EFFECTS OF BEAM-TUBE PERMEABILITY ON THE MAGNETIC FIELD UNIFORMITY

King-Yuen Ng

(SSC CDG, Lawrence Berkeley Laboratory 90/4040, Berkeley, CA 94720 and Fermi National Accelerator Laboratory, Batavia, IL 60510)

and

Jack Peterson

(SSC CDG, Lawrence Berkeley Laboratory 90/4040, Berkeley, CA 94720)

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The beam tube of the SCC dipoles has been designed as a circular cylinder with 3.45 cm O.D., with 1-mm wall thickness, made of Nitronic-40, and coated on the inside with about a 2-mil layer of copper. Recently there has been a consideration that the flange welding between Nitronic-40 materials might be brittle. One possible solution is to use stainless steel SS 304N type as either the flange or the tube material or both. If SS 304N is to be considered as the tube material, one concern is whether its higher susceptibility χ ($\chi = \mu - 1$, where μ is the permeability) would cause undesirable beam dynamics effects. This note is to study this susceptibility effect. We conclude that the previous specification of χ, although still applies to the collars, can be relaxed for the beam tube. This allows SS 304N to be considered as tube material.

The Nitronic-40 χ is in the range 0.0022 (J. D. Jackson, SSC-N-193) to 0.0025 (R. Gupta, SSC-MD-116), and for SS 304N χ is about 0.008 (Gupta) or higher.

Jackson gives the multipole strengths a'_n, b'_n produced in an applied field $B^{(0)}(r,\varphi)$ by perturbation due to a region whose susceptibility distribution is given by $\chi(r,\varphi)$, which is nonvanishing only in the volume of the bore tube

$$b'_n = \frac{n+1}{2\pi B_0}\int_0^{R_0} dr \int_0^{2\pi} d\varphi\, \chi(r,\varphi) B_r^{(0)} \left(\frac{1}{r^{n+1}} + \frac{r^{n+1}}{R_0^{2n+2}}\right) \sin[(n+1)\varphi]$$

$$- B_\varphi^{(0)} \left(\frac{1}{r^{n+1}} - \frac{r^{n+1}}{R_0^{2n+2}}\right) \cos[(n+1)\varphi]\ , \qquad (1)$$

$$a'_n = \frac{n+1}{2\pi B_0}\int_0^{R_0} dr \int_0^{2\pi} d\varphi\, \chi(r,\varphi) B_r^{(0)} \left(\frac{1}{r^{n+1}} + \frac{r^{n+1}}{R_0^{2n+2}}\right)\cos[(n+1)\varphi]$$
$$+ B_\varphi^{(0)}\left(\frac{1}{r^{n+1}} - \frac{r^{n+1}}{R_0^{2n+2}}\right)\sin[(n+1)\varphi]\ , \qquad (2)$$

For $r > R_0$, the radius of the circular iron yoke, χ is assumed to be infinite. If the beam tube is round and uniform, then the interior multipoles are just those of the applied, unperturbed field $B^{(0)}$, excepting that the magnitudes are changed slightly. However, if the beam tube is not uniform in thickness or in susceptibility, or if the tube is out of round, then there are in general feed-down and feed-up effects in the multipole strengths.

Here we consider the effects of an elliptical distortion of the beam tube, whose inner radius we describe as $r_1 = a + \Delta\cos 2\varphi$, and the outer radius $r_2 = b + \Delta\cos 2\varphi$. There is no variation of χ within the wall of the beam tube.

Jackson's equations (1) and (2) then lead to the perturbed inner field (whose coefficients are a'_n, b'_n) in terms of the applied, unperturbed field (described by a_m, b_m)

$$\left\{\begin{matrix} b'_n/b_m \\ a'_n/a_m \end{matrix}\right\} = \frac{\chi(b^{2n+2}-a^{2n+2})}{2R_0^{2n+2}}\delta_{n,m} + \frac{(n+1)\chi\Delta}{2R_0^{2n+2}}\left(b^{m+n+1} - a^{m+n+1}\right)(\delta_{n,m+2}+\delta_{n,m-2})$$
$$\pm\frac{\chi\Delta(b-a)}{2ab}\delta_{n0}\delta_{m0}\ , \qquad (3)$$

$$\left\{\begin{matrix} b'_n/a_m \\ a'_n/b_m \end{matrix}\right\} = 0\ , \qquad (4)$$

where $\delta_{i,j} = 0$ $(i \neq j)$ and $= 1$ $(i = j)$. If the beam tube were skewed, the (b'_n/a_m) and (a'_n/b_m) terms would not in general be zero.

We evaluated the case of a beam tube of SS 304N with

$$\begin{aligned} \chi &= 0.008 \\ a &= 1.625 \text{ cm} \\ b &= 1.775 \text{ cm} \\ \Delta &= 1 \text{ mm} \\ R_0 &= 5.57 \text{ cm}\ . \end{aligned}$$

Here, the thickness of the beam tube has been taken as 1.5 mm, which is necessary to withstand the stress in the case of a quench. The resulting perturbed dipole coefficients are

$$\left\{\begin{matrix} b'_0/b_m \\ a'_0/a_m \end{matrix}\right\} = (6.6 \pm 2.1)\times 10^{-5}\delta_{m,0} + 1.7\times 10^{-5}(\text{cm}^2)\delta_{m,2}\ ,$$

and the perturbed sextupole coefficients are

$$\left\{ \begin{array}{c} b_2'/b_m \\ a_2'/a_m \end{array} \right\} = 5.2 \times 10^{-8}(\mathrm{cm}^{-2})\delta_{m,0} + 1.7 \times 10^{-6}\delta_{m,2} + 1.0 \times 10^{-6}(\mathrm{cm}^2)\delta_{m,4} \ .$$

Or

$$\begin{aligned} b_0' &= 0.87 + 1.7 \times 10^{-5} b_2 \ , \\ a_0' &= 4.5 \times 10^{-5} a_0 + 1.7 \times 10^{-5} a_2 \ , \\ b_2' &= 5.2 \times 10^{-4} + 1.7 \times 10^{-6} b_2 + 1.0 \times 10^{-6} b_4 \ , \\ a_2' &= 5.2 \times 10^{-8} a_0 + 1.7 \times 10^{-6} a_2 + 1.0 \times 10^{-6} a_4 \ , \end{aligned} \tag{5}$$

In Eq. (5), we used $b_0 = 1$. The other unperturbed and perturbed multipole coefficients a_n, b_n, a_n', and b_n' are in units of $10^{-4}\mathrm{cm}^{-n}$.

Thus a b_0' of 0.87 unit is the only perturbation that might be observable. About 24% is due to the 1-mm elliptical deformation. For b_2', there is about 0.0017 unit due to the deformation. It is possible that SS 304N may have a χ bigger than 0.008, but the effects to the multipoles are still small.

We see that the perturbed b_0' has a positive sign, implying that the dipole strength increases with the beam tube in place, contrary to the usual argument that the interior of the beam tube should be shielded from the exterior field. But the interior field can also increase because the beam tube contributes magnetic reluctance. Since the susceptibility is infinite in the iron yoke, a line integral of the magnetic field H from the bottom to the top of the yoke interior should be constant as a result of Ampere Law. When a beam tube of positive χ is introduced, H inside the tube wall should decrease and therefore the average H in the along the empty space part of the path should increase. If the beam tube has a very small radius r_2, it is possible that H or B increases outside the beam tube but decreases inside so that the net line integral of H in the empty space part of the path is still bigger. However, if we take the extreme case of letting the beam tube have the same radius as R_0, the average H or B inside the beam tube will definitely become bigger. Thus, the sign of the perturbed b_0' depends on the size of the beam tube. It was shown by Steve Marks (LBL Mechanical Engineering Note M6531) that when

$$\left(\frac{r_2}{R_0}\right)^2 > \frac{\mu - 1}{\mu + 1} \ , \tag{6}$$

b_0' is positive and when

$$\left(\frac{r_2}{R_0}\right)^2 < \frac{\mu - 1}{\mu + 1} \ , \tag{7}$$

b_0' is negative. Equations (3) and (4) do not exhibit this property because they are accurate up to first order in χ only. Since $\chi = 0.008$ in our case, Eq. (6) will always be satisfied with a beam tube of reasonable radius, or the perturbed b_0' is always positive.