NET TRANSVERSE MAGNETIC LOAD ON AN SSC DIPOLE COIL

R. I. Schermer
SSC Central Design Group
January 1988

If the collared coil in an SSC dipole is centered within the iron yoke there is zero net transverse force on the coil, but the situation is one of unstable static equilibrium. That is, if the coil is displaced laterally it will experience a force directed so as to increase the displacement. It has been postulated that this force might be the major factor in determining the frictional force acting along the collar-yoke interface.

The harmonics generated by a displaced coil have been calculated[1] and the forces could have been computed at the same time, but were not. Rather than attempt to resurrect this computer model I have chosen to estimate the forces from the calculated harmonics by a method that appears to be sufficiently precise. Because the method involves a lot of hand-waving and only a little actual calculation, I will present the arguments and result first and save mathematical detail for the Appendix to this note.

Consider a set of coils, with inside radius r_i ,shaped to produce a perfect dipole field. If these coils are placed coaxially inside a perfect cylinder of iron with radius R and constant permeability, the resultant field inside the coils is still a perfect dipole. If the coils are now displaced transversely toward the iron cylinder by a distance δr, however, the field inside the coil will be found to have harmonics. Where do these harmonics come from?

The field anywhere inside the iron, $r<R$, is a superposition of the fields from the coils and the field due to the iron. Because the latter may be perfectly represented by the field due to image currents (located at $r>R$), the field at $r<R$ is the vector sum of the fields due to two current distributions, with the iron ignored. I will refer to these as the direct field and the image field. If the coils are centered in the iron, the image field will be a perfect dipole. If the coil is displaced transversely, however, the image currents become asymmetric and generate harmonics. Within the coil bore, $r< r_i$, the image field is the *only* source of harmonics, because the direct field from a displaced coil is still perfectly uniform. Therefore, by computing harmonics in the region $r< r_i$, Morgan has specifically computed only the harmonics of the image field. Further, these harmonics are valid over the entire region $r<R$ and not just within $r< r_i$. It is shown theoretically in the next section that only quadrupole is generated, to lowest order in δr ,for the simplest case of a thin coil carrying a current distribution that varies as $\cos\theta$. Even for a real coil cross section, Morgan's computations[1] verify that quadrupole is by far the largest contribution.

I now show that Morgan's computed harmonics are sufficient to compute the net force on the coil. I write the force on any one conductor as the sum of three terms:

(coil current) **X** (direct field)

+ (coil current) **X** (image dipole field)

+ (coil current) **X** (image quadrupole field)

These terms are to be summed over all the conductors. The sum of the direct field terms is certainly zero, as a coil will exert no net force on itself in the absence of iron. Also, the uniform dipole component of the image field will exert no net force on the symmetric coil. Thus, the net force is due completely to the quadrupole component of the image field.

The argument is exact, so far, but the actual computation involves an appproximation. I represent the coil as a sheet of current of radius ρ, distributed azimuthally as $\cos\theta$, in which case the force per unit length is given by Eqn. (A13)

$$f= \pi \, NI \, B_2(\rho)$$

Morgan's calculations show that a transverse displacement of 1 mil produces a quadrupole harmonic slightly less than 0.4 unit. This translates to 2.6×10^{-4} T at 1.0 cm, or 7.8×10^{-4} T at an average coil radius of 3.0 cm. With N=(16+20)=36 and I=6500A, the force can be written in the following equivalent forms:

570 N/m per mil of displacement

3.2 Lb/in " " " "

2200 Lb on a long coil, per mil of displacement.

REFERENCES

1. G.Morgan, private communication of calculation performed 1/16/85.

2. K.Halbach, "Fields and First Order Perturbation Effects in Two-Dimensional Conductor Dominated Magnets". Nucl. Inst. and Meth. 78, 185, 1970.

I follow a notation and phase convention due to Halbach[2] , in which a field along the positive x-axis has zero phase. By contrast, multipole measurements on dipoles asssign zero phase to a field along the positive y-axis. Further, I use a multipole index n=1 for dipole, n=2 for quadrupole, etc, rather than the index (n-1) common in dipole work. Calculations are performed in the complex plane and complex quantities will be denoted in boldface, e.g. **B**, **B***.

Consider a current element dI located at **z**=x+jy and directed out of the plane. The element is inside the bore of a steel ring with infinite permeability and radius R ; $|\mathbf{z}| < R$. The field at location $\mathbf{z}_0$ is given by

$$d\mathbf{B}^* = \frac{\mu_0 j\, dI}{2\pi} \frac{1}{\left(\mathbf{z}-\mathbf{z}_0\right)} + \frac{\mu_0 j\, dI}{2\pi} \frac{1}{\left(\frac{R^2}{\mathbf{z}^*} - \mathbf{z}_0\right)} \tag{A1}$$

where $\mathbf{B}^* = B_x - jB_y$. The first term on the right hand side is the direct field and the second term is the image field. Suppose the coil is a thin, cylindrical sheet of current of radius ρ and thickness δρ ,coaxial with R and with current distribution

$$dI = J_0 \operatorname{Cos}\theta\, \rho\, d\theta\, \delta\rho$$

where J_0 is the current density at θ=0. As an illustrative calculation, I will show that the direct field in the coil bore is a perfect dipole. Expand the direct field in a power series in $(\mathbf{z}_0/\mathbf{z})$

$$d\mathbf{B}^*_{dir} = \frac{\mu_0 j\, dI}{2\pi \mathbf{z}} \sum_{n=1}^{\infty} \left(\frac{\mathbf{z}_0}{\mathbf{z}}\right)^{n-1} = \sum_{n=1}^{\infty} d\mathbf{B}^*_{n,dir} \tag{A2}$$

and integrate over the current distribution. Set $\mathbf{z} = \rho\, e^{j\theta}$ and obtain

$$\mathbf{B}^*_{n,dir} = \frac{\mu_0 j\, J_0\, \delta\rho}{2\pi} \left(\frac{\mathbf{z}_0}{\rho}\right)^{n-1} \int_0^{2\pi} \operatorname{Cos}\theta\, e^{-jn\theta}\, d\theta \tag{A3}$$

The integral is a delta-function

$$\int_0^{2\pi} \mathrm{Cos}\theta\, e^{-jn\theta}\, d\theta \begin{array}{ll} = \pi & \text{if } n=1 \\ = 0 & \text{otherwise} \end{array} \qquad \text{(A4)}$$

It is more convenient to express the result in terms of current rather than current density by using the relation

$$\int_0^{2\pi} J_0 \,\mathrm{Cos}\theta\, d\theta\, \rho\, \delta\rho \;=\; J_0\, \rho\, \delta\rho \;=\mathrm{NI} \qquad \text{(A5)}$$

where NI is the number of ampere-turns per pole of the magnet. Inserting (A4) and (A5) into (A3) I obtain

$$\mathbf{B}^*_{1,\mathrm{dir}} = j\frac{\mu_0\, \mathrm{NI}}{2\rho} \;\; ; \text{ or } B_x = 0\,,\; B_y = -\frac{\mu_0\, \mathrm{NI}}{2\rho} \qquad \text{(A6)}$$

As noted in the main text of this Note, the direct field creates no net force on the coils and I will now treat only the image field. Again the calculation proceeds by performing a power series expansion in (A1) but, because the image currents are located at $|\mathbf{z}| > R$, the expansion converges over the entire region $|\mathbf{z}| < R$. The analog of (A2) is

$$d\mathbf{B}^*_{\mathrm{image}} = \frac{\mu_0 j\, dI\, \mathbf{z}^*}{2\pi R^2} \sum_{n=1}^{\infty} \left(\frac{\mathbf{z}_0\, \mathbf{z}^*}{R^2}\right)^{n-1} = \sum_{n=1}^{\infty} d\mathbf{B}^*_{n,\mathrm{image}} \qquad \text{(A7)}$$

This time we are interested in the field from a displaced coil $\mathbf{z}^* = (\delta\mathbf{z} + \rho\, e^{j\theta})^*$,and the analog of (A3) becomes

$$\mathbf{B}^*_{n,\mathrm{image}} = \frac{\mu_0 j\, J_0\, \delta\rho}{2\pi} \left(\frac{\rho}{R}\right)^{n+1} \left(\frac{\mathbf{z}_0}{R}\right)^{n-1} \left[\int_0^{2\pi} \mathrm{Cos}\theta\, e^{-jn\theta}\, d\theta \;+\; \frac{n\, \delta\mathbf{z}^*}{\rho} \int_0^{2\pi} \mathrm{Cos}\theta\, e^{-j(n-1)\theta}\, d\theta \right] \qquad \text{(A8)}$$

Where I have retained only terms linear in $\delta\mathbf{z}^*$ .The first integral on the right side vanishes unless n=1 and the second integral vanishes unless n=2.

Inserting (A5) into (A8) produces two non-zero terms

$$\mathbf{B}^*_{1,im} = j\frac{\mu_0 NI}{2\rho}\left(\frac{\rho}{R}\right)^2 \tag{A9}$$

and

$$\mathbf{B}^*_{2,im} = j\frac{\mu_0 NI}{\rho}\left(\frac{\rho}{R}\right)^3\left(\frac{\delta z^*}{\rho}\right)\left(\frac{\mathbf{z_0}}{R}\right) \tag{A10a}$$

which can be written as

$$\mathbf{B}^*_{2,im} = \mathbf{b}_{2,im}\left(\frac{\mathbf{z_0}}{R}\right) \tag{A10b}$$

Eqn.(A9) gives the dipole part of the image field. The total dipole field in the coil bore is given by adding (A6) and (A9). Eqns.(A10) give the quadrupole part of the image field.

With all currents directed perpendicular to the x-y plane, the force on a conductor element can be written

$$\mathbf{dF^*} = -j\, dI\, \mathbf{B^*} \tag{A11}$$

As argued in the main text, only the image field can contribute to the force. Integrating (A11) over the current distribution yields

$$\mathbf{F^*} = -j\int_0^{\delta\rho}\int_0^{2\pi}\left[\mathbf{B}^*_{1,im} + \mathbf{B}^*_{2,im}\right] J_0\, \mathrm{Cos}\theta\, \rho\, d\rho\, d\theta \tag{A12}$$

The dipole term integrates to zero, because $\mathbf{B}^*_{1,im}$ does not depend on θ. For the quadrupole term insert (A10b) into (A12) and set $\mathbf{z_0} = (\delta\mathbf{z} + \rho e^{j\theta})$ to integrate over a displaced coil. Using (A4) and (A5), the final result is

$$\mathbf{F^*} = -j\pi NI\, \mathbf{b}_{2,im}\left(\frac{\rho}{R}\right) = -j\pi NI\, \mathbf{b}_2(\rho) \tag{A13}$$

where $\mathbf{b}_2(\rho)$ is the quadrupole field at the coil radius. By substituting for $\mathbf{b}_2$ from (A10a) it may be verified that the net force is in the same direction as $\delta\mathbf{z}$.