified multipole errors in the SSC dipoles are being compensated for. The table lists the smear resulting from tracking the particle launched at $x = 5mm, y = 5mm$ and $\delta = 0.0$. The complete sets of results are given in

Appendix A. The same five random seeds are used in these tracking runs as in the above described study. Five cases have been studied:

(i) correction for b_2 only,

(ii) correction for b_2, a_2,

(iii) correction for b_2, a_3,

(iv) correction for b_3, a_3,

(v) correction for b_2, a_2, b_3, a_3.

Table 3. 5 seed average smear $x = 5mm, y = 5mm$ and $\delta = 0.0$ for the Neuffer scheme correcting only for the indicated multipole errors. The random sextupole through decapole errors are present in all cases.

Corrected Random Multipole Errors	Smear	Smear Reduction Factor
none	10.2 ± 1.1	
b_2	$4.24 \pm .44$	2.4
b_2, a_2	$3.29 \pm .51$	3.1
b_2, a_3	$2.65 \pm .20$	3.8
b_3, a_3	8.96 ± 1.0	1.1
b_2, a_2, b_3, a_3	$0.88 \pm .14$	11.6
all corrected	$0.74 \pm .11$	13.8

From the table it is evident that correcting only for the normal sextupole reduces the smear by more than half. Furthermore the additional correction of the skew sextupole component amounts to a lesser gain in the smear improvement than the additional correction of the skew octupole errors. In the later case the smear improves by a factor of almost four. This reinforces the notion that the normal sextupole and the skew octupole errors contribute the most to the smear due to the random multipole errors. The last conclusion which can be drawn out of Table 3. is that the random decapole errors contribute very little to the smear. The analytical calculations by Forest[9] show an improvement of the smear from correcting only the b_2's by a factor of approximately 2 in agreement with the numerical results presented here.

5. Conclusions.

This study shows that any of the two-corrector per half cell lump corrector schemes is capable of compensating for the random errors specified for the SSC satisfactory. Furthermore the correction scheme proposed by Neuffer appears to be the most effective compensation scheme. The correction improvements of the various schemes resulting from this numerical study agree in general with previous analytical calculation.

6. Appendix A

This appendix lists the smears for the Neuffer scheme for correcting selected multipole errors. The results are obtained from an average of five random seeds. The uncorrected smears are listed for comparison.

No correction:

$8.71 \pm .94$	11.1 ± 1.3
$6.28 \pm .94$	10.2 ± 1.1
7.80 ± 1.0	10.5 ± 0.7

$0.16 \pm .02$	6.73 ± 1.2
$0.15 \pm .02$	5.52 ± 1.2
$0.17 \pm .01$	7.40 ± 1.0

b_2 corrected only:

$4.49 \pm .52$	$6.43 \pm .74$
$3.38 \pm .92$	$4.24 \pm .44$
$4.76 \pm .88$	$5.22 \pm .56$

$.080 \pm .007$	$4.59 \pm .78$
$.048 \pm .006$	$2.89 \pm .49$
$.061 \pm .001$	$5.23 \pm .74$

a_2, b_2 corrected only:

$3.93 \pm .39$	$4.95 \pm .40$
$3.45 \pm .97$	$3.29 \pm .51$
$3.62 \pm .79$	$5.54 \pm .35$

$.052 \pm .004$	$4.32 \pm .98$
$.019 \pm .004$	$2.57 \pm .50$
$.047 \pm .004$	$4.85 \pm .47$

b_2, a_3 corrected only:

		$2.73 \pm .32$			$3.91 \pm .36$
	$1.54 \pm .19$			$2.65 \pm .20$	
$2.33 \pm .45$			$3.90 \pm .16$		

		$.049 \pm .004$			$3.09 \pm .52$
	$.042 \pm .005$			$1.78 \pm .18$	
$.050 \pm .003$			$3.07 \pm .39$		

a_3, b_3 corrected only:

		7.11 ± 1.2			$9.00 \pm .81$
	$5.80 \pm .71$			8.96 ± 1.0	
$5.91 \pm .63$			$9.42 \pm .75$		

		$.150 \pm .018$			5.82 ± 1.0
	$.147 \pm .015$			5.31 ± 1.0	
$.160 \pm .013$			$6.59 \pm .88$		

a_2, b_2, a_3, b_3 corrected only:

		$1.36 \pm .25$			$2.50 \pm .11$
	$.637 \pm .13$			$.880 \pm .14$	
$1.49 \pm .29$			$2.50 \pm .06$		

		$.019 \pm .001$			$1.29 \pm .19$
	$.021 \pm .001$			$.454 \pm .03$	
$.018 \pm .002$			$1.36 \pm .20$		

7. Appendix B

Table B.1. lists the smear before and after correcting for the normal multipole errors only, that is b_2, b_3, and b_4. The point of this exercise is to demonstrate that the Neuffer scheme does not have an deficiency in this case rather than to present accurate tracking results. Thus only one random seed has been run. For comparison sake, the smears resulting from correcting all the multipole errors are listed in parallel with the smears from correcting only the normal multipole errors. The correcting power of the Neuffer scheme in this case is around 5, whereas that of the $2 * 2 * 2$ scheme is around 4. The $1 * 4 * 1$ scheme exhibits similar results as the $2 * 2 * 2$ scheme.

Finally we studied various cases of the Neuffer scheme correcting all the normal and various skew multipole errors. Table B.2. summarizes the smear results for one random seed. Again, for comparison sake, the smears resulting from correcting all the multipole errors are also given. Taking into account Table B.1., we notice, that while correcting all the b_i's and none of the a_i's does show an appreciable improvement in smear, further correction of either the skew sextupole or skew octupole error by itself does not show appreciable gain in the smear reduction. The improvement is around 30 percent. On the other hand, the correction ratio for correcting both a_2 and a_3 but not a_4, approaches that of the ideal case.

No correction:

		5.845			16.04
	4.266			13.20	
5.885			10.66		

		.2073			10.33
	.1804			9.655	
.1750			9.454		

$*3*3$ Scheme:

		.708 | 1.98			.826 | 4.70
	.762 | 1.33			.873 | 2.87	
.816 | 1.45			.942 | 2.87		

		.008 | .055			.309 | 3.91
	.007 | .030			.378 | 2.09	
.006 | .054			.566 | 4.76		

$2*2*2$ Scheme:

		1.47 | 2.34			1.28 | 5.35
	.905 | 1.17			1.32 | 3.31	
1.35 | 2.07			1.73 | 5.01		

		.023 | .061			.862 | 4.38
	.022 | .043			.749 | 2.12	
.023 | .064			.879 | 4.98		

Table B.1. Smear with Uncorrected Skew Multipole Errors

The smears are given for one random seed. For comparison sake, the smear resulting from correcting all random errors is given to the left of each box. To the right of each box the results from correcting only the random b_i's are given. The $1*4*1$ compensation scheme exhibits a similar behavior as the $2*2*2$ scheme, and is not presented.

$*3*3$ Scheme, b_i's and a_2 corrected:

.708	2.89	.826	3.88
.762	1.26	.873	2.09
.816	1.42	.942	3.69

.008	.041	.309	5.84
.007	.012	.378	1.89
.006	.049	.566	4.06

$*3*3$ Scheme, b_i's and a_3 corrected:

.708	1.53	.826	2.07
.762	1.06	.873	1.76
.816	1.64	.942	2.29

.008	.030	.309	1.80
.007	.025	.378	1.82
.006	.026	.566	1.81

$*3*3$ Scheme, b_i's, a_2 and a_3 corrected:

.708	1.63	.826	1.07
.762	.759	.873	.895
.816	1.53	.942	1.18

.008	.010	.309	1.32
.007	.007	.378	.409
.006	.009	.566	1.71

Table B.2. Smear with Selected Uncorrected Skew Multipole Errors

The smears are given for one random seed. For comparison sake, the smear resulting from correcting all random errors is given to the left of each box. To the right of each box the results from correcting only the random b_i's and selected a_i are given. Studies on the $1*4*1$ and the the $2*2*2$ compensation schemes have similar behavior and are not presented.

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