December 9, 1987

MEMORANDUM

To: V. N. Karpenko

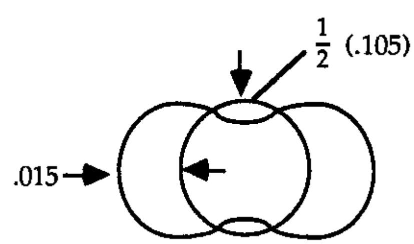
From: M. Zaslawsky

Subject: Distortion of the Beam Tube — Beam Tube on Magnet DD00010 was Distorted during Initial Assembly

---

The calculations on pages 1–4 illustrate the fact that the beam tube cannot be considered a ring, unsupported, with a load applied at two points (90° and 270°) and obtain the deflections observed. It only takes 125 lbs. at A for pt. B to go plastic. To achieve .015" def. on a side requires 16,000 lbs., and this would either exceed σ_{ult} at B or the tube would collapse. Therefore, the model must be as shown on page 4, where the beam tube is supported. In practice, it is supported by the G-10 bumpers.

Given the following observation of the beam tube



The beam tube has the following dimensions

$$t = .035$$

$$r_i = .645$$

$$r_0 = .68$$

$$r_m = .662$$

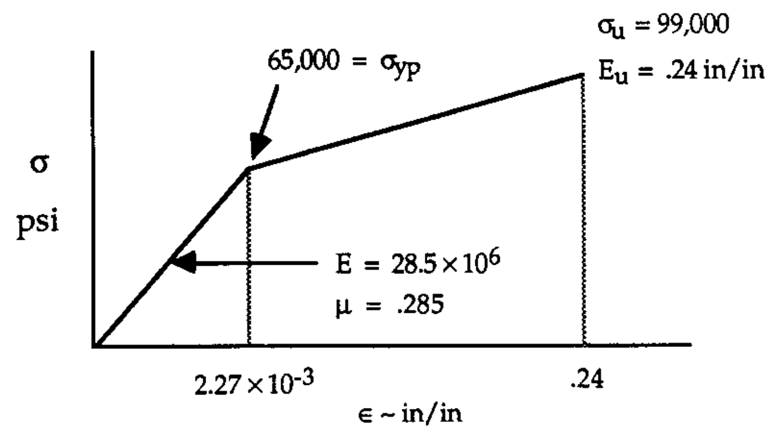
where t is the thickness, r_i is the inside radius, r_0 is the outside radius, and r_m is the mean radius. Then

$$r_m^3 = .29$$

$$I_z = \frac{\pi(d_m)^4}{64} = .15$$

$$Z = \frac{I}{C} = .228$$

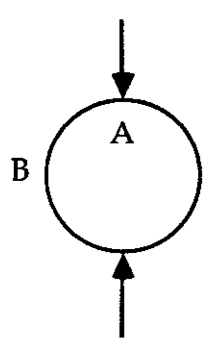
where I is the moment of inertia, d_m is the mean diameter, Z is the section modulus, and C is the distance from neutral axis to the outermost fiber. Since these deflections are plastic deformations, it is necessary to use the entire stress strain curve for Nitronic 40, the material of the beam tube.



$$E_p = \frac{99{,}000 - 65{,}000}{.24 - .0023} = \frac{34 \times 10^3}{.24} = 142 \times 10^3$$

$\sigma_{yp} = 65{,}000$ psi	yield stress
$\sigma_u = 99{,}000$ psi	ultimate stress
$E_{el} = 28.5 \times 10^6$ psi	elastic modulus
$E_{pl} = 142 \times 10^3$ psi	work-hardening modulus
$\mu_{el} = .285$	Poisson's ratio

From either Timshenko, Part I, *Strength of Materials* p. 380, Roark p. 221, or Seely and Smith p. 181, all of which have the same equations



$$M_A = .318\, Pr_m$$

$$M_B = \frac{Pr}{2}\left(1 - \frac{2}{\pi}\right) = 1.82\, Pr_m$$

where M_A is the moment at A and M_B is the moment at B

$$\delta_A = .149\frac{Pr^3}{EI}$$

$$\delta_B = .137\frac{Pr^3}{EI} \quad .$$

where δ_A is the $^1/_2$ deflection at A and δ_B is the $^1/_2$ deflection of B, σ_A is the stress at A, σ_B is the stress at B, and a is the cross-sectional area.

$$\sigma_A = K\frac{Mc}{I} \quad \text{where} \quad K = 1 \text{ for } \frac{R}{c} = 37$$

$$\sigma_B = \frac{P}{2a} + \frac{Mc}{I}K$$

K is the correction factor to apply to curved flexural members for use in the straight beam formula. See pages 149–151 of Seely and Smith for Table of K factor, or graph on page 152.

At pt. A

If σ = 65,000 psi

$$\sigma = \frac{Mc}{I}$$

M = 65,000 (.228) = 14,820 in lbs.

$$P = \frac{M_A}{(.318)(r_m)} = \frac{14{,}820}{(.318)(.662)} = 70{,}398 \text{ lbs.}$$

the elastic load is then 70,398 lbs., since M_A is the greatest moment. However, if pt. B is considered

$$65{,}000 = \frac{P}{2a} + \frac{Mc}{I}$$

$$= \frac{P}{2(9.6 \times 10^{-4})} + \frac{.182\,P(.662)}{.228}$$

$$a = \left(\frac{.035}{2}\right)^2 \pi = 9.6 \times 10^{-4}$$

65,000 = P(520.8) + .528 P

P = 124.7

If $P \geq 125$ lbs., pt. B goes plastic, while pt. A is still elastic.

$$\delta_B = (.015)2 = .137\frac{Pr^3}{EI}$$

$$.030 = \frac{.137(.29)P}{142 \times 10^3(.15)}$$

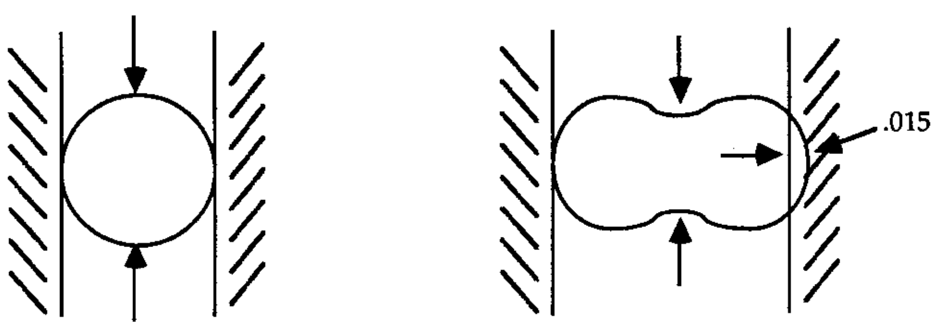
$$P = \frac{(.030)(142 \times 10^3)(.15)}{(.137)(.29)} = 16.08 \times 10^3 = 16{,}000 \text{ lbs.}$$

In other words, it takes 16,000 lbs at A to displace B .015. Since P > 125 lbs. causes B to go plastic, 16,000 lbs. would be way beyond σ_{yp} of material.

$$M_B = (.182)(.662)(16{,}000) = 1927.7 \text{ in lbs.}$$

$$\sigma_B = \frac{16{,}000}{2(9.6 \times 10^{-4})} + \frac{1927.7}{.228} >> \sigma_{y.p.}$$

Therefore, δ_B must be restrained, and we have a contact problem.



It is assumed that the parallel planes are fixed — limiting case as the bumpers and coil will deflect. For a cylinder between two flat plates, see Roark p. 516.

$$\Delta D = \frac{4p(1-\mu^2)}{\pi E}\left(\frac{1}{3} + \ln\frac{2D}{b}\right)$$

$$E = 142 \times 10^3 \quad \mu = .5 \quad \text{in plastic range}$$

$$b = 2.15\sqrt{\frac{pK_D}{E}} = 2.15\,\frac{p(1.324)^{1/2}}{142 \times 10^3}$$

where K_D is a numerical geometric coefficient.

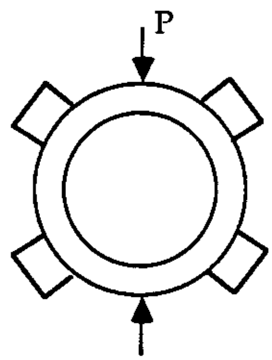
$$2.22 \times 10^{-6}\,p + 6.73 \times 10^{-6}\,p \ln\frac{403.47}{p^{\frac{1}{2}}} = \Delta D = .105$$

Solving by trial and error,

p = 10,000 lbs. satisfies the above equation.

$$.0222 + .0673 \ln \frac{403.47}{100} = .116 > .105$$

A load of p = 10,000 lb/in. will cause a permanent deflection of .105 on the diameter. The question is then, what is the stress at B? Let p = 10,000 lbs/in.



Q = total force or pressure exerted by body 1 (beam tube) on body 2 (G–10 bumper).

$E_1 = 28.5 \times 10^6$ psi

$E_2 = 4.1 \times 10^6$ psi

$\mu_1 = .285$

$\mu_2 = .15$

a = semi major axis of ellipse of contact

b = semi minor axis of ellipse of contact

$R_1 = .68$

$R_1' = .68$

$R_2 = -.68$

$R_2' = -.68$

$\alpha = 0°$

$k \frac{Z}{b} = .015$

$$B = \frac{1}{4}\left[\left(\frac{4}{.68}\right)^2\right]^{1/2} = \frac{1}{.68^{1/2}} = 1.22$$

A = 1.22 .

From Fig. 183, Seely and Smith, p. 356.

$\frac{B}{A} = 1$ $\quad C_G = .195$ $\quad C_\sigma = .64$ $\quad k = 1.0$

$C_\tau = .21$ $\quad C_b = .9$

$C_Z = .48$ $\quad C_\delta = 2.3$

$$\Delta \equiv \frac{1}{A+B}\left[\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right] = \frac{1}{1.22 + 1.22}\left[\frac{1-.285^2}{28.5 \times 10^6} + \frac{1-.15^2}{4.1 \times 10^6}\right]$$

$$\Delta = \frac{1}{2.44}\left[\frac{.03223 \times 10^{-7}}{.0032 \times 10^{-6}} + \frac{.2384 \times 10^{-6}}{2384}\right] = .099 \times 10^{-6}$$

$p = 10{,}000$ lbs. $\quad \frac{p}{4} = 2500$ $\quad$ (4 pumpers taking the load)

Analysis ignores friction

$$b = C_b \quad (PD)^{1/3} = .9\sqrt[3]{2500 \ \times .099 \times 10^{-4}} = .9 \times 10^{-1}\ (.628)$$

$$2.475 \times 10^{-4}$$

$$2.475 \times 10^{-3}$$

$b = .565 \times 10^{-1}$ $\quad b = .0565$

$\sigma_{max} = -c_\sigma(b/\Delta) = (-.64)(.57) \times 10^6 = -.36 \times 10^6 = -360{,}000$ psi

$\tau_{max} = c_\tau(b/\Delta) = .21 \times .57 \times 10^6 = 119{,}700$ psi

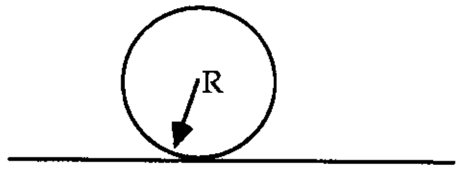
$\tau_{Gmax} = C_G(b/\Delta) = .195\ (.57 \times 10^{-6}) = 111{,}150$ psi

$Z_s = C_{Zs}b = .48 \times .057 = .027$ <u>reasonable</u>

$$\frac{b}{\Delta} = \frac{.0565}{.099 \times 10^{-6}} = .57 \times 10^6 = 570{,}000$$

The fact that these stresses are large is consistent with the discussion in Seely and Smith. "As is all contact stress problems, the three principal stresses at the point where they have their maximum values have the same sense, i.e., they are all compressive stress." The values obtained here are all similar to the values in the illustrative problem used in Seely and Smith, p. 361.

If we consider the following case (in previous case bumper and beam tube had same contour)



$R_2 = \infty$ Rigid

Cylinder on a plane $\quad B = \frac{1}{2}\frac{1}{R_1}$, $A = 0$, $B = \frac{1}{2}\frac{1}{.68} = .735$

If $B/A = \infty$, $k = 0$.

$$\Delta = \frac{1}{.735}(.2416 \times 10^{-6}) = .3287 \times 10^{-6}$$

$$q = \frac{\text{load}}{\text{length of contact area}} = \frac{10{,}000}{.680} = 14705.9$$

Bumper .170 width

$\frac{4}{.680}$ (do not consider upper and lower bumpers)

$$b = \sqrt{\frac{2q\,\Delta}{\pi}} = \sqrt{\frac{2(14705)(.329 \times 10^{-6})}{3.14}}$$

$$b = 55.5 \times 10^{3}$$

$$\sigma_{xz} = -2(.285)\,\frac{b}{\Delta}$$

$$= -2(.285)\,\frac{55.5 \times 10^{-3}}{.329 \times 10^{-6}} = -96.2 \times 10^{3} = -96{,}000 \text{ psi.}$$

$$\sigma_{yz} = -\frac{b}{\Delta} = -168{,}000$$

$$\sigma_z = -168{,}000 \text{ psi}$$

$$\sigma_1 = -168{,}000 \text{ psi}$$

Conclusion

It is clear that a load of only 125 lbs. produces a stress on the sides that will result in the beginning of plastic flow. A ring solution is thereby inappropriate in view of the observed plastic deformations. Therefore, the problem is a contact stress problem. The solution to the contact problem is that $p \approx 10{,}000$ lbs to produce the deflection observed. If we then proceed to determine the stress between tube and the G-10 bumpers per Hertz's contact stress and using the coefficients in Seely and Smith, $\sigma_{max} = -360{,}000$ psi. A less rigorous solution of cylinder of cylinder on a flat plate gives

σ_{max} = 168,000 psi. The consequence of such high stresses is that material will flow. Considering the yield strength of G-10, Epoxy, and Kapton, is it probable that N_iT_i cable will short out.

These stresses are order of magnitude higher than materials in general can sustain without massive plastic flow. This plastic flow will result in immediate failure of insulation and thus a short will develop in the coil or under subsequent cycling. Analysis indicates that a high probability exists to develop a short in the coil.

When the above was presented, it raised more questions regarding the effect on the coil and what happens to the bumpers. A finite element calculation was being avoided because of demands to work on other problems. However, in view of the discussion, a FE calculation was performed. Two sets of calculations were analyzed:

A. a 52 mil displacement at the pole of the beam tube

B. standard model — no distortion of beam tube

A third set of calculations whereby the beam tube was forced to have both the observed deflections, i.e. at A and B, was performed but is not discussed in this report.

Case A produced a side deflection of only 5 mils, instead of the 15 mils observed. Therefore, the third case, whereby this side deflection was found to be 15 mils, was also run. The reason for the difference is that the side deflection depends on the clearance. Unlimited clearance results in "unlimited" side deflection. Therefore, the clearance in practice is greater than in this model, which is 3 mils. In the hand calculation, no clearance was permitted between the beam tube and bumper or bumper an coil. Another difference is that the thickness of the beam tube in the model is 40 mils, whereas in the experiment at BNL it is 35 mils. The properties used in the model for G-10, beam tube, and collar were elastic-plastic σ_{yield} and work-hardening modulus, as functions of temperature. The following correlations are made:

$0.0 \leq t \leq .01$ sec.	Assembly + 10 mils distortion of beam tube at pole
$0.0 \leq t \leq .025$ sec.	Assembly + complete distortion of beam tube at pole
$.025 \leq t \leq .1$	300° K to 4.35° K
$t \leq .025$	T = 300° K
$t \geq .1$	T = 4.35° K
$.1 \leq t \leq .6$ sec.	Application of Lorentz forces
$t = .6$	6.6 Tesla
$t = .4$	5.1 Tesla

The results are tabulated below, in Table 1.

Table 1 was compiled from fringe plots, rather than elemental or nodal plots. It should be noted that, except for element 129, max. stress locations may not consistently be at the same location. Fringe plots are not as accurate as elemental or nodal data, but show an entire component more directly.

Conclusion based on the computer FE calculations:

- Stresses in beam tube exceed yield.
- Stresses in G-10 bumpers exceed interlaminar shear stress strength
- The main difference is at 6.6 Tesla, where the circumferential stress is -2900 psi without the distortion and approximately zero (-400) with the distortion. In addition, the maximum stress in the coil is 9500 psi with no distortion and 13,000 psi with distortion.
- This table does not show deflections or separation between the coil and the collar. In the no-distortion case, no separation is observed any any time (< 6.6 Tesla). In the distortion case, separation was observed at 5 Tesla.

While the beam tube moves away from the coil at the pole, it moves and applies pressure on the coil at the bumper adjacent to the equilateral plane. This impacts cooldown and application of Lorentz forces. The computer calculations enforce the conclusion drawn as a result of the hand calculations.

	Su	Max. σ_{eff} Beam Tube	G-10 Bumper $\bar{\sigma}$	G-10 Bumper τ_g	(σ_r) Max. Principal Stress in Coil	Max. (σ_θ) Min. Principal Stress in Coil	Max. Principal Stress in Element 129 (σ_r)	Max. Min. Principal Stress in Element 129 (σ_θ)
Distortion	t = .01 Assembly + 10 mils Dist.	78,000 *	5000	4000	-4400	-11,500	+210 / -600	-11,500 / ... / -6500
Distortion	t = .025 Dist.	80,000 *	20,000	6100 *	-4500	-13,000 *	-550 / +1800	-1300 / -11,000 / -9300
Distortion	t = .1 End of C.D.	224,000 *	26,000	7500 *	-5800	-13,000 *	-700 / +100 / +1600	-1300 / -11,000 / -4200
Distortion	t = .6 6.6 Tesla	250,000 *	27,000	26000 *	+1000	-11,000	-4500	-400
No Distortion	t = .01	1600	1000	550	-4800	-10,500	-1000 / -1900 / -2700	-8500 / ... / -5700
No Distortion	t = .025	1600	1100	560	-4900	-10,200	-1000 / -1900 / -2700	-8900 / ... / -5800
No Distortion	t = .1	95,000	5700	1950	-5100	-11,000	-4200 / -3200	-8300 / -9500
No Distortion	t = .6	90,000	6700	3350	-4550	-10,400	-100 / -610	-2900

* EXCEEDS YIELD STRENGTH OR MAX. SHEAR STRESS