SSC-N-492

Comparison of Numerical and Analytical Results for the Systematic Tune Variation of Various Lumped Compensation Schemes for the SSC.

DAVID NEUFFER

AT-6, Los Alamos National Laboratory

and

RICHARD TALMAN

SSC Central Design Group

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ABSTRACT

The dependence of tune on transverse amplitude and on momentum for various lumped compensation schemes for the SSC is analysed. Only those systematic multipole elements which are due to persistent currents in the dipoles are considered. Comparisons are made between analytic and quasi-analytic calculations, using methods based on theoretical lattice functions, and results from Fourier analysis of the motion obtained numerically using the tracking program TEAPOT. In all cases studied the methods have been found to agree to an accuracy much better than is required for SSC planning.

1. Introduction.

This report has two main purposes. The first is to compare the results of two completely different calculational methods of obtaining the systematic tune variation of the SSC when lumped correctors are used to compensate for magnetic field errors. This is done on lattices which are simple enough to make the calculations and comparisons easy, with the intention of enhancing the credibility of similar calculations for more complicated configurations. The methods compared are on the on hand analytic and quasi-analytic, using methods based on theoretical lattice functions, and described by Neuffer,[1] and on the other hand TEAPOT tracking,[2] followed by Fourier analysis of the motion. In all cases studied, and after the not necessarily easy task of assuring that solutions to identical problems were being compared, the two methods have been found to agree to an accuracy much better than is required for SSC planning.

The second purpose is to compare various possible lumped compensation schemes,[3][4][5] and to gain insight into the general problem of compensation with lumped elements. Reasonably thorough results on several schemes are given, and most of them meet the SSC specifications for acceptably weak tune variation. In the present paper no attempt is made to select the best possible scheme but the fact that any scheme with a sufficient number of correctors can provide adequate compensation indicates the general validity of the lumped correction technique, without any fine sensitivity to the exact corrector locations. Choice of the best scheme will include consideration of the success with which the lumped correctors can be used for the "binning" compensation of random magnet errors as well as economic considerations. Those issues are not addressed in the present report.

2. Analysis Procedures.

In this report magnetic multipole errors in the dipoles are studied, assuming that there are no field errors in other magnetic elements and no survey errors. The systematic errors used are $b_2 = -4.7$ and $b_4 = 0.30$, coming from persistent currents at the injection energy.[6] These are fractional field errors in the usual units of parts per 10^4 at one centimeter. They are larger than the saturation multipoles though the latter may demand more powerful correctors since they appear at full field.

A lattice consisting only of 320 simple 90° FODO cells, with parameters identical to those in the regular arcs of the SSC, is assumed.

All quantitative results are contained in the tables, which have code numbers 3.1 through 6.3. In this table designation code the first number refers to the number of lumped correctors per cell; it can be regarded as a measure of the "cost" of the scheme. The second number distinguishes among different dispositions of the correctors.

The degradation of accelerator performance due to field errors is usually discussed by quantifying the tune variation and the smear. The former tends to be dominated by systematic errors and the latter by random errors. Though this separation is not perfectly clean, in the present report which is concerned only with systematic behavior, only tune variation will be studied.

The independent variables are A_x, the horizontal, and A_y, the vertical transverse amplitude and the momentum offset δ. For this report calculations are performed with these variables set to all combinations of the following extremes:

(*i*) $\delta = \pm 0.001$.

(*ii*) $A_x = 5$ mm at the point in the lattice where β_x is maximum.

(*iii*) $A_y = 5$ mm at the point in the lattice where β_y is maximum.

as well as near the origin for reference. In the tables, values of the tune difference

from the nominal small-amplitude value $Q_x^{(nom)}(0,0,0)$ at these extreme points

$$\Delta Q_x(A_x, A_y, \delta) = Q_x(A_x, A_y, \delta) - Q_x^{(nom)}(0,0,0)$$

will be presented in the following format, in units of 0.001:

$$\begin{pmatrix} \Delta Q_x(0,5,+0.001) & \Delta Q_x(5,5,+0.001) \\ \Delta Q_x(0,5,0.000) & \Delta Q_x(5,5,0.000) \\ \Delta Q_x(0,5,-0.001) & \Delta Q_x(5,5,-0.001) \\ & \\ \Delta Q_x(0,0,+0.001) & \Delta Q_x(5,0,+0.001) \\ \Delta Q_x(0,0,0.000) & \Delta Q_x(5,0,0.000) \\ \Delta Q_x(0,0,-0.001) & \Delta Q_x(5,0,-0.001) \end{pmatrix} \tag{2.1}$$

and $\Delta Q_y(A_x, A_y, \delta)$ will be exhibited similarly. This array is intended to suggest a coordinate system in which A_x points right, A_y points up, and δ points into the page. One final complication in the tables is that as many as three different determinations are written side-by-side for easy comparison. Arrows in the upper left corner distinguish the different cases.

The CDR specifies, for values of the variables in the interior of the region defined by the above extremes, that the maximum tune variation should remain in the range ± 0.005. The units have been chosen in the tables of tune variation such that 10 marks the boundary between acceptable and unacceptable performance according to this specification. That is, the difference between the largest and the smallest entry must not exceed 10. It is assumed that the only errors present are systematic (with the values given above.) These errors are large enough, if completely uncompensated, to give unacceptably large smear, but after compensation by any of the following schemes the smear is less than 1 percent and will not be exhibited.

The arrangements of lumped correctors are shown in the figures accompanying the tables. The strengths of assumed multipole errors in the dipoles and

the strengths of the lumped compensators are given in the tables at the bottom. In every case there is a further adjustment of the sextupoles to bring both the horizontal and vertical chromaticities to zero and then the analysis is completed. For the analysis using TEAPOT each boundary-defining particles is tracked to obtain 256 "time-domain" values for the transverse coordinates x and y. These are transformed by FFT into the "tune-domain" from which the tunes are determined by locating peaks. By making a parabolic fit to data in the channel with the peak and its two nearest neighbours a precision much better than the FFT tune-bin-spacing is achieved.

Analytic and "quasi-analytic" evaluations of the tune shifts have also been calculated. First-order tune shifts can be calculated by averaging the phase advance caused by the field perturbation in the Hamiltonian

$$\Delta\nu_{x,y} = \frac{1}{2\pi}\int \frac{d\phi_{x,y}}{ds} \equiv < \frac{dH}{dI_{x,y}} > \tag{2.2}$$

where $I_{x,y}$ are the x and y action variables given by $A^2_{x,y}/(2\beta_{max})$ and β is the usual lattice function. To first order in the coefficients b_n and a_n, only systematic normal multipoles $< b_n >$ contribute; the resulting expressions for the tune shifts as a function of I_x, I_y, and δ due to sextupole (b_2), octupole (b_3), and decupole (b_4) components are

$$\begin{aligned}\Delta\nu_x =< & b_2\beta_x\eta\delta + \frac{3}{4}b_3\beta_x^2 I_x - \frac{3}{2}b_3\beta_x\beta_y I_y + \frac{3}{2}b_3\beta_x\eta^2\delta^2 \\ & + 3b_4\beta_x^2\eta I_x\delta - 6b_4\beta_x\beta_y\eta I_y\delta + 2b_4\beta_x\eta^3\delta^3 >\end{aligned} \tag{2.3}$$

$$\begin{aligned}\Delta\nu_y =< & -b_2\beta_y\eta\delta + \frac{3}{4}b_3\beta_y^2 I_y - \frac{3}{2}b_3\beta_x\beta_y I_x - \frac{3}{2}b_3\beta_y\eta^2\delta^2 \\ & + 3b_4\beta_y^2\eta I_y\delta - 6b_4\beta_x\beta_y\eta I_x\delta - 2b_4\beta_y\eta^3\delta^3 >\end{aligned} \tag{2.4}$$

In the cases we consider, only first-order effects are important for octupole and decapole. This has been verified by evaluation of equation (2.2) and comparison with short-term and long-term tracking results.

First-order sextupole tune shifts are completely corrected by the lumped sextupoles, but second-order sextupole tune shifts can be quite large. From perturbation theory these second-order sextupole tune-shift terms are of the same form as the first-order octupole terms:

$$\Delta\nu_x = aI_x + bI_y + c\delta^2 \quad \text{and} \quad \Delta\nu_y = dI_y + bI_x + e\delta^2 \tag{2.5}$$

Analytical evaluation of the second-order tune shifts can be obtained from perturbation theory expressions or extrapolated from test orbit parameters, as will be explained next.

In the "DN" results presented below, tune shifts are calculated by following test orbits through a full cycle of 320 cells and adding phase advances through that cycle. The unperturbed tune/cell is displaced from 90° sufficiently to provide one additional oscillation per complete revolution; thus the $\phi_x = 90°$ resonance is avoided and an averaging over phases is achieved. Coupling terms are similarly avoided since the x and y phase advances per cell are separated.

This method provides a fast (a minute or less on a PC) and accurate tune shift determination. It is somewhat vulnerable to orbit distortion and coupling effects, but the presence of these effects is readily detected visually in the graphical display of the orbits. The results of this method, which we call "quasi-analytic", are in close agreement with the analytic formula (2.2) for all cases which have been checked. They are also in agreement with analytic results of E. Forest[4] and, as will be shown, with the TEAPOT-Fourier determinations. This latter method is capable of handling arbitrarily complicated and realistic lattices including errors and misalignments. It has possible errors due to finite statistics, resonant tune choices and unresolved coupling but the close agreement among the various methods provides evidence of the absence of these errors and of substantially linear motion.

3. Results.

The tables below are supposed to be self-explanatory, with numbers which are to be compared placed side-by-side. Analytical results are designated DN and TEAPOT results are designated RT. In many cases, results of analysing the same configuration by these two methods are shown. In other cases, other dependencies are exhibited, such as upon the number of thin elements (kicks) into which the 17m SSC dipoles are broken for the analysis. Since TEAPOT requires such a replacement, the same was done in the analytic calculation. Results below will show that adequate accuracy is obtained with 2 kicks per dipole. Other dependencies exhibited will be explained as we next comment on each table individually.

The Tables 3.1 to 3.5 give results for five different schemes having only three lumped correctors per cell. They share the inelegance that the pattern of correctors is different in the two half-cells making up a full cell.

(3.1) The DN results with 2 and 4 kicks per dipole agree rather well. In most subsequent case 2 kicks per dipole will be used. Deviation between DN and RT results never exceed 1 unit. (Remember that the numbers are in units of 0.001.)

(3.2) DN results with two different b_4 weightings for the lumped correctors. With the second weighting the scheme is brought "into specs" since the deviation between maximum entry and minimum entry does not exceed 10 (but just barely.) In the second weighting scheme, the weights are set by requiring that an integration over the cell be exact when averaged over full cells, as in the three-point Simpson's Rule. This type of weighting greatly improves b_4 correction in all cases.

(3.3) By introducing an octupole compensation element the tune variation can be reduced by another order of magnitude. Compensation using octupoles is explained by Neuffer[3] .

(3.4, 3.5) show a different configuration which can similarly be brought into specifications with the use of octupoles.

Next are listed results for schemes using four lumped correction elements per cell.

(4.1) This shows results with the so-called Simpson's Rule scheme for the locations and weightings of the lumped elements. Only b_2 errors are included. Agreement between RT and DN is excellent, with the greatest deviation being 0.3 units.

(4.2) This shows results with two slightly different locations for the lumped correctors — one with a corrector at the quad center and one with the corrector beside the quad. There is almost no detectable change. Essential to obtaining "Simpson's Rule" with only two lumped correctors in each half-cell is that the lumped corrector value which is calculated to be needed at the left end of the half-cell on the right and the one needed at the right end of the half-cell on the left can, with impunity, be combined into a single lump on one or the other side of the quadrupole. This is true in the limit of zero length elements, but needs to be supported for realistic lattice geometry. The present results support the concept.

(4.3) With b_4 only there is again excellent agreement between DN and RT.

(4.4) Dependence on number of kicks per dipole. Going from one to four makes a noticeable difference.

(4.5) With both b_2 and b_4 errors present and compensated there is excellent agreement between DN and RT. These numbers also show that two kicks per dipole is an adequately fine granularity.

(4.6) This shows various fine-tunings, including both deviation from the Simpson's Rule weighting and the inclusion of octupole compensation. Second-order b_2 correction can be greatly improved by using three independent *octupoles*, as demonstrated in this case. It can be seen that extremely tight tune variation tolerances can be met. In this table disagreements be-

tween DN and RT for $\delta \neq 0$ should be discounted as decupole corrector strengths were different in the comparison.

(4.7) Another scheme meeting the specs (barely) and showing good agreement between DN and RT. With the inclusion of octupoles much tighter tolerances could undoubtedly be met.

Finally, three schemes are listed which have six lumped correctors in each cell. Since the preceding schemes with fewer lumps have easily met the tune variation tolerances there is little motivation for studying more expensive schemes as far as systematic compensation is concerned. But, as has been mentioned previously, the important issue of compensating random errors could dictate the use of more elements.

(6.1) There is good agreement between DN and RT, and the tune variation tolerances are met with simple-minded uniform strength-weighting.

(6.2) Similar results are obtained with displaced elements.

(6.3) By judicious strength-weighting much tighter tolerances are met.

4. Conclusions.

The results show a set of benchmark cases in which different evaluation methods have calculated essentially identical results, showing that the different methods have substantial validity for tuneshift calculations. In all cases in which identical lattice configurations were compared, tune determinations agree within ± 0.001; in most cases the accuracy is several times better than this. Both of the methods "RT" and "DN" are adequate for further use in SSC design. As always, this use must be accompanied by continued vigilance against program bugs, data set errors, and unexpected lattice and orbit properties. This seemingly self-evident statement, already not superfluous for the special lattice studied, will be all the more true for more detailed lattices.

REFERENCES

1. Neuffer, D. *Lumped Correction of Systematic Multipoles in Large Synchrotrons*, SSC-132, June 1987.

2. Schachinger, L. and Talman R. *TEAPOT: A Thin Element Accelerator Program for Optics and Tracking.* **Particle Accelerators 22**, 35 (1987).

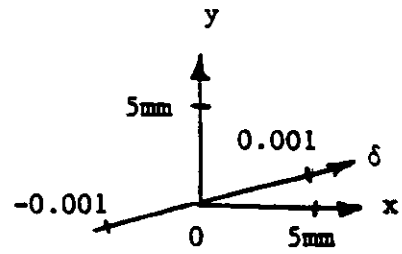
3. Neuffer, D. *Correction of Residual Second-Order Sextupole Tune Shifts by the Simpson's Rule Octupoles.* SSC-N-384, Sept. 1987.

4. Private communication with E. Forest.

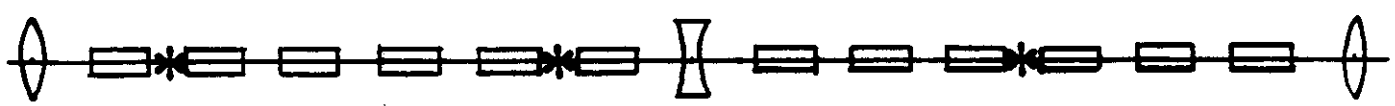
5. Talman, R. *Systematic Compensation of the SSC With Two Lumped Correctors Per Half-Cell.*, SSC-N-413, December, 1987.

6. The values listed are from CDR, p. 148. To reduce complexity the small term $b_6 = -0.07$ has been neglected. More recent calculations of M. Green, SSC-N-376, give somewhat larger values, $b_2 = -7.6, b_4 = 0.65,\ b_6 = -0.13$.

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS



Footnote										
Method		RT	DN	DN						
Kicks/dipole		2	2	4						
		↓	↓	↓						
		9.0	9.9	9.9			16.4	17.4	17.5	
	2.4	2.4	2.4			5.1	5.1	5.1		
3.9	4.0	3.9			1.3	1.0	0.8			ΔQ_x
		6.5	6.9	6.9			13.9	14.9	15.0	$^o/_{oo}$
	0.2	0.05	0.05			2.9	2.9	2.8		
2.0	0.00	1.8			-0.5	-0.9	-1.2			
		5.9	6.3	6.3			8.3	8.9	8.8	
	3.0	3.2	3.1			5.2	5.5	5.4		
2.5	2.7	2.6			4.2	4.7	4.6			ΔQ_y
		1.3	1.5	1.4			3.6	3.9	3.9	$^o/_{oo}$
	-0.1	0.05	0.05			2.0	2.3	2.3		
1.2	1.3	1.3			2.9	3.3	3.2			

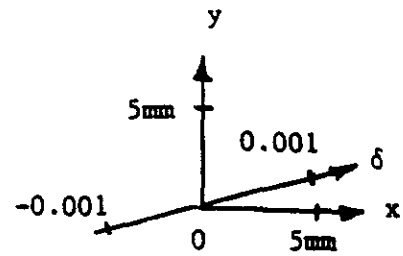


				Dipole Error
b_2	3	3	6	$\times \frac{2\pi}{320\times12} \times -4.7\times10^{-4}$
b_3				
b_4	3	3	6	$\times \frac{2\pi}{320\times12} \times 0.3\times10^{-4}$

Lattice designations: 1*4*1,3*3 cls4sl3s3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

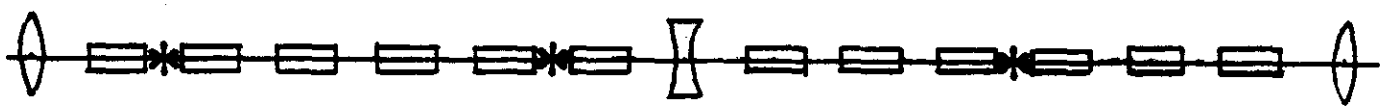
Footnote		$b_4^{(1)}$	$b_4^{(2)}$						
Method		DN	DN						
Kicks/dipole		2	2						
		↓	↓	↓					
		9.9	6.9	.			17.5	9.6	.
	2.4	2.4	.			5.1	5.1	.	
3.9	6.9	.			0.8	8.8	.		
		6.9	4.6	.			15.0	7.6	.
	0.05	0.05	.			2.8	2.8	.	
1.8	4.1	.			-1.2	6.3	.		
		6.3	4.3	.			8.8	6.4	.
	3.1	3.1	.			5.4	5.3	.	
2.6	4.5	.			4.6	7.1	.		
		1.4	1.3	.			3.9	3.4	.
	0.05	0.05	.			2.25	2.3	.	
1.3	1.4	.			3.2	3.9	.		



Different b_4 weightings

ΔQ_x $^o/_{oo}$

ΔQ_y $^o/_{oo}$



					Dipole Error
b_2	3	3	6	$\times \frac{2\pi}{320\times12}$	$\times -4.7 \times 10^{-4}$
b_3					$\times 10^{-4}$
b_4 (1)	3	3	6	$\times \frac{2\pi}{320\times12}$	$\times \quad \times 10^{-4}$
b_4 (2)	4.5	4.5	3		

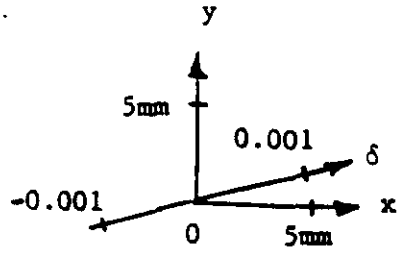
Lattice designations: 1*4*1,3*3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Footnote

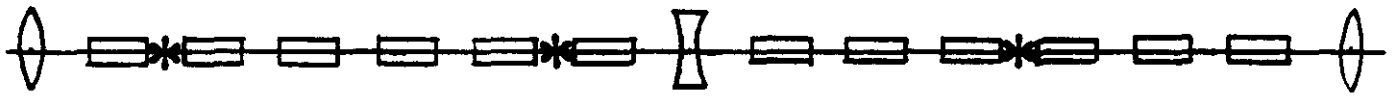
Method DN

Kicks/dipole 2



Octupole used

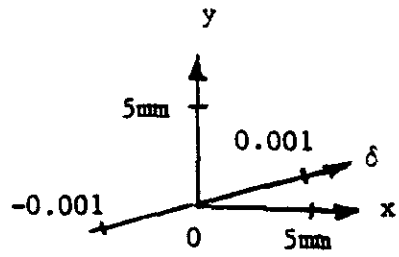
		0.34	.	.			0.41	.	.		
	-0.46	.	.			-0.61	.	.			
-0.3	.	.			-0.9	.	.				ΔQ_x
		0.86	.	.			1.05	.	.		$^o/_{oo}$
	0.00	.	.			-1.0	.	.			
0.3	.	.			-0.5	.	.				
		0.48	.	.			-0.1	.	.		
	0.92	.	.			0.45	.	.			
0.42	.	.			0.54	.	.				ΔQ_y
		-0.5	.	.			-1.09	.	.		$^o/_{oo}$
	0.04	.	.			-0.47	.	.			
-0.46	.	.			-0.83	.	.				



				Dipole Error
b_2	3	3	6	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3	0.0696	0.0620	-0.1584	$\times 10^{-4}$
b_4	4.5	4.5	3	$\times \frac{2\pi}{320\times12} \times 0.3 \times 10^{-4}$

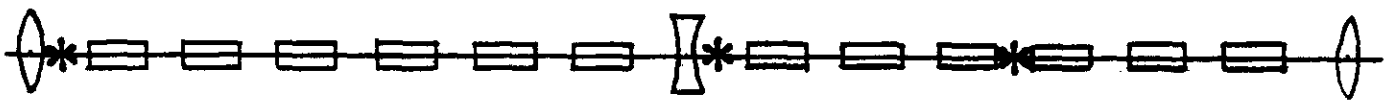
Lattice designations: 1*4*1,3*3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS



Simpson rule -- only one center lump

Footnote									
Method		RT	DN						
Kicks/dipole		2	2						
		.	16.0	.		.	19.7	.	
	7.0	7.2	.		13.2	13.3	.		
15.2	16.4	.		22.0	22.6	.			ΔQ_x
		.	8.0	.		.	12.8	.	$^o/_{oo}$
	0.2	0.12	.		6.6	6.6	.		
9.0	9.3	.		15.9	16.5	.			
		.	12.8	.		.	20.0	.	
	9.8	9.5	.		16.3	16.3	.		
13.0	12.9	.		19.0	19.1	.			ΔQ_y
		.	3.6	.		.	10.4	.	$^o/_{oo}$
	-0.1	0.15	.		6.4	6.7	.		
3.2	3.4	.		9.0	9.4	.			

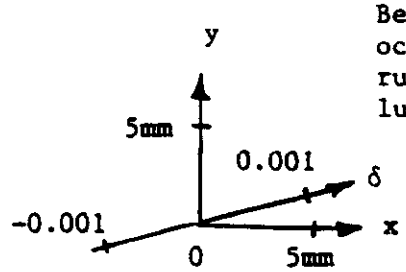


				Dipole Error
b_2	2	2	8	$\times \frac{2\pi}{320\times 12} \times -4.7 \times 10^{-4}$
b_3				$\times 10^{-4}$
b_4	2	2	8	$\times \frac{2\pi}{320\times 12} \times 0.3 \times 10^{-4}$

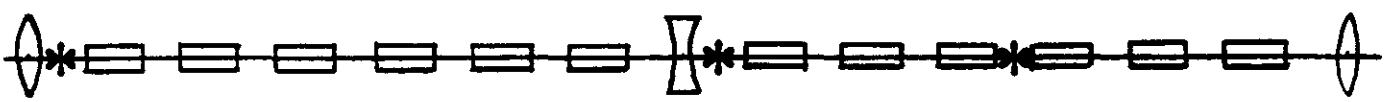
Lattice designations: *6,*3*3 as6s3s3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Best case including octupoles, Simpson rule -- one center lump



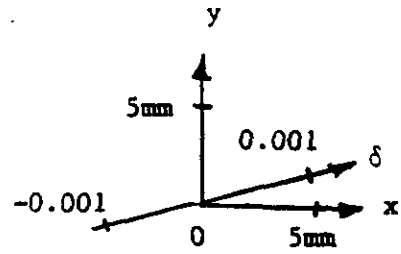
Footnote										
Method		DN								
Kicks/dipole		2								
		-0.32	.	.			-0.69	.	.	
	-0.14	.	.			-0.08	.	.		
-0.60	.	.			-0.96	.	.			ΔQ_x
		-0.33	.	.			-0.60	.	.	$^o/_{oo}$
	0.00	.	.			0.12	.	.		
-0.16	.	.			-0.28	.	.			
		0.00	.	.			-0.04	.	.	
	-0.06	.	.			-0.23	.	.		
-0.03	.	.			-0.58	.	.			ΔQ_y
		0.10	.	.			0.05	.	.	$^o/_{oo}$
	0.00	.	.			-0.17	.	.		
0.06	.	.			-0.51	.	.			



				Dipole Error
b_2	4	4	4	$\times \frac{2\pi}{320\times 12} \times -4.7 \times 10^{-4}$
b_3	-0.032	-0.036	0.088	$\times 10^{-4}$
b_4	1.80	2.02	8	$\times \frac{2\pi}{320\times 12} \times 0.3 \times 10^{-4}$

Lattice designations: *6,*3*3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS



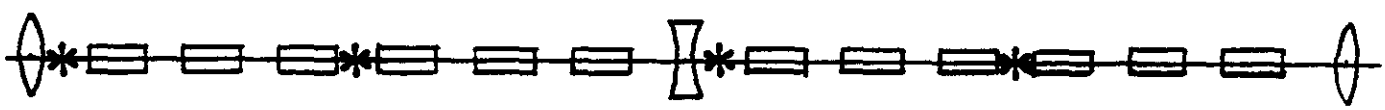
Standard Simpson -- b_2 only

Footnote									
Method		RT	DN						
Kicks/dipole		1	1						
		↓	↓	↓					
		3.1	3.1	.			4.7	4.4	.
	0.9	0.8	.			2.5	2-2	.	
3.1	3.1	.			4.7	4.5	.		
		2.4	2.2	.			3.9	3.7	.
	0.2	0.02	.			1.7	1.5	.	
2.4	2.2	.			3.9	3.7	.		

ΔQ_x $^o/_{oo}$

		2.0	2.2	.			2.8	2.9	.
	1.5	1.5	.			2.1	2.2	.	
2.0	2.2	.			2.8	2.9	.		
		0.6	0.7	.			1.3	1.4	.
	-0.1	0.02	.			0.7	0.7	.	
0.6	0.7	.			1.4	1.4	.		

ΔQ_y $^o/_{oo}$

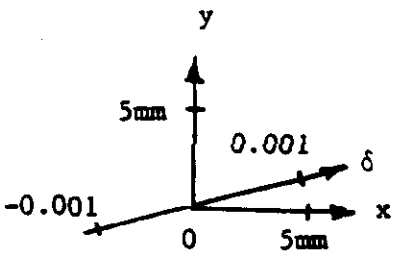


					Dipole Error
b_2	2	4	2	4	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3					$\times 10^{-4}$
b_4					$\times \frac{2\pi}{320\times12} \times \quad \times 10^{-4}$

Lattice designations: *3*3 ds3s32826

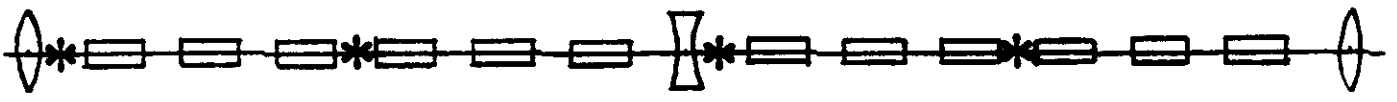
LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Footnote	F,D corr. beside quad	corr. at center of quad
Method	RT	RT
Kicks/dipole	1	1



Sensitivity to small translation of lumped corrector

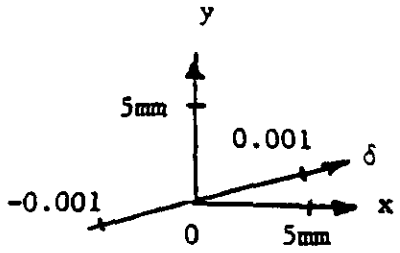
		3.1	3.1	.			4.7	4.6	.	
	0.9	0.9	.			2.5	2.5	.		
3.1	3.1	.			4.7	4.7	.			ΔQ_x
		2.4	2.4	.			3.9	3.9	.	$^o/_{oo}$
	0.2	0.2	.			1.7	1.7	.		
2.4	2.4	.			3.9	3.9	.			
		2.0	2.0	.			2.8	2.7	.	
	1.5	1.5	.			2.8	2.7	.		
1.5	1.5	.			2.1	2.0	.			ΔQ_y
		0.6	0.6	.			1.3	1.3	.	$^o/_{oo}$
	-0.1	-0.1	.			0.7	0.6	.		
0.6	0.6	.			1.4	1.3	.			



					Dipole Error
b_2	2	4	2	4	$\times \frac{2\pi}{320\times 12} \times -4.7 \times 10^{-4}$
b_3					$\times 10^{-4}$
b_4					$\times \frac{2\pi}{320\times 12} \times \quad \times 10^{-4}$

Lattice designations: *3*3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS



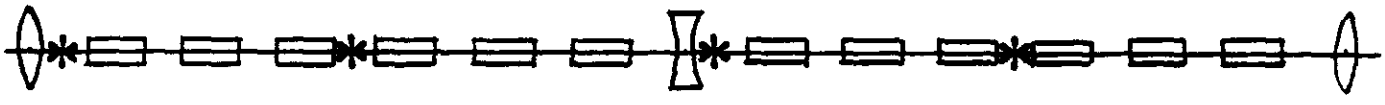
Standard Simpson -- b_4 only

Footnote									
Method		RT	DN						
Kicks/dipole		1	1						
		↓	↓	↓					
		-0.3	-0.45	.			-1.8	-2.1	.
	0.1	-0.1	.			0.1	-0.1	.	
0.4	0.2	.			1.9	1.9	.		
		-0.7	-0.9	.			-2.3	-2.5	.
	0.2	0.00	.			0.2	0.02	.	
1.0	0.84	.			2.6	2.5	.		

ΔQ_x $^o/_{oo}$

		-0.1	0.02	.			0.4	0.46	.
	-0.1	-0.01	.			-0.2	-0.1	.	
0.0	0.07	.			-0.5	-0.56	.		
		0.1	0.13	.			0.5	0.57	.
	-0.1	-0.00	.			-0.2	-0.1	.	
-0.2	-0.06	.			-0.7	-0.7	.		

ΔQ_y $^o/_{oo}$

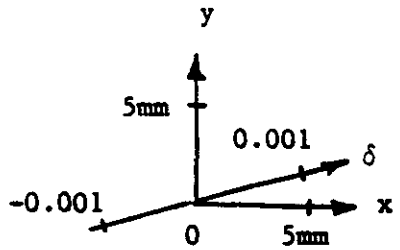


					Dipole Error
b_2				$\times \frac{2\pi}{320\times12} \times$	$\times10^{-4}$
b_3					$\times10^{-4}$
b_4	2	4	2	4	$\times \frac{2\pi}{320\times12} \times 0.3$ $\times10^{-4}$

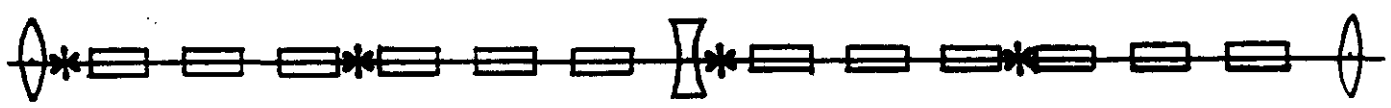
Lattice designations: *3*3 es3s32826

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Dependence on number of kicks/dipole, b_4 only



Footnote									
Method		DN	DN						
Kicks/dipole		1	4						
		↓	↓	↓					
		-0.45	-0.2	.		-2.1	-1.4	.	
	-0.1	-0.1	.		-0.1	-0.1	.		
0.2	-0.03	.			1.9	1.1	.		ΔQ_x
		-0.9	0.00	.		-2.55	-1.8	.	$^o/_{oo}$
	0.00	0.00	.		0.02	-0.02	.		
0.84	0.6	.			2.5	1.7	.		
		0.02	0.16	.		0.46	0.59	.	
	0.01	0.00	.		-0.1	-0.1	.		
0.07	-0.07	.			-0.56	-0.68	.		ΔQ_y
		0.13	0.001	.		0.57	0.55	.	$^o/_{oo}$
	-0.001	0.00	.		-0.1	-0.07	.		
-0.06	-0.06	.			-0.7	-0.68	.		

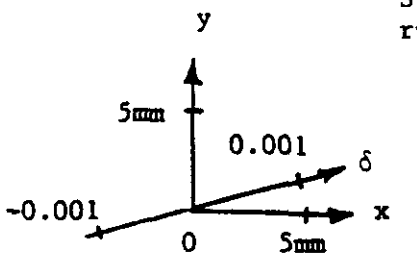


						Dipole Error
b_2					$\times \frac{2\pi}{320\times12} \times$	$\times10^{-4}$
b_3						$\times10^{-4}$
b_4	2	4	2	4	$\times \frac{2\pi}{320\times12} \times 0.3$	$\times10^{-4}$

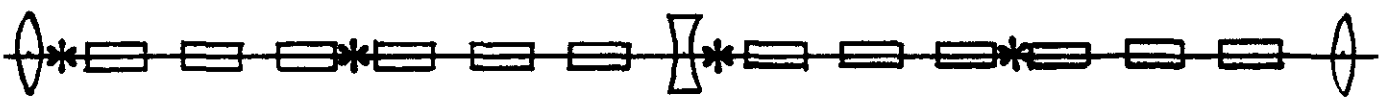
Lattice designations: *3*3

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Standard Simpson rule



Footnote										
Method		RT	DN	DN						
Kicks/dipole		2	2	4						
		↓	↓	↓						
		2.7	2.5	2.5			2.7	2.3	2.3	
	0.9	0.7	0.7			2.2	1.9	1.8		
2.9	2.8	2.7			5.2	5.1	4.9			ΔQ_x
		1.5	1.3	1.2			1.5	1.1	1.2	$^o/_{oo}$
	0.2	0.02	0.02			1.5	1.3	.		
2.8	2.6	2.5			5.1	5.0	4.7			
		1.9	2.1	2.0			3.1	3.2	3.2	
	1.4	1.4	1.3			1.9	2.0	2.0		
1.8	1.9	1.8			2.0	2.1	2.0			ΔQ_y
		0.6	0.7	0.7			1.7	1.9	1.8	$^o/_{oo}$
	-0.1	0.02	0.02			0.6	0.66	0.65		
0.4	0.5	0.5			0.6	0.7	0.6			

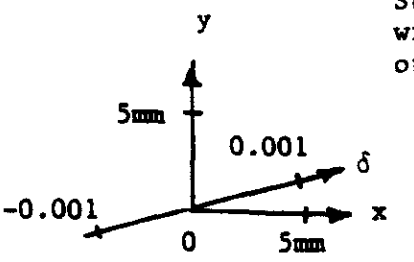


					Dipole Error
b_2	2	4	2	4	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3					$\times 10^{-4}$
b_4	2	4	2	4	$\times \frac{2\pi}{320\times12} \times 0.3 \times 10^{-4}$

Lattice designations: *3*3 is3s32828

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

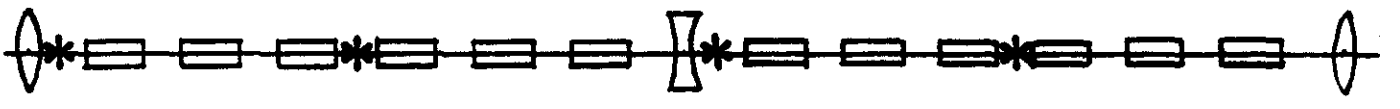
Standard Simpson -- with octupole and other tuning



Footnote	(1)	(2)	(3)
Method	RT	DN	DN
Kicks/dipole	2	2	4

		0.3	0.03	0.18			-0.8	-1.4	-0.08	
	0.0	-0.02	-0.04			0.1	-0.05	-0.11		
0.5	0.3	-0.04			1.6	1.4	-0.20			ΔQ_x
		-0.2	-0.5	0.03			-1.2	-1.9	-0.22	$^o/_{oo}$
	0.0	0.0	-0.00			0.2	-0.02	-0.07		
1.1	0.8	0.19			2.2	1.9	0.05			

		0.3	0.3	0.15			0.7	0.8	0.30	
	0.1	0.08	0.03			0.0	0.06	0.00		
0.2	0.2	0.23			-0.3	-0.4	0.00			ΔQ_y
		0.2	0.3	-0.4			0.6	0.7	0.30	$^o/_{oo}$
	0.0	0.00	0.00			-0.1	-0.03	-0.05		
0.0	0.07	0.13			-0.5	-0.5	-0.11			

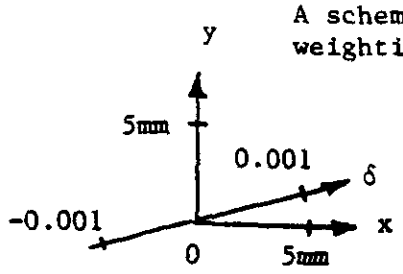


					Dipole Error
b_2	2	4	2	4	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3	-0.0122 (1,2) -0.0122 (3)	0.0156 0.308	-0.0122 -0.0122	0.0156	$\times 10^{-4}$
b_4	2 (1) 1.9 (2,3)	4 4	2 2.02	4 4	$\times \frac{2\pi}{320\times12} \times 0.3 \times 10^{-4}$

Lattice designations: *3*3 b33octm

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

A scheme with uniform weighting.

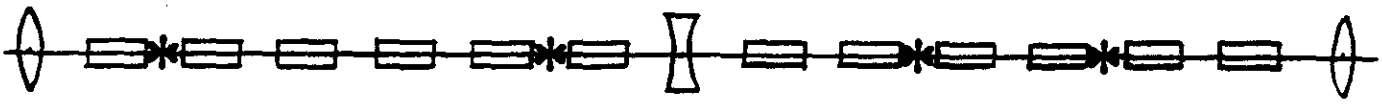


Footnote									
Method		RT	DN						
Kicks/dipole		2	2						
		↓	↓	↓					
		4.9	5.2	.			9.1	9.6	.
	1.2	1.2	.			2.9	2.8	.	
2.4	2.3	.			1.1	0.6	.		
		3.8	4.0	.			8.1	8.6	.
	0.2	0.03	.			1.8	1.7	.	
1.3	1.1	.			0.1	-0.5	.		

ΔQ_x $^o/_{oo}$

		3.0	3.2	.			4.0	4.2	.
	1.6	1.6	.			2.6	2.7	.	
1.4	1.4	.			2.3	2.5	.		
		0.6	0.7	.			1.6	1.7	.
	-0.1	0.03	.			1.0	1.1	.	
0.6	0.7	.			1.6	1.7	.		

ΔQ_y $^o/_{oo}$

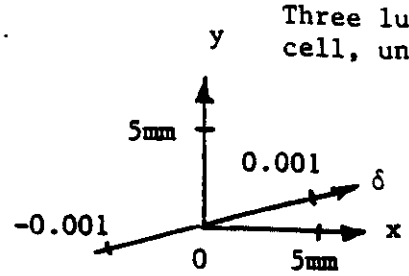


					Dipole Error
b_2	3	3	3	3	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3					$\times 10^{-4}$
b_4	3	3	3	3	$\times \frac{2\pi}{320\times12} \times 0.3 \times 10^{-4}$

Lattice designations: 1*4*1,2*2*2 a1s4s12s2s2

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Three lumps per half-cell, uniform weighting.

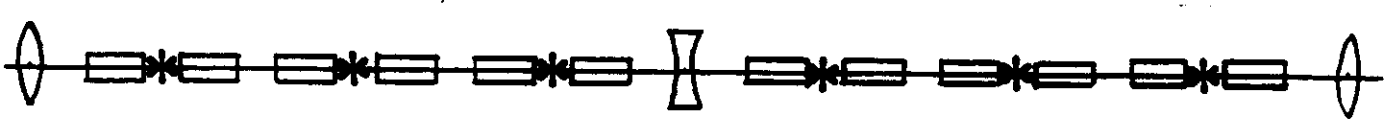


Footnote									
Method		RT	DN						
Kicks/dipole		2	2						
		2.4	2.3	.			4.8	4.7	.
	0.5	0.3	.			1.2	0.9	.	
0.8	0.4	.			-0.5	-1.0	.		
		2.0	2.0	.			4.5	4.5	.
	0.2	0.01	.			0.9	0.65	.	
0.4	0.1	.			-0.8	-1.3	.		

ΔQ_x $^o/_{oo}$

		1.4	1.3	.			1.6	1.5	.
	0.6	0.5	.			0.9	0.7	.	
0.3	0.2	.			0.6	0.5	.		
		0.2	0.2	.			0.5	0.4	.
	-0.1	0.01	.			0.2	0.25	.	
0.2	0.25	.			0.5	0.5	.		

ΔQ_y $^o/_{oo}$

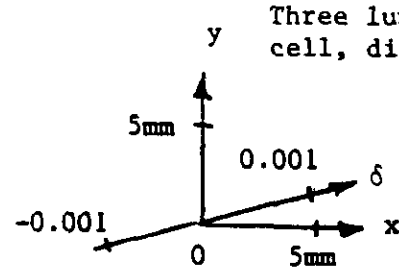


						Dipole Error
b_2	2	2	2	2	2	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3						$\times 10^{-4}$
b_4	2	2	2	2	2	$\times \frac{2\pi}{320\times12} \times 0.3 \times 10^{-4}$

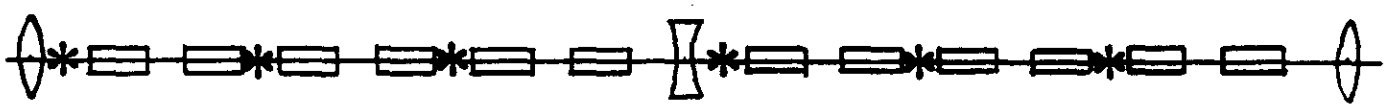
Lattice designations: 1*2*2*1 als2s2sl

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Three lumps per half-cell, displaced.



Footnote										
Method		RT	DN							
Kicks/dipole		2	2							
		↓	↓	↓						
		-0.8	-1.2	.			-4.3	-5.0	.	
	0.2	0.04	.			0.9	0.7	.		
3.0	2.9	.			7.6	7.8	.			ΔQ_x
		-0.8	-1.3	.			-4.4	-5.2	.	$^o/_{oo}$
	0.2	0.01	.			0.9	0.6	.		
3.0	2.9	.			7.6	7.8	.			
		-0.5	-0.6	.			-0.4	-0.5	.	
	0.4	0.4	.			0.4	0.4	.		
1.8	1.9	.			1.8	1.8	.			ΔQ_y
		0.2	0.2	.			0.3	0.4	.	$^o/_{oo}$
	-0.1	0.00	.			0.0	0.03	.		
0.1	0.2	.			0.1	0.1	.			

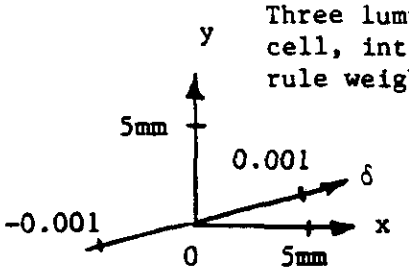


							Dipole Error
b_2	2	2	2	2	2	2	$\times \frac{2\pi}{320\times12} \times -4.7 \times 10^{-4}$
b_3							$\times 10^{-4}$
b_4	2	2	2	2	2	2	$\times \frac{2\pi}{320\times12} \times 0.3 \times 10^{-4}$

Lattice designations: *2*2*2 as2s2s2

LUMPED COMPENSATION OF SYSTEMATIC DIPOLE ERRORS: DIFFERENT TUNE DETERMINATIONS

Three lumps per half-cell, interpolation rule weighting.

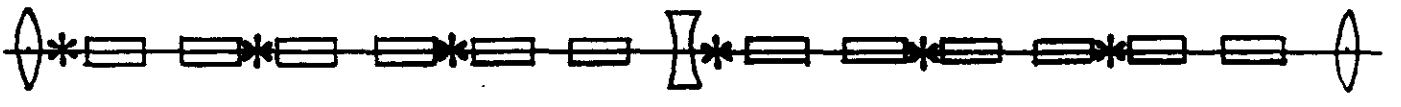


Footnote

Method DN

Kicks/dipole 4

		0.9	.	.			0.8	.	.	
	-0.2	.	.			0.7	.	.		
1.0	.	.			2.0	.	.			ΔQ_x
		0.5	.	.			0.4	.	.	$^o/_{oo}$
	0.01	.	.			0.51	.	.		
1.1	.	.			2.0	.	.			
		0.85	.	.			1.2	.	.	
	0.56	.	.			0.73	.	.		
0.76	.	.			0.70	.	.			ΔQ_y
		0.3	.	.			0.7	.	.	$^o/_{oo}$
	0.01	.	.			0.01	.	.		
0.20	.	.			0.12	.	.			



								Dipole Error
b_2	$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$\times \frac{2\pi}{320\times 12} \times -4.7$	$\times 10^{-4}$
b_3								$\times 10^{-4}$
b_4	$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$\times \frac{2\pi}{320\times 12} \times$	$\times 10^{-4}$

Lattice designations: *2*2*2