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OPERATING PARAMETERS FOR THE 90° SSC LATTICE

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INTRODUCTION

The purpose of this report is to illustrate and document the determination of the operating parameters of the current SSC lattice (90° per cell, 114.25 meter half-cell length, circumference 83.631 km, SSC-146).

PROCEDURE

From the report SSC-146 ("The 90° (September 1987) SSC Lattice," A. A. Garren and D. E. Johnson, September 1987), we extract the following lattice parameters (for the case of collision optics):

C_0	=	83.631 km	circumference
ν_x, ν_y	=	95.285, 95.265	betatron tunes
$L_{1/2}$	=	114.250 m	half-cell length
B_0	=	6.613449 T	dipole field at 20 TeV/c
ρ	=	10.08744 km	bend radius
α	=	1.563×10^{-4}	compaction factor
$\xi_{x,y}$	=	–218.55, –218.55	natural chromaticity
β^*	=	0.50 m	beta function at high-luminosity IP
β^*	=	10.0 m	beta function at med-luminosity IP
L_{CI}	=	2×79.5 m	close-interaction distance (at low β^*)
L_{CI}	=	2×166 m	close-interaction distance (at med β^*)
$\left(\frac{B''L}{B\rho}\right)_{SF}$	=	0.00999210	chromaticity sextupole strength
$\left(\frac{B''L}{B\rho}\right)_{SD}$	=	–0.02001794	chromaticity sextupole strength

From the CDR (SSC-SR-2020) we take the following parameters:

proton momentum, max	P	=	20.000 TeV/c
proton kinetic energy, max	E	=	19.99906 TeV
proton momentum, injection	P	=	1.000 TeV/c
proton kinetic energy, injection	E	=	0.99906 TeV
average luminosity ($\beta^* = 0.5$ m)	$\bar{L}$	=	1.0×10^{33} cm^{-2} s^{-1}
normalized rms emittance (x&y)	ϵ_N	=	1.0×10^{-6} rad-m
longitudinal rms emittance	ϵ_L	=	0.035 eV-s at 1 TeV
		=	0.233 eV-s at 20 TeV
nominal crossing angle	α_c	=	75×10^{-6} rad
rf voltage, peak	V	=	20 MV
accelerating period	T_a	=	1,000 s

From the rf report SSC-N-333 ("Choice of the Accelerating Frequency for the 90-Degree Lattice and . . ." J. M. Peterson, May 1987), we take the following parameters:

bunch spacing	S_B	=	5.07778 m
beam filling factor	FF	=	0.93843
accelerating frequency	f_{rf}	=	354.2405 MHz
accelerating harmonic	h	=	98,820

From the CDR we take the following relationships: The average luminosity per crossing if the beam gaps are uncorrelated

$$\bar{L}\,(\mathrm{cm}^{-2}\,\mathrm{s}^{-1}) = \frac{N_B^2\, \beta c\, R_L\, (FF)^2}{4\pi\, S_B\, (\sigma^*)^2\, 10^4}\ , \tag{1}$$

where N_B is the number of protons per bunch,

βc = proton velocity

σ^* = rms beam radius at the IP

$$= \left(\frac{\varepsilon_N \beta^*}{\beta\gamma}\right)^{1/2} \tag{2}$$

R_L = luminosity reduction factor

$$= \left[1 + \left(\frac{\alpha_c \sigma_z}{2\sigma^*}\right)^2\right]^{-1/2} \tag{3}$$

where σ_z is the rms bunch length.

The average number of reactions per bunch crossing:

$$<n> = \frac{N_B^2 R_L \Sigma_{\text{inel}}}{4\pi (\sigma^*)^2} , \tag{4}$$

where Σ_{inel} is the inelastic p-p cross-section at 20 TeV, and is estimated to be 90×10^{-27} cm^2.

$$L_1 = L <n> \exp(-<n>) \qquad \text{1-event luminosity} \tag{5}$$

$$N_T = C_0 N_B \frac{FF}{S_B} \qquad \text{total no. protons/ring} \tag{6}$$

$$P = \frac{377 e^2 c^2 \gamma^4}{3\rho C_0} N_T \qquad \text{synch radiation power/ring,} \tag{7}$$

Beam-beam tune shifts:

$$\text{(head-on)} \qquad \Delta\nu_{HO} = \frac{N_B r_p}{4\pi \varepsilon_N} \left(\frac{2R_L^2}{1 + R_L}\right) \text{ in crossing plane} \tag{8}$$

$$= \frac{N_B r_p}{4\pi \varepsilon_N} \left(\frac{2R_L}{1 + R_L}\right) \text{ in non-crossing plane,}$$

$$\text{(long-range)} \qquad \Delta\nu_{LR} = \frac{N_B r_p n_{LR}}{2\pi \gamma\beta^* \alpha_c^2} \tag{9}$$

where r_p is the classical proton radius (1.5347×10^{-18} m), n_{LR} is the number of long-range beam-beam interactions per crossing

$$n_{LR} = \frac{2L_{CI}}{S_B} . \tag{11}$$

Finally, from SSC-M-156 ("RF and Longitudinal-Phase-Space Parameters in the SSC," J. M. Peterson, April 1986) we use

$$\sigma_z = \left(\frac{\epsilon_L c^2}{f_0}\right)^{1/2} \left(\frac{\alpha}{2\pi h\, EeV \cos\phi_s}\right)^{1/4}$$

$$\frac{\sigma_E}{E} = \left(\frac{\beta f_0 \epsilon_L}{E}\right)^{1/2} \left(\frac{2\pi h\, eV \cos\phi_s}{\alpha E}\right)^{1/4} , \tag{12}$$

where f_0 is the revolution frequency $\beta c / C_0$, h is the rf harmonic number, V the peak rf voltage, and ϕ_s the synchronous phase.

From these input data and parameter relationships we derive the following parameters of interest:

max. no. of bunches	$n_{B,\max}$	=	$2\pi R/S_B$ = 16,470
no. of bunches used	n_B	=	$n_{B,\max} \times FF$ = 15,456
rms bunch length	$\sigma_{z(\mathrm{inj})}$	=	5.580 cm
	σ_z (20 TeV)	=	6.808 cm
rms beam radius at IP, 20 TeV	σ^* (β^* = 0.5 m)	=	4.843×10^{-6} m
	σ^* (β^* = 10 m)	=	21.660×10^{-6} m
luminosity reduction factor	R_L (β^* = 0.5 m)	=	0.8846
	R_L (β^* = 10 m)	=	0.9931
protons per bunch	N_B	=	8.006×10^9
protons per ring	N_T	=	1.237×10^{14}
peak current	$\hat{I}_{\mathrm{inj}}$	=	2.749 A
	$\hat{I}_{20\,\mathrm{TeV}}$	=	2.253 A
average current	I_{avg}	=	71.07 mA
beam energy	U_B	=	$N_T eE$ = 396.50×10^6 j
reactions per crossing, average	$\langle n \rangle$ (β^* = 0.5 m)	=	1.731
	$\langle n \rangle$ (β^* = 10 m)	=	0.097
average reaction power	P_R (β^* = 0.5 m)	=	576.8 W
	P_R (β^* = 10 m)	=	32.38 W
proton loss rate from collisions	$\dot{N}_T$	=	$1.901 \times 10^8\ \mathrm{s}^{-1}$ per ring
partial luminosity lifetime	$N_T/\dot{N}_T$	=	180.8 hr
synch rad power (H + V dipoles)	P_{TOT}	=	8.870 kW
	$\Delta E_{\mathrm{synch}}$	=	124.82×10^3 eV/turn
	U_{crit}	=	284.18 eV
synch. rad. time constant	$\tau_{x,y}$, τ_E	=	24.838, 12.415 hr
head-on beam-beam tune shifts:			
crossing plane (β^* = 0.5 m)	$\Delta\upsilon_{HO}$	=	0.812×10^{-3}
non-crossing plane (β^* = 0.5 m)		=	0.918×10^{-3}
crossing plane (β^* = 10 m)		=	0.968×10^{-3}
non-crossing plane (β^* = 10 m)		=	0.974×10^{-3}

long-range beam-beam tune shifts:				
($\beta^* = 0.5$ m)	$\Delta\upsilon_{LR}$	=	2.044×10^{-3}	
($\beta^* = 10$ m)		=	2.133×10^{-4}	
space charge tune shifts, inj	$\Delta\upsilon_x$	=	-3.1×10^{-4}	(SSC-N-403)
	$\Delta\upsilon_y$	=	-4.3×10^{-4}	(SSC-N-403)
rms energy spread, inj	σ_E / E	=	1.88×10^{-4}	
rms energy spread, 20 TeV		=	5.13×10^{-5}	
synchrotron tune, inj	υ_s	=	7.0118×10^{-3}	
synchrotron frequency, inj	$\upsilon_s f_0$	=	25.135 Hz	
synchrotron tune, 20 TeV	υ_s	=	1.5679×10^{-3}	
synchrotron frequency, 20 TeV	$\upsilon_s f_0$	=	5.6204 Hz	
energy gain per turn (during ramp)	ΔE	=	5.300 MeV/turn	
bend angle between adjacent IR's	ϕ_{IR}	=	81.983×10^{-3} rad (4.6973°)	
tangential offset between IR's	$\frac{1}{2}\phi_{IR} \times 2285$ m	=	93.6657 m	