SSC DIPOLE MAGNETS: ASYMMETRIES INVOLVED IN THE ANALYSIS OF MANUFACTURING ERRORS*

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INTRODUCTION.

As an aid to the diagnosis of the ills of a dipole magnet, as an aid to the establishment of manufacturing tolerances, and for predicting field aberrations it is useful to examine the various kinds of asymmetry that can occur as a result of manufacturing errors and their effects on field aberrations.

DEFINITIONS AND CONVENTIONS.

For a dipole array of conductors we define positive position radii and angles, currents, and perturbations of these quantities, as shown in Fig. 1.

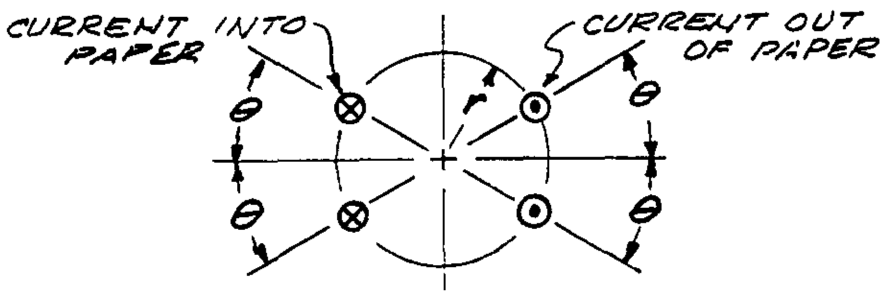

Figure 1. Positive sign convention for radius, angle, current, and perturbations of those quantities.

We define "multipoles" as the coefficients a-sub-n and b-sub-n in the following equations.

$$B^* = B_x - iB_y = \sum_{n=1}^{\infty} C_n (z/\rho)^{n-1}$$

where $C_n = B_1(a_n + ib_n)$, $z = x+iy$, or

$$B_{\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right|} = B_1 \sum_{n=1}^{\infty} (r/\rho)^{n-1} \left[a_n \left|\begin{matrix}\cos(n-1)\theta \\ -\sin(n-1)\theta\end{matrix}\right| - b_n \left|\begin{matrix}\sin(n-1)\theta \\ \cos(n-1)\theta\end{matrix}\right| \right]$$

x,y or r,theta are the coordinates of a point in the aperture at which the field components are B-sub-x and B-sub-y, rho is

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an arbitrary reference radius, usually the radius of the good-field region, B-sub-1 is the dipole component of the field, and n is the multipole order.

The multipole a-sub-n is called a "skew" multipole, b-sub-n a "regular" multipole.

Note that the definition of n is different from the one often used. Here n is the number of pole pairs: n=1 for dipole, n=2 for quadrupole, etc.

A non-uniform perturbation in current cannot occur in normal magnet operation. However since positive perturbations in radius, angle, and current in any quadrant all have the same effect on the multipoles relative to those produced by a positive perturbation in the first quadrant, as we will see later, the concept of current perturbations is a useful one. We use the following symbols to represent perturbations of either of the three.

Positive perturbations   Negative perturbations

Figure 2. Symbols used for designating perturbation in radius, angle, and current relative to those in the first quadrant.

FORMULAS FOR MULTIPOLES

The following formulas give the multipoles produced by a single conductor situated at r,theta and having a current I.

$$\begin{vmatrix} a_n \\ b_n \end{vmatrix} = \frac{\mu_0}{2\pi}\,\frac{I}{r}\left(\frac{\rho}{r}\right)^{n-1}\left[1+\left(\frac{r}{r_{Fe}}\right)^{2n}\right]\begin{vmatrix} \sin n\theta \\ \cos n\theta \end{vmatrix} \Big/ B_1$$

From these formulas, for conductors arranged as shown in Fig. 1 the effect of a conductor in each quadrant, and perturbations in its position and current, can be obtained. These are shown in the following paragraph.

EFFECT OF PERTURBATIONS IN EACH QUADRANT

The multipoles produced by the conductor in each quadrant relative to those of the corresponding conductor in the first quadrant, and their derivatives, are shown in Table 1.

Table 1. Multipoles produced by conductor in each quadrant relative to those in the first quadrant

The following applies to the multipoles and to their partial derivatives with respect to radius, angle, and current.

Quadrant	a		b	
	n odd	n even	n odd	n even
1	+1	+1	+1	+1
2	-1	+1	+1	-1
3	+1	-1	+1	-1
4	-1	-1	+1	+1

If we add the columns in Table 2 we find that all are zero except b-odd. A symmetrical dipole can produce only b-odd multipoles, and symmetrical perturbations do not break the symmetry.

SOME SIMPLE FORMS OF ASSYMMETRY.

Let us examine some simple forms of asymmetric perturbations. This is most easily done by constructing tables similar to Table 2 but having the numbers in each row multiplied by +1, zero, or -1 according to the perturbation in each quadrant.

First we consider three kinds of left-right (LR) asymmetry, the third of which is the sum of the first two. See Table 2.

Table 2. Simple forms of left-right asymmetry.

	Quadrant	Perturbation	a-odd	a-even	b-odd	b-even
	1	+1	+1	+1	+1	+1
	2	0	0	0	0	0
	3	0	0	0	0	0
	4	+1	-1	-1	+1	+1
	Total		0	0	+2	+2
	1	0	0	0	0	0
	2	-1	+1	-1	-1	+1
	3	-1	-1	+1	-1	+1
	4	0	0	0	0	0
	Total		0	0	-2	+2
	Total of above two		0	0	0	+4

Next we do the same for simple top-bottom (TB) asymmetry. See

Table 3.

Table 3. Simple forms of top-bottom asymmetry.

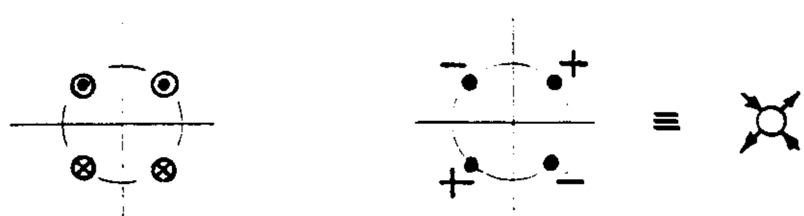
	Quadrant	Perturbation	a-odd	a-even	b-odd	b-even
	1	+1	+1	+1	+1	+1
	2	+1	-1	+1	+1	-1
	3	0	0	0	0	0
	4	0	0	0	0	0
	Total		0	+2	+2	0
	1	0	0	0	0	0
	2	0	0	0	0	0
	3	-1	-1	+1	-1	+1
	4	-1	+1	+1	-1	-1
	Total		0	+2	-2	0
	Total of above two		0	+4	0	0

It would be tempting at this point to draw conclusions regarding the effects of LR or TB asymmetry on the multipoles, but we don't yet have enough information to do that correctly.

ASYMMETRIES THAT PRODUCE PURE MULTIPOLES.

Next we turn things around and examine the effect on the asymmetry of adding perturbations that produce only a single, pure multipole.

a-odd perturbation. Adding a perturbation of the a-odd multipole to a dipole is like adding a skew dipole currents to the regular dipole currents. (Adding a skew component of any odd order produces the same effect on symmetry, but it is simpler to just look at the lowest-order one.) The effect is illustrated in Fig. 3.



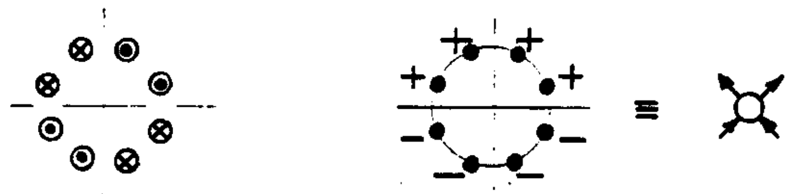
Currents

Signs of currents according to convention of Fig. 1

Figure 3. Added skew-dipole component.

We see that this has both LR and TB asymmetry.

a-even perturbation. To illustrate this we add a skew quadrupole currents to the dipole currents. See Fig. 4.



Currents

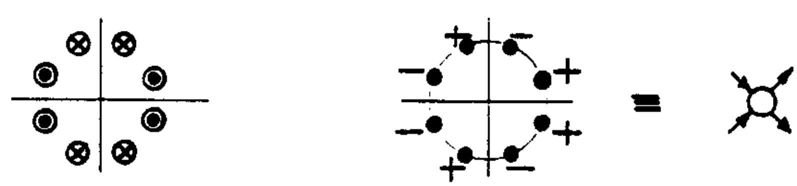
Signs of currents according to convention of Fig. 1

Figure 4. Added skew-quadrupole component.

This produces only TB asymmetry.

b-odd perturbation. This is like simply adding a regular dipole perturbation to a regular dipole; it produces neither LR nor TB asymmetry.

b-even perturbation. This is illustrated by adding regular quadrupole currents to the dipole. See Fig. 5.



Currents

Signs of currents according to convention of Fig. 1

Figure 5. Added regular-quadrupole component.

Only LR asymmetry is produced.

OTHER KINDS OF ASYMMETRY.

So far we have examined only a few of the possible kinds of asymmetry involving correlated perturbations in the various quadrants. If we allow the perturbation in each quadrant to be either +1, zero, or -1 then there are 3 to the fourth power or 81 combinations. One has zeros in all four quadrants, so we eliminate it. Of the 80 remaining, half are simply the negatives of the others, so 40 remain. Fifteen involve either a +1 or a -1 in three quadrants, which seems unlikely to occur, leaving 25. Five of these having both +1's and -1's have unequal numbers of them; again, unlikely. The remaining 20 are listed in Table.

The effects of the perturbations having these kinds of asymmetry on the multipoles, obtained by a procedure similar to that used in constructing Tables 2 and 3, are also shown in Table 5. Finally, the right two columns indicate whether there is LR or TB asymmetry.

EFFECT OF LR OR TB ASYMMETRY.

Having examined all valid forms of asymmetry, we can now generalize on the effect of LR or TB asymmetry on the multipoles.

a-odd might be generated if there is both LR and TB asymmetry.

a-even might be generated if there is TB asymmetry.

b-odd requires no asymmetry, but either LR or TB asymmetry might generate b-odd.

b-even might be generated if there is LR asymmetry.

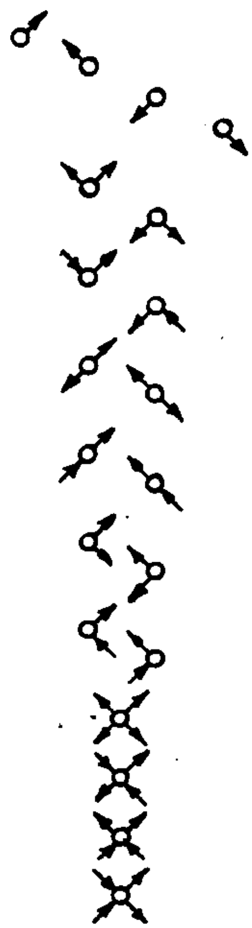
I don't find this sort of thing particularly useful. There is no information regarding signs, there are too many "mights", and simply LR or TB is inadequate to describe a kind of asymmetry.

CONCLUSIONS.

Table 5 shows all forms of asymmetry that are needed for manufacturing error analysis. For each, the multipoles, including their signs, are presented. It remains to calculate the multipoles produced by specific kinds of manufacturing errors, having these kinds of asymmetry, for a particular dipole magnet design

Table 5*. Asymmetrical correlated error modes and the multipoles produced by them.

Multipole factors listed are factors by which multipoles produced by a perturbation in the first quadrant are to be multiplied in order to obtain the multipoles for the entire magnet having the perturbations indicated.



CODE	POSITION ERRORS QUADRANT				MULTIPOLE FACTORS				ASYMMETRY	
					a		b		LR	TB
	1	2	3	4	odd	even	odd	even		
1A	+	0	0	0	1	1	1	1	*	*
1B	0	+	0	0	-1	1	1	-1	*	*
1C	0	0	+	0	1	-1	1	-1	*	*
1D	0	0	0	+	-1	-1	1	1	*	*
2A	+	+	0	0	0	2	2	0		*
2B	0	0	+	+	0	-2	2	0		*
3A	+	-	0	0	2	0	0	2	*	*
3B	0	0	+	-	2	0	0	-2	*	*
4A	+	0	+	0	2	0	2	0	*	*
4B	0	+	0	+	-2	0	2	0	*	*
5A	+	0	-	0	0	2	0	2	*	*
5B	0	+	0	-	0	2	0	-2	*	*
6A	+	0	0	+	0	0	2	2	*	
6B	0	+	+	0	0	0	2	-2	*	
7A	+	0	0	-	2	2	0	0	*	*
7B	0	+	-	0	-2	2	0	0	*	*
8	+	+	+	+	0	0	4	0		
9	+	-	+	-	4	0	0	0	*	*
10	+	+	-	-	0	4	0	0		*
11	+	-	-	+	0	0	0	4	*	

* Perhaps this should really be called "Figle 5". A figle is a combination of a figure and a table. I'll bet you didn't know that.