SSC-N-494

EFFECTS OF GAPS IN THE COPPER COATING OF THE SSC BEAM TUBE

King-Yuen Ng

SSC Cental Design Group

c/o Lawrence Berkeley Laboratory 90/4040, Berkeley, CA 94720

and

Fermi National Accelerator Laboratory

Batavia, IL 60510

(March 1988)

I. INTRODUCTION

In order to minimize transverse resistive wall instability and parasitic heating, the SSC beam tube has been designed with a layer of inside copper coating. However, due to technical difficulties, there may be a one inch gap of the copper coating for every 15 feet of the beam tube. This note reports an investigation[1] of the effects of the gaps on coupling impedances, parasitic heating, and beam instabilities.

In the following, we assume the stainless steel beam tube of radius $b = 1.65$ cm has a wall thickness of $t_s = 1$ mm and a conductivity of $\sigma_s = 2.0\times10^6$ $(\Omega\text{-m})^{-1}$. The copper layer has a thickness of $t_c = 2$ mil and a conductivity[2] of $\sigma_c = 2.27\times10^9$ $(\Omega\text{-m})^{-1}$ corresponding to bulk copper with a residual resistance ratio (RRR) of 200 at 4.35 K in a magnetic flux density of 6.6 tesla. The SSC ring radius is taken as $R = 13200$ m.

II. LONGITUDINAL IMPEDANCE AND PARASITIC HEATING

At extremely low frequencies, the wall current fills up the whole thickness of the beam tube wall. Therefore, the longitudinal wall impedance $Z_\parallel$ is inversely proportional to the product of the conductivity and the thickness of the wall material.

However, since copper is nearly 1000 times more conductive than stainless steel, we can assume that, except for the gaps, all the wall current resides in the copper layer only. Therefore, the increase in longitudinal impedance due to the gaps is

$$\frac{\Delta Z_{\|}}{(Z_{\|})_{\rm wall}} \simeq \frac{1}{180}\frac{t_c\sigma_c}{t_s\sigma_s} = 32.1\% \ , \tag{2.1}$$

where the factor 1/180 represents the gap contribution of one inch per 15 ft.

At higher frequencies, the wall current penetrates only a skin depth of the wall material, the longitudinal impedance is inversely proportional to the square root of the wall material. Therefore, the increase in longitudinal impedance is

$$\frac{\Delta Z_{\|}}{(Z_{\|})_{\rm wall}} = \frac{1}{180}\left(\frac{\sigma_c}{\sigma_s}\right)^{1/2} = 18.7\% \ . \tag{2.2}$$

As a result, the bunch-mode level shiftings due to wall resistivity will be 18.7% bigger, but they pose no threat to mode-coupling instability because the longitudinal mode-coupling threshold[3] is $\mathcal{Im}(Z_{\|}/n) = 87\ \Omega$ while the estimated $|Z_{\|}/n|$ (not including the gap contribution but including the contributions of bellows, beam position monitors, etc) is only about 0.2 Ω.

Parasitic heating arises from the resistive wall of the beam tube can be approximated by assuming that the wall current flows in a skin depth of the wall material since this assumption breaks down only in a small region where the frequency is extremely low. Then the heating power P is directly proportional to the longitudinal impedance. Therefore the fractional increase in parasitic heating is

$$\frac{\Delta P}{P} = \frac{1}{180}\left(\frac{\sigma_c}{\sigma_s}\right)^{1/2} = 18.7\% \ . \tag{2.3}$$

According to the SSC Conceptual Design,[4] power lost to the resistive wall amounts to only 9.1% (7.7%) of the total parasitic heating if inner bellows (bellows with sliding contact) are assumed. Thus the gaps in the copper coating will increase the total parasitic heating by only 1.7% or 1.4% for the two types of bellows.

III. TRANSVERSE IMPEDANCE AND INSTABILITIES

Unlike the monopole case, the wall currents need not flow across the high-resistance stainless steel gap, because they can turn around before the gap as shown in Fig. 1a. This can be represented by an equivalent circuit with two resistances Z_g and two inductances L_g as depicted in Fig. 1b.

At very high frequencies, I_W passes through the stainless steel gap only. Assume that each side of the dipole wall currents flows in one quarter of the beam tube wall or a width of $\pi b/2$ of the tube circumference. Then

$$Z_g = (1+j)\frac{2g}{\pi\delta_s\sigma_s} \simeq (1+j)\frac{2g}{\pi b}\sqrt{\frac{nZ_0}{2R\sigma_s}} \ , \tag{3.1}$$

where $g = 1$ in is the gap width, $Z_0 = 377\ \Omega$ is the impedance of free space, δ_s is the stainless steel skin depth, and $f = nf_0$ is the frequency under consideration measured in terms of the revolution frequency f_0. Numerically, this gives

$$Z_g = (1+j)\sqrt{n}\,0.827 \times 10^{-4}\ \Omega \ . \tag{3.2}$$

At extremely low frequencies, since the wall currents will fill up the whole stainless steel wall thickness at the gap, Eq. (3.1) should be replaced by

$$Z_g = \frac{2g}{\pi b t_s \sigma_s} = 4.90 \times 10^{-4}\ \Omega \ . \tag{3.3}$$

When the frequency approaches zero, all the wall currents will return through the inductance L_g. This situation as shown in Fig. 2a is equivalent to the situation in Fig. 2b. Thus the total inductance for one gap is $2L_g$ because the two inductances are in series in the wall-current loop. This is in fact the inductance of a loop of size $\sim b \times g$, because, as shown in Fig. 2c, the addition of this loop to the dipole wall currents is exactly the same as wall currents returning in front of the gap. Another way to understand the introduction of this loop and the resulting contribution to the impedance is through the change in space-charge impedance due to the absence of the wall currents I_W in the gap. This alternate derivation is given in Appendix B.

To compute the inductance of the additional loop, we assume the wall current as two plate currents of height $\pi b/2$ separated by distance b. These dimensions are chosen because they produce the correct formula for the space-charge impedance. The magnetic flux density inside and perpendicular to the loop is approximately constant, and from Ampere's law it is roughly

$$B = \frac{2\mu I_W}{\pi b} \ , \tag{3.4}$$

where μ is the magnetic susceptibility of free space. The flux linking the loop is

$$\phi = \frac{2\mu g I_W}{\pi} \ . \tag{3.5}$$

[The derivation appears nonrigorous. If one wishes, however, one can consider the loop to have a size $2b \times g$ instéad or two plate currents of height $\pi b/2$ separated by distance

$2b$, and compute exactly the flux intensity in the median plane as demonstrated in Appendix B. One finds the average B to be approximately one half of that shown in Eq. (3.4) and the flux ϕ linking the circuit in the median plane very nearly the same as given by Eq. (3.5).]

From Eq. (3.5), the inductance is

$$2L_g = \frac{2\mu g}{\pi} \, . \tag{3.6}$$

Therefore the reactance of the inductance L_g is

$$Z_L = j\omega L_g = jn\frac{Z_0 g}{\pi R} = jn\,2.31 \times 10^{-4}\ \Omega \, . \tag{3.7}$$

The impedance Z_W seen by the wall current I_W is the parallel combination of Z_g and Z_L.

The transverse wall impedance at extremely low frequencies can drive a transverse couple-bunch instability. The lowest frequency of interest is $(\nu - m)$ times the revolution frequency, where m is the integer nearest to the betatron tune ν. If we take $n = |\nu - m| \sim 0.4$, $Z_L = j0.924 \times 10^{-4}\ \Omega$. Then, together with Eq. (3.3), the extra impedance seen by I_W is

$$\Delta Z_W = \begin{cases} (0.168 + j0.892) \times 10^{-4}\ \Omega & \text{for 1 gap} \\ (0.305 + j1.62)\ \Omega & \text{for whole ring} \, . \end{cases} \tag{3.8}$$

The transverse impedance for the gap resistivity is given by

$$\Delta Z_\perp = \frac{c}{\omega b^2}\left[\frac{\Delta Z_W}{2}\right] \, , \tag{3.9}$$

where c is the velocity of light and $\omega/2\pi$ is the frequency under consideration. Equation (3.9) is derived in Appendix A for reference. On the other hand, the transverse impedance for the resistive wall of a round beam tube is related to the corresponding longitudinal impedance by

$$(Z_\perp)_{\text{wall}} = \frac{2c}{\omega b^2}(Z_\parallel)_{\text{wall}} \, . \tag{3.10}$$

From the SSC Conceptual Design Report,[5] $(Z_\parallel)_{\text{wall}}$ has been estimated to be $\sim 3.5\ \Omega$ at extremely low frequencies. [Actually in that estimation, a more conservative copper conductivity of $\sigma_c = 1.8 \times 10^9\ (\Omega\text{-m})^{-1}$ has been used. With our value of σ_c, $(Z_\parallel)_{\text{wall}}$ should be smaller.] Therefore, the contribution of the gaps is

$$\frac{\Delta Z_\perp}{(Z_\perp)_{\text{wall}}} = \frac{\Delta Z_W}{4(Z_\parallel)_{\text{wall}}} = 2.17\% \, . \tag{3.11}$$

In other words, the growth rate of the transverse coupled-bunch oscillations will be increased by $\sim 2\%$ only, which should be well within the damping power of the damper to be installed.

When the frequency becomes bigger, Z_g is given by Eq. (3.2). Since

$$\left|\frac{Z_g}{Z_L}\right| = \frac{0.506}{\sqrt{n}} , \tag{3.12}$$

which is pretty small even when n is not too big, most of the wall currents will pass through the gap instead. The increase in impedance ΔZ_W encountered by each I_W is

$$\Delta Z_W = 4\Delta Z_{\parallel} , \tag{3.13}$$

where $\Delta Z_{\parallel}$ is just the extra longitudinal impedance across the gap. The above relation is true because we have assumed each wall current I_W flows along one quarter of the beam tube circumference, and $\Delta Z_{\parallel}$ *is the parallel combination of four* ΔZ_W's. Therefore, the increase in transverse impedance is

$$\frac{\Delta Z_{\perp}}{(Z_{\perp})_{\mathrm{wall}}} = \frac{\Delta Z_{\parallel}}{(Z_{\parallel})_{\mathrm{wall}}} = 18.7\% \tag{3.14}$$

as given by Eq. (2.2). Again this poses no threat to transverse mode-coupling instability, because the threshold[3] is $\mathcal{I}m(Z_{\perp}) = 250$ MΩ/m, whereas the estimated $|Z_{\perp}|$ is only about 40 MΩ/m.

In principle, there should be a capacitor in parallel with Z_g across the gap due to charge buildups at both sides of the gap. This capacitor should be small, however, because of the thin thickness of the copper layer. This capacitor will block the flow of wall currents across the gap at low frequencies and therefore will only encourage more wall currents to turn back through L_g. The effective $|\Delta Z_W|$ of Eq. (3.8) will be reduced lessening the growth of the transverse coupled-bunch oscillations. At higher frequencies, it will encourage more wall current flowing through the gap, or effectively lowering $|Z_g|$ in Eq. (3.12) and also lowering the extra transverse impedance $\Delta Z_{\perp}$ in Eq. (3.14). For the longitudinal impedance, this capacitor will lower the impedance across the gap and therefore lower also the extra parasitic heating in Eq. (2.3). In any case, all these effects should be small.

IV. CONCLUSION

From the above analysis, it appears that the possible gaps in the copper coating will not affect significantly the purpose of the introduction of the coating. For parasitic heating, the increase is only about 2% of the total. For the transverse couple-bunch instability, the increase in growth rate is only about 2% (and much smaller if $|\nu - m|$ is assumed to be smaller). This small increase in growth rate is due to the fact that this particular instability is driven by impedance at extremely low frequencies. At those frequencies, the wall currents tend to turn around in front of the gaps instead of traversing through them.

APPENDIX A

The derivation here follows closely that of Sacherer.[6] Assume that the beam tube is rectangular in cross section having a width of $2b$ and each the dipole wall currents I_W is concentrated in a strip in the tube wall on either side of the displaced beam, as shown in Fig. 4. With the beam displaced by an amount Δ in the y-direction from the beam tube axis, the beam current density is

$$J_0(x, y - \Delta e^{j\omega t}, z) \simeq J_0(x, y, z) - \frac{\partial J_0}{\partial y} \Delta e^{j\omega t} , \tag{A.1}$$

where only terms up to the dipole contribution have been kept, and $\omega/2\pi$ is the transverse oscillation frequency of the beam.

The longitudinal electric field produced by the dipole term is

$$E_z(y) = \frac{E_W y}{b} e^{j\omega t} \tag{A.2}$$

at the median plane, where E_W is the dipole electric field at the wall.

The wall currents can be found by equating the power lost per unit length by the beam due to E_z,

$$\int E \cdot J^* dx dy = -\frac{E_W \Delta}{b} \int y \frac{\partial J_0}{\partial y} dx dy = \frac{E_W \Delta}{b} I_0 \tag{A.3}$$

to the power flow into the walls, $-2 I_W E_W$, namely

$$I_W = -\frac{1}{2} \frac{\Delta}{b} I_0 , \tag{A.4}$$

where $I_0 = \int J_0(x, y, z) dx dy$ is the total beam current.

The electric field at the wall is given by

$$E_W = \frac{I_W Z_W}{\ell} , \tag{A.5}$$

where Z_W is the impedance seen by one wall current I_W for a length ℓ of the beam tube. The deflecting magnetic field at the beam is (from Faraday's law)

$$B_x = -j \frac{I_W Z_W}{\omega b \ell} e^{j\omega t} , \tag{A.6}$$

where Eqs. (A.2) and (A.5) have been used. Substituting into the definition of $Z_\perp$,

$$Z_\perp = \frac{j}{\beta I_0 \Delta} \int_\ell [E + v \times B]_\perp d\ell , \tag{A.7}$$

and using Eq. (A.4), we arrive at

$$Z_{\perp} = \frac{c}{\omega b^2}\left[\frac{Z_W}{2}\right] \tag{A.8}$$

for the length of the pipe under consideration. If we assume that I_W flows in a strip of width one quarter of the tube circumference, the longitudinal impedance of this length of the tube $Z_{\parallel}$ equals $4Z_W$. We then recover the relation between the transverse and longitudinal wall impedances for a round beam tube as given in Eq. (3.10).

APPENDIX B

If the beam has a radius a and the transverse displacement Δ is infinitesimal, we have a real dipole beam of width Δ only but separated by $2a$. The magnitude of the current I_Δ is given by

$$I_\Delta 2a = I_0\Delta \ . \tag{B.1}$$

Each I_Δ sees an impedance Z_Δ. Similar to the derivation from Eqs. (A.5) to (A.8), the transverse impedance can also be written as

$$Z_\perp = \frac{c}{\omega a^2}\left[\frac{Z_\Delta}{2}\right] \ . \tag{B.2}$$

We now compute the magnetic part of the space-charge impedance. First the self-inductance. We assume that each current I_W fills one quarter of the circumferential surface of the beam. The magnetic flux density due to I_Δ is assumed to be uniform inside the beam and is given approximately through Ampere's law by

$$B = \frac{2\mu I_\Delta}{\pi a} \ . \tag{B.3}$$

(The exact result[7] at the center of a round beam tube is one half of the above.) We next assume that this magnetic flux will link only a width a of the circuit. This is because the currents I_Δ being on the beam surface are curved so that not all the flux will be linked. In other words, we assume the distance between the two surface currents to be a. Therefore, amount of flux linked is

$$\phi = \frac{2\mu \ell I_\Delta}{\pi} \ , \tag{B.4}$$

where ℓ is the length of beam considered.

We can also take the dipole currents I_Δ seriously as two plate currents of height h separated by $2a$, the vertical magnetic flux density in the median plane is

$$B(x) = \frac{\mu I_\Delta}{\pi h}\left[\tan^{-1}\frac{h}{2(a+x)} + \tan^{-1}\frac{h}{2(a-x)}\right] \ , \tag{B.5}$$

for $-a < x < a$. With $h = \pi a/2$, it reduces to

$$B(x) = \frac{\mu I_\Delta}{\pi a}F(x) \ , \tag{B.6}$$

where the form factor is

$$F(x) = \frac{2}{\pi}\left[\tan^{-1}\frac{\pi b}{4(b+x)} + \tan^{-1}\frac{\pi b}{4(b-x)}\right] \ , \tag{B.7}$$

which varies from 0.848 at the center $x = 0$ to 1.238 at the plate current $x = \pm a$ with an average of 0.978. The flux linking the circuit in the median plane can be integrated easily to get

$$\phi = \frac{2\mu\ell I_\Delta}{\pi}\left[\frac{\pi a}{2h} + \frac{1}{4}\ln\frac{4a^2 + h^2/4}{h^2/4} - \frac{a}{h}\tan^{-1}\frac{4a^2 - h^2/4}{2ah}\right] , \tag{B.8}$$

which reduces to

$$\phi = \frac{2\mu\ell I_\Delta}{\pi} \times (0.978) \tag{B.9}$$

when $h = \pi a/2$ and agrees extremely well with the estimated value in Eq. (B.4). We use the expression in Eq. (B.4) because it produces the correct expression for space-charge impedance in below.

In any case, the self-inductance is

$$L_{\rm self} = \frac{2\mu\ell}{\pi} , \tag{B.10}$$

giving a self reactance of

$$Z_{\rm self} = j\omega L_{\rm self} , \tag{B.11}$$

which is equal to $Z_\Delta/2$ which is two Z_Δ's seen by the dipole current I_Δ in series. Therefore, using Eq. (B.2), we get

$$(Z_\perp)_{\rm self\ mag} = \frac{Z_0\ell}{2\pi a^2} . \tag{B.12}$$

Now the wall current contribution. From Eq. (3.3), the flux due to the wall current I_W linking the current loop I_Δ is

$$\phi = \frac{2\mu\ell a I_W}{\pi b} = -\frac{2\mu\ell I_\Delta}{\pi}\frac{a^2}{b^2} , \tag{B.13}$$

giving a mutual inductance of

$$M = -\frac{2\mu\ell}{\pi}\frac{a^2}{b^2} , \tag{B.14}$$

and a reactance of

$$Z_{\rm image} = j\omega M , \tag{B.15}$$

which is again equal to $Z_\Delta/2$. Therefore, the total magnetic contribution to the space-charge impedance is

$$(Z_\perp)_{\rm mag} = j\frac{Z_0\ell}{2\pi}\left[\frac{1}{a^2} - \frac{1}{b^2}\right] . \tag{B.16}$$

Note that the image contribution is in fact capacitive. Now with the presence of a high resistive gap of length g, a length g of wall currents I_W will be absent and it therefore contributes an inductive impedance of

$$Z_{\perp} = j\frac{Z_0 g}{2\pi b^2} \tag{B.17}$$

which is exactly the amount due to the additional loop in Fig. 2c computed using Eq. (3.8) by letting ΔZ_W equal $Z_L/2$ of Eq. (3.6).

REFERENCES

1. Parts of the investigation in Sections II were done in collaboration with R. Meller.
2. A.W. Chao, SSC Note SSC-N-434, 1987.
3. SSC Conceptual Design, ed. J.D. Jackson, SSC-N-110, 1986, p.181.
4. SSC Conceptual Design, ed. J.D. Jackson, SSC-N-110, 1986, p.179.
5. SSC Conceptual Design, ed. J.D. Jackson, SSC-N-110, 1886, p.169.
6. F. Sacherer, *Theoretical Aspects of the Behaviour of Beams in Accelerators and Storage Rings,* CERN 77-13, 1977, p.198.
7. K.Y. Ng, Fermilab Report UPC 149, 1980.

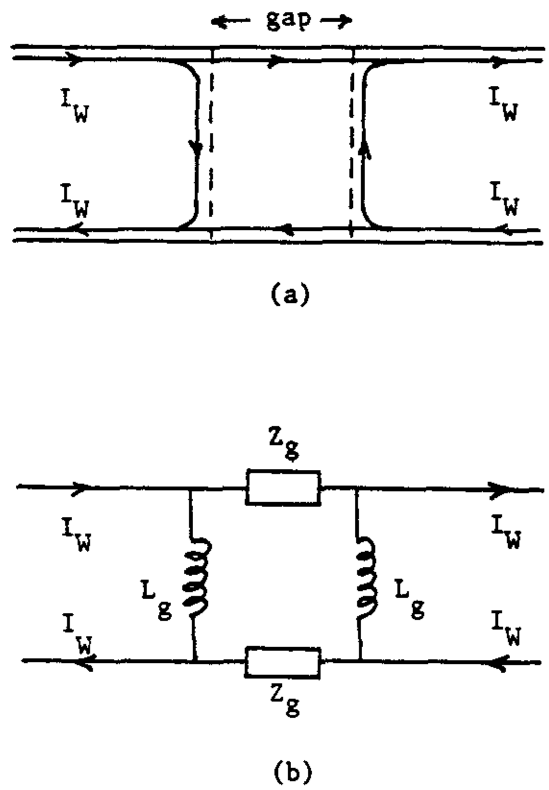

Fig. 1

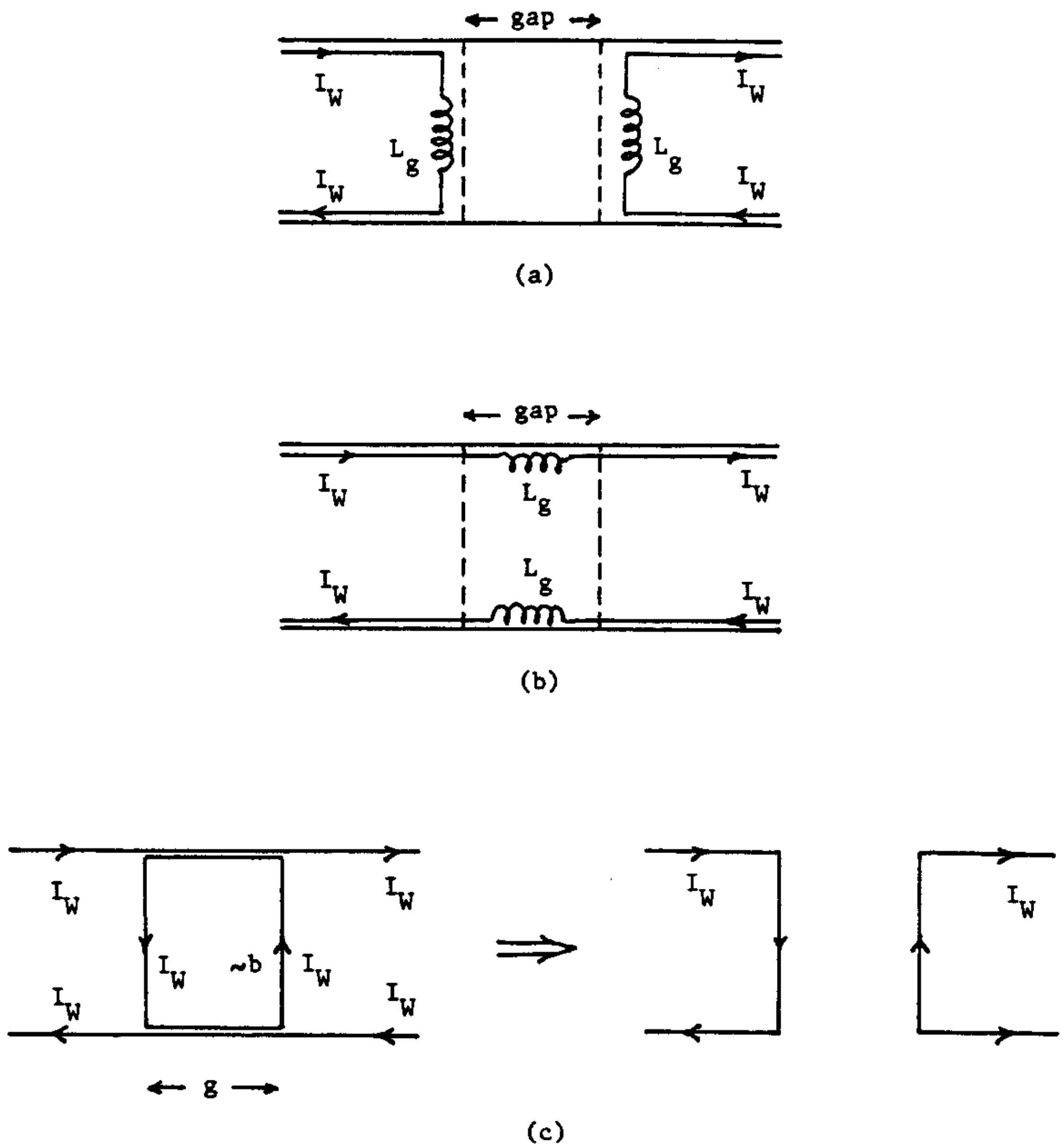

Fig. 2

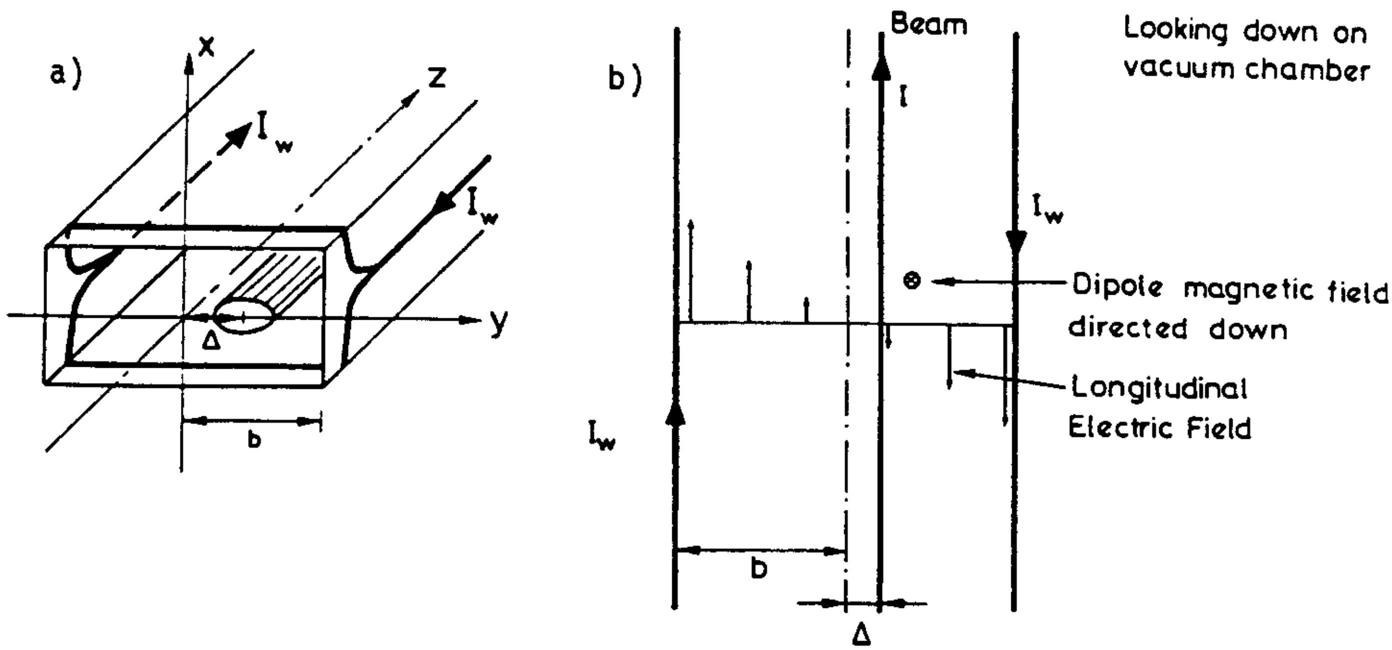
a)
x
z
I_w
I_w
y
Δ
b
b)
Beam
I
Looking down on
vacuum chamber
I_w
Dipole magnetic field
directed down
Longitudinal
Electric Field
I_w
b
Δ

Fig. 3