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MIITS Integrals for Copper and for Nb - 46.5 wt% Ti

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INTRODUCTION

In the adiabatic, one dimensional approximation the temperature rise of a conductor carrying a current is governed by the simple heat balance of a unit length of cross-sectional area A and density d.

$$\frac{I^2(t) \cdot \rho(T)}{A} \cdot dt = A \cdot d \cdot C(T) \cdot dT$$

Here I is the current carried which is a function of time, and d is the density, ρ the resistivity and C the heat capacity of the conductor which are in general functions of the local temperature.

This expression can be rearranged and integrated to give:

$$\int_0^t I^2 dt = A^2 \cdot d \cdot \int_{T_0}^{T} \frac{C(T)}{\rho(T)} dT$$

Here $T = T_0$ when $t = 0$. Note that thermal expansion is ignored, and the area and the density should be specified at the same temperature. The advantage to this formulation is that the left hand integral in this expression is a function of circuit parameters while the right hand integral contains all of the materials parameters of the problem. It is therefore possible using a table of the value of the right hand integral as a function of temperature to find the final temperature reached during an episode of conduction characterized by the integral on the left if the adiabatic conditions are met. In quenching magnets that operate very far from the cryostable condition, the resistive power levels are very high, and the time of decay of the current short compared to the thermal time constants of the winding. The adiabatic assumption is in these cases, therefore, a very good one.

It is usual to use the units of MIITS for the values of these integrals. MIITS are units of 10^6 Amps^2 – sec in the left hand side of the equation or units of 10^6 J/Ω in the right.

MIITS INTEGRALS

The use of these integrals is, of course, a staple of the superconducting magnet business[1] and it hardly seems that anything can be added to the subject here. However, a MIITS tabulation for copper and for niobium-titanium does not appear in the literature of the CDG[2], and it is a convenient thing to have readily available. Yet another calculation has been made, therefore, and the results appear in Tables I and II below.

Table I

MIITS Integral for Copper as a Function of Temperature for Residual Resistivity Ratios of 100, 50, and 25

$$d_{copper} \cdot \int_0^T \frac{C_{copper}}{\rho_{copper}} dT$$

Tabulated is MIITS/cm^4

T(K)	100	50	25	T	100	50	25
10	1.393	.6964	.3482	380	1585	1415	1240
20	19.49	9.911	4.999	400	1616	1445	1270
30	94.71	51.78	27.22	420	1645	1475	1299
40	219.7	134.1	76.39	440	1673	1503	1327
50	352.7	236.8	147.0	460	1700	1530	1354
60	469.4	336.2	223.8	480	1726	1556	1379
70	568.2	425.1	297.9	500	1751	1580	1404
80	653.0	503.7	366.6	550	1810	1639	1461
90	726.7	573.3	429.3	600	1863	1692	1514
100	791.7	635.4	486.3	650	1913	1742	1563
120	902.0	742.0	586.1	700	1960	1788	1609
140	993.9	831.5	671.2	750	2003	1831	1652
160	1073	909.0	745.6	800	2044	1872	1693
180	1143	977.4	811.7	850	2083	1911	1732
200	1205	1039	871.1	900	2120	1948	1768
220	1261	1094	925.0	950	2156	1983	1804
240	1312	1145	974.5	1000	2190	2018	1838
260	1359	1191	1020	1050	2223	2051	1870
280	1403	1235	1063	1100	2255	2082	1902
300	1444	1275	1102	1150	2286	2113	1932
320	1482	1313	1140	1200	2315	2142	1962
340	1518	1349	1175	1250	2344	2171	1990
360	1552	1383	1208	1300*	2372	2199	2018

*Copper melting point is 1357.6 K.

Table II

MIITS Integral for NbTi as a Function of Temperature
for Copper Residual Resistivity Ratios of 100, 50, and 25

$$d_{NbTi} \cdot \int_0^T \frac{C_{NbTi}}{\rho_{copper}} dT$$

Tabulated is MIITS/cm^4

T(K)	100	50	25	T	100	50	25
10	1.393	.6964	.3482	380	1585	1415	1240
10	6.363	3.182	1.591	380	1744	1473	1228
20	58.23	29.56	14.89	400	1768	1497	1251
30	211.6	114.7	60.04	420	1791	1520	1274
40	423.4	253.9	142.9	440	1812	1541	1295
50	611.8	399.0	242.6	460	1833	1562	1316
60	757.7	523.1	338.5	480	1854	1582	1336
70	869.8	624.0	422.5	500	1873	1602	1355
80	958.6	706.3	494.4	550	1918	1647	1400
90	1031	774.9	556.1	600	1960	1689	1441
100	1092	833.5	609.9	650	1999	1727	1479
120	1192	929.5	699.8	700	2035	1763	1515
140	1271	1007	773.1	750	2069	1797	1549
160	1337	1071	835.3	800	2101	1829	1581
180	1394	1127	889.3	850	2132	1859	1611
200	1444	1177	937.3	900	2161	1888	1639
220	1489	1221	980.4	950	2188	1916	1667
240	1530	1261	1020	1000	2214	1942	1693
260	1567	1298	1056	1050	2240	1967	1718
280	1602	1332	1089	1100	2264	1991	1742
300	1634	1364	1120	1150	2287	2014	1765
320	1664	1394	1150	1200	2309	2037	1787
340	1692	1422	1177	1250	2331	2058	1808
360	1719	1448	1203	1300	2352	2079	1829

The total MIITS at any particular temperature and resistivity ratio value for a composite conductor is given by the sum of the tabulated difference value of the integral for copper and the tabulated difference value of the integral for NbTi divided by the copper-to-superconductor ratio all multiplied by the square of the copper area. As an example take the SSC dipole inner conductor. This has an area of copper of 0.06612 sq-cm and a copper-to-superconductor ratio of 1.3 : 1. In a magnetic field of 6.6T the resistivity ratio of the copper is about 25, and on quench the conductor warms beyond the residual region before the field decays significantly. The total MIITS needed to warm this conductor from the operating temperature to 500 K is:

$$\text{MIITS} = (.06612)^2 \cdot (1404 + 1355/1.3) = 10.69$$

The values of the integrals at 4.35 K are negligible in this case. This value is in good agreement with Figure 5.5-5 of the Conceptual Design Report which shows under the same conditions that between 10.5 and 11 MIITS gives a maximum temperature of 500K. Measurements of current as a function of time made during the quench testing of several dipoles give MIITS values of from 6 to 9 depending on conductor and on current.

Take as a second example the copper diode bypass bus in the dipole. This has an area of .188 sq-in or 1.213 sq-cm and is shielded from the magnetic field. The resistivity ratio, therefore, could probably be taken as 100. On the quench of the magnet this bus must carry the decaying current of the ring as the magnet system is ramped down. The time constant of this decay is nominally 20 seconds, but if one of the dump resistors fails, the time will be about 26 seconds. In this case the MIITS are:

$$(6600)^2 \cdot \int_0^{\infty} e^{-2t/26}\,dt = 566 \times 10^6 \text{ amp}^2 - \text{sec} = 566 \text{ MIITS}$$

This is 385 MIITS/cm^4 and from Table I it can be seen that this current pulse in the conductor should produce a temperature rise to about 53 K. The cross-section chosen for this conductor is clearly ample and could be reduced by a factor of 1.4 if the room were needed for something else.

It is important to assess the limit of error of the figures in these tables. In the case of the integral for copper there is a large body of accurate specific heat and electrical conductivity data, and it is likely that the tabulated values are correct to a few percent. A significant source of error in the use of the table will lie, therefore, in uncertainties in the resistivity ratio and in the cross-sectional area. In the case of the NbTi integral, the main uncertainty lies in the knowledge of the heat capacity. As will be described in the following paragraphs, the Debye formula with a correction on the order of 10% was used to represent the heat capacity. This will only be correct if the alloy system is a simple one. However, the heat capacity at the 46.5 wt% Ti point is not a strong function of composition and there seem to be no structural change at low temperatures. It is quite likely that the result is within 10% of the true curve.

PREPARATION OF THE TABLES

Copper Resistivity

The copper electrical resistivity can be represented by the following semi-empirical expression to an accuracy of better than a percent over the temperature range 0 to 1000 K.

$$\rho(T, RRR) = \frac{1.545}{RRR} + \left(\frac{2.32547 \times 10^9}{T^5} + \frac{9.57137 \times 10^5}{T^3} + \frac{1.62735 \times 10^2}{T} \right)^{-1}$$

Here the units are micro-ohm – cm. The parenthesis on the right is an approximation to the Grüneisen integral formula for the phonon scattering resistivity.

Copper Heat Capacity

The copper heat capacity is known very accurately from measurements. For the purposes here, therefore, 53 values were taken from the literature and fit with cubic splines. Below 300 K the NBS smoothed values[3] were used. From 300 to 1300 K values were taken from the compilation noted below[4].

NbTi Specific Heat

Measurements of the specific heat of niobium-titanium over the range of temperatures needed here are not available in the literature. Values of Debye temperature and electronic specific coefficient from low temperature specific heat measurements have been compiled by Collings[5], and the properties listed below for Nb - 46.5 wt% Ti come from this source.

Table III

Some Properties of Nb - 46.5 wt% Ti

Debye θ	223 K
Electronic Specific Heat Coefficient γ	1.61×10^{-4} J/g-K^2
Mol Weight	64.66
Density	6.63 g/cu-cm
at% Nb	37.23
at% Ti	62.77

From this information the specific heat of NbTi at low temperatures in the normal conducting state is given by:

$$Cp\ (T < 10\ K) = (1.61 \times 10^{-4} \cdot T + 2.711 \times 10^{-6} \cdot T^3)\ J/g\text{-}K$$

This form agrees reasonably well with the measurements of Zbasnik[6] made on Nb - 48wt% Ti.

An approximate expression for the heat capacity in the superconducting state at 6 T is:

$$Cp\,(6T, T < Tc) = 1.228 \times 10^{-5} \cdot T^3 \;\; J/g\text{-}K$$

The specific heat at higher temperatures can be calculated from the Debye theory. The accuracy of this procedure can be judged by comparing the Debye specific heat for pure niobium with measured values. Pure niobium is similar to NbTi in having a low Debye temperature and a high electronic heat coefficient. Again from Collings[7] the parameters for pure niobium are θ = 273 K and $\gamma = .008$ J/mol-K^2. If the ad hoc correction factor $(1 - .00013 \cdot T)$ is applied, the Debye specific heat with the electronic contribution agree with measured values[8] to within a few percent. This is in general the level of the reported error of the measurements, so the agreement is good.

Table IV gives values for the specific heat of NbTi calculated from the Debye function with the electronic component and with the same correction factor mentioned above.

Table IV
Heat Capacity of Nb – 46.5 wt% Ti
as a Function of Temperature
Cp in J/g-K

T	Cp	T	Cp
10	.00432	360	.416
20	.0246	380	.419
30	.0685	400	.421
40	.124	420	.424
50	.176	440	.426
60	.219	460	.428
70	.253	480	.430
80	.279	500	.432
90	.300	550	.437
100	.317	600	.442
120	.341	650	.447
140	.357	700	.451
160	.369	750	.456
180	.378	800	.460
200	.385	850	.464
220	.391	900	.468
240	.396	950	.471
260	.400	1000	.475
280	.404	1050	.478
300	.407	1200	.488
320	.410	1250	.491
340	.413	1300	.494

At 1000 K, a temperature considerably above the Debye temperatures of both niobium and titanium, it can be expected that the heat capacity of the alloy can be found from a weighted average of the molar heat capacities of the constituents (Kopp-Neumann rule). The heat capacity derived in this way is 0.481 J/g-K which can be seen to be in good agreement with the value in the table. This confirms the use of the correction factor and suggests that the values derived here are reliable.

It should be remarked that the difference of Cp and Cv for this system can be estimated from the elastic properties and the thermal expansion and is on the order of only a few percent at 1000 K. In addition, at high temperature the difference is of the same form as the correction factor, so an appropriate allowance has been made.

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[1]For a discussion, a graph of values, and references to original work see: M. N. Wilson, Superconducting Magnets, Oxford University Press (1983), Chapter 9

[2]A graph of estimated values for superconducting composite is presented in: J. D. Jackson, "Scaling of Quench Behavior to Higher Temperature -- Impact of higher Temperatures on Magnet Design Parameters", SSC – N – 379 (9/87)

[3]"A Compendium of the Properties of Materials at Low Temperatures", Victor J. Johnson, General Editor, WADD Technical Report 60-56 (1960)

[4]"Specific Heat – Metallic Elements and Alloys", Vol. 4; Y. S Touloukian, Editor; Plenum Press, New York (1970)

[5]Collings, E. W., Applied Superconductivity, Metallurgy, and Physics of Titanium Alloys, Volume I, Plenum Press, New York (1986). See Table 8-4

[6]J. Zbasnik, Private Communication

[7]Ibid. note 5. See Table 8-3

[8]Ibid. note 4