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LUMPED CORRECTION OF THE MULTIPOLE CONTENT IN LARGE SYNCHROTRONS*

David Neuffer
AT-6, Los Alamos National Laboratory, Los Alamos, NM 87545

and

SSC Central Design Group, Lawrence Berkeley Laboratory, Berkeley, CA 94720

A new method of correcting dynamic nonlinearities due to the multipole content of the dipoles in a large alternating-gradient synchrotron is discussed. The method uses lumped multipole elements placed at the center (C) of the accelerator half-cells as well as elements near the focusing (F) and defocusing (D) quadrupoles at the half-cell ends. In a first approximation, the strengths of the correctors follow Simpson's Rule for three-point integration. Correction of second-order sextupole nonlinearities may also be obtained through use of the F, C, and D octupoles. Correction of nonlinearities by more than two orders of magnitude is obtained.

A large synchrotron such as the Superconducting Super Collider (SSC)[1] requires an adequate linear aperture for beam stability and reliable operation. Linear motion is required over a working region in amplitude and momentum space sufficient to include the beam size and momentum spread with closed orbit deviations and injection errors. Orbit nonlinearity can be measured by the amplitude and momentum-dependent tune shifts per turn $\Delta\nu_x$, $\Delta\nu_y$. The SSC linear aper-

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ture tolerances (SLAT) have been set by requiring $\Delta\nu_x$, $\Delta\nu_y \leq \pm 0.005$ for orbits with amplitudes A_x, A_y, up to 0.5 cm in the SSC arcs and with momentum offsets $\delta \equiv \frac{\Delta p}{p} \leq \pm 0.001$.[1]

Except for relatively short utility and interaction regions (IR), the SSC circumference is composed of ~320 alternating gradient (FODO) cells consisting of long dipoles, short focusing (F) and defocusing (D) quadrupoles, and short corrector magnets (see Fig. 1). In an excellent first approximation, the ring can be assumed to consist only of such cells, and the nonlinear fields in the dipoles dominate the nonlinear motion. Finite-length effects of nondipole elements may also be ignored.

The magnetic fields in the dipoles may be represented by the complex expression

$$B_y + iB_x = B_o\{1 + \sum[b_n(s) + ia_n(s)](x + iy)^n\} \quad ,$$

where B_o is the bending field and $b_n(s)$ and $a_n(s)$ are the normal and skew multipole components. The transverse motion may be described by a Hamiltonian, which includes the linear focusing

$$H = \frac{I_x}{\beta_x(s)} + \frac{I_y}{\beta_y(s)} + \Re\sum_n \frac{B_o}{B\rho}\frac{[b_n(s) + ia_n(s)](x + iy)^{n+1}}{n+1} \quad ,$$

where I_x and I_y are the action coordinates and $\beta_x(s)$ and $\beta_y(s)$ are the Courant-Snyder[2] betatron functions of the linear motion. The coordinates x and y of particle motion are represented in action-angle variables by $x = \sqrt{2\beta_x I_x}\cos(\phi_x) + \eta\delta, y = \sqrt{2\beta_y I_y}\cos(\phi_y)$. The terms ϕ_x and ϕ_y are the angle variables (betatron phases), and the off-momentum orbit displacement at δ determined by the dispersion function $\eta(s)$ is included. The terms $A_x = \sqrt{2\beta_x I_x}$ and $A_y = \sqrt{2\beta_y I_y}$ are the amplitudes. The tune shifts are obtained to first order by averaging the phase advance caused by the magnetic field perturbation

$$\Delta\nu_{x,y} = \frac{1}{2\pi}\int \frac{d\phi_{x,y}}{ds} ds = \Re\langle\frac{dH}{dI_{x,y}}\rangle \quad . \tag{1}$$

In first order in the coefficients b_n and a_n only systematic normal multipoles ($\overline{b_n}$) contribute; the resulting expressions for the tune shifts as a function of I_x, I_y, and δ due to sextupole (b_2), octupole (b_3) and decupole (b_4) components are

$$\begin{aligned}
\Delta\nu_x &= \langle b_2\beta_x\eta\delta\rangle + \langle\frac{3}{4}b_3\beta_x^2 I_x - \frac{3}{2}b_3\beta_x\beta_y I_y + \frac{3}{2}b_3\beta_x\eta^2\delta^2\rangle \\
&\qquad + \langle 3b_4\beta_x^2\eta I_x\delta - 6b_4\beta_x\beta_y\eta I_y\delta + 2b_4\beta_x\eta^3\delta^3\rangle \\
\Delta\nu_y &= -\langle b_2\beta_y\eta\delta\rangle + \langle\frac{3}{4}b_3\beta_y^2 I_y - \frac{3}{2}b_3\beta_x\beta_y I_x - \frac{3}{2}b_3\beta_y\eta^2\delta^2\rangle \\
&\qquad + \langle 3b_4\beta_y^2\eta I_y\delta - 6b_4\beta_x\beta_y\eta I_x\delta - 2b_4\beta_y\eta^3\delta^3\rangle \quad .
\end{aligned} \tag{2}$$

The SLAT may be applied to Eqs. (1) and (2) to obtain tolerance limits[3] on uncorrected $|\bar{b}_n|$; the values for the current SSC design lattice are in Table I.

The SSC dipoles are expected to have significant multipole content, particularly in the $\bar{b}_n$ with n even, which are allowed in dipole symmetry. Estimates[5] of the expected systematic and rms random multipole strengths have been extrapolated from the measurements[6] of the similar Tevatron dipoles. Multipole strengths caused by saturation effects at high field and by persistent current effects at injection field have also been calculated.[1] These estimates have been collected in Table I where they can be compared with SLAT values. Serious possible deficiencies in $\bar{b}_2, \bar{b}_3$, and $\bar{b}_4$ are obtained. Consequently, initial SSC design included sets of multipole trim coils within every dipole for local correction of $\bar{b}_2, \bar{b}_3$, and $\bar{b}_4$. However, these trim coils greatly complicate the dipoles.

Multipole correction is simplified if it is implemented using short correctors separate from the dipoles. Initial attempts used correctors placed near F and D quads, where chromatic (momentum-dependent) correction sextupoles are placed. Because there are only two first-order b_2 terms, F and D sextupoles can completely correct them; however, second-order sextupole effects are also important (see below). For b_3, b_4, and higher multipoles, there are five or more terms and they cannot be completely corrected. In Ref. 7, F and D multipole strengths are set by correcting the chromatic terms; the SLAT b_3 and b_4 nonlinearities are reduced only by a factor of ≤ 2. (See Table II.) Other optimizations are not much more effective.[8]

A dramatic improvement is obtained by adding a corrector to the center (C) of each half-cell (see correctors provide only three-parameter correction, an empirical optimization converges very close to a characteristic solution (Simpson's Rule)[9] and reduces tune shifts by more than two orders of magnitude. For example, the first-order amplitude-dependent 1-D tune shift caused by octupoles may be written as

$$\Delta\nu_x = \frac{3I_x}{4L}\left\{\int_0^L b_3(s)\beta_x^2(s)\ ds + \left[\frac{S_{3,F}}{B_o}\beta_x^2(0) + \frac{S_{3,C}}{B_o}\beta_x^2(L/2) + \frac{S_{3,D}}{B_o}\beta_x^2(L)\right]\right\} \tag{3}$$

in a simplified lattice. The corrector multipole strengths $S_{n,i}$ are defined by $S_{n,i} \equiv B_{n,i}l_i = -f_{n,i}B_o\bar{b}_nL$, where $B_{n,i}$ and l_i are the corrector lengths and strengths, and L is the half-cell length. The Simpson's Rule solution is $f_F = f_D = 1/6$ and $f_C = 4/6$ per half cell; that corrects b_3 and b_4 nonlinearities by approximately two orders of magnitude. Optimization about that solution changes the strengths slightly, permitting another order of magnitude reduction (see Table II). The correction is more than adequate for the SSC.

Lumped correction [see Eq. (3)] is equivalent to approximating integrals of powers of betatron functions by a sum over discrete points, and Simpson's Rule is a third order integration that is very accurate for smoothly varying functions. Figure 2 shows β_x, β_y, and η, over a full cell; they are smoothly varying over half-cells with a derivative discontinuity at the quadrupoles. Figures 2 and 3 show the specific functions that appear in b_2 and b_3 tune shifts. These functions are smoothly varying on the half-cell level; F, C, and D correctors provide three-point, Simpson's Rule type cancellation on the half-cell level. The discontinuities at the half-cell ends also provide an explanation for the failure of two-point (F, D) correction (Ref. 8 provides a more detailed discussion).

The first-order correction is insensitive to betatron function perturbations, because Simpson's Rule requires only smooth variation on the half-cell level. Second-order effects are also reduced by first-order correction. The method is easily extended to include other multipoles (b_5, b_6, etc.), if necessary. Random multipole effects can be reduced by varying the correctors to follow the local multipole content.[10] The method can also be extended to correct quadrupole multipole content, particularly in the long IR quadrupoles.

Superconducting dipoles can have a very large b_2 content, and second-order terms are important. Identification of second-order terms requires perturbation theory.[11-13] As a result, second-order sextupole tune-shift terms are of the same form as the first-order octupole terms:

$$\Delta\nu_x = aI_x + bI_y + c\delta^2 \text{ and } \Delta\nu_y = dI_y + bI_x + e\delta^2 \quad . \tag{4}$$

The coefficients a through e scale as b_2^2. The expression for a in a simplified lattice is

$$a = \frac{C}{32\pi L}\int_0^{2L} ds\beta_x(s)^{3/2}B_2(s)\int_s^{s+2L} ds'\beta_x(s')^{3/2}B_2(s') \times\left\{3\frac{\cos[\psi_x(s')-\psi_x(s)-\pi\nu_x]}{\sin(\pi\nu_x)}+\frac{\cos 3[\psi_x(s')-\psi_x(s)-\pi\nu_x]}{\sin(3\pi\nu_x)}\right\} \quad ,$$

where C is the ring circumference, $B_2(s)$ is the normalized sextupole strength, $\psi_x(s)$ is the betatron phase, ν_x is the cell tune, and $\Delta\nu_x$ is the full-ring tune shift. These terms are double integrals with phase factors and a discontinuity at $s = s'$ and are not closely fitted to Simpson's Rule half-cell integration. Because of the b_2^2 dependence, all of these terms are naturally positive after first-order correction. The first-order b_2 correction with only F and D elements reduces second-order terms by a factor of ~ 5; however, the b_2 SLAT is only increased to 2.7×10^{-4} cm^{-2}, somewhat below the desired level of $\sim 5 \times 10^{-4}$ cm^{-2}. Addition of a C sextupole corrector reduces these terms by a factor of > 4, increasing b_2 tolerance by a factor > 2 to a level above the estimated SSC dipole levels. Setting the C corrector strength by Simpson's Rule ($f_C = 2/3$) or "equal-weights" ($f_C = 1/2$) obtains similar correction (See Table III).

Second-order b_2 correction can be greatly improved by using the F, C, D octupoles. Unlike b_2^2 terms, first-order octupole terms have opposing signs [see Eq. (2)], and there are only three correctors for five independent terms. However, the negative terms in Eq. (2) have similar dependence with relatively enhanced values at the C correctors (see Fig. 3). The correction strategy is to use the C octupoles to correct the b and e terms in Eq. (4) and the F and D octupoles to correct the others; the ratios of F, C, and D strengths per half-cell are $\sim (1 : -2.7 : 1)$.

The reduction in nonlinearity can be very impressive. The correctors can be tuned to correct completely either amplitude or chromatic $\Delta\nu$ with the remanent terms reduced by $\geq 10\times$. The SLAT tune shifts can be reduced by a factor of ≥ 30, increasing $\bar{b}_2$ tolerances to $\geq 30 \times 10^{-4}$cm^{-2}, providing an extremely large safety margin (see Table III). Other nonlinear effects such as orbit distortion remain small, provided cell resonances are avoided. Because the octupole tune shifts are linear, there is no interference between their b_2^2 and b_3 correction roles. The use of F, C, and D octupoles to correct second-order sextupole nonlinearities adds an extra operational dimension, conceptually similar to the use of F and D sextupoles to correct quadrupole chromaticity. Their use may be extended to control nonlinearities from other elements such as quadrupoles; for example, all A_i^2 tune shifts can be cancelled to zero, regardless of their sources.

With first order b_2, b_3, b_4 and second order b_2 correction, the SSC dipoles should satisfy the SLAT with a safety margin of more than an order of magnitude. This gives the SSC design the ability to correct unexpectedly large nonlinearities. The safety margin may also be used to obtain a linear aperture much larger than minimal, which can be exploited to improve operational reliability and increase luminosity. Although the specific numerical results are SSC values, the same correction method can be applied to any synchrotron with large nonlinear fields, such as the CERN Large Hadron Collider,[14] with similar improvements in linearity.

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TABLE I. Tolerances and estimated strengths of systematic multipole content in the SSC dipoles. All multipole strengths are in units of 10^{-4} cm^{-n}. The tolerances are obtained from the SLAT. Estimated strengths are extrapolated from Tevatron data or calculated from the magnet properties.

Multipole	Tolerance in SSC Lattice (230 m, 90° Cells)	Estimated Random Error (Tevatron)	Systematic Strength (Tevatron)	Persistent Current Multipole Strength	Saturation Multipole Strength
b_2	0.0097	2.0	0.45	-4.7	1.2
b_3	0.017	0.35	-0.14	–	–
b_4	0.031	0.60	-0.33	0.30	-0.05
b_5	0.054	0.06	-0.024	–	–
b_6	0.096	0.08	1.57[a]	0.07	-0.01
b_7	0.17	0.16	0.009	–	–
b_8	0.29	0.02	-2.1[a]	<0.02	0.02

[a] The higher allowed multipoles (b_6, b_8) were not minimized in the Tevatron design; the SSC conductor placement should reduce these within tolerances.[4]

TABLE II. First-Order Correction of b_3, b_4. The correction factor is the ratio of uncorrected to corrected $\Delta\nu$ in the SLAT aperture. The tolerance is the maximum corrected b_n permitted under the SLAT criteria.

Correction Condition	Correction Factor	Tolerance (10^{-4}cm^{-n})
	b_3 (Octupole) Correction	
No correction	1.0	0.018
F, D, correction of chromaticity[7] ($f_F = 0.28$, $f_D = 0.70$)	1.9	0.033
F, C, D correction, Simpson's Rule (f_F, f_C, f_D) = (1/6, 4/6, 1/6)	93	1.6
F, C, D correction (f_F, f_C, f_D) = 0.99 (1/6, 4/6, 1/6)	370	6.7
	b_4(Decupole) Correction	
No correction	1.0	0.029
F, D correction of chromaticity[7] (f_F, = 0.24, f_D = 0.93)	1.4	0.04
F, C, D, correction, Simpson's Rule (f_F, f_C, f_D) = (1/6, 4/6, 1/6)	31	0.9
F, C, D corrections (f_F, f_C, f_D) = 0.99 (0.16, 0.67, 0.17)	800	24

TABLE III. Second-Order b_2 Correction

Correction Condition	Correction Factor	b_2 Tolerance ($10^{-4}cm^{-2}$)
No correction	1.0	1.2
F, D chromatic b_2 correction	5.1	2.7
F, C, D chromatic b_2 correction, equal weights ($f_C = 0.5$)	24	5.9
F, C, D chromatic correction, Simpson's Rule ($f_C = 0.667$)	23	5.7
F, D first-order b_2 correction ($f_{C,2} = 0$), F, C, D octupole second-order correction	120	13
F, C, D first-order b_2 correction ($f_{C,2} = 0.5$ to 0.67), F, C, D octupole second-order correction	700	32

Fig. 1. A symmetrical SSC cell. The element labels are: B - dipoles, F, D - quadrupoles, S - slots for correctors, C - center corrector slot. The correctors on opposite sides of the F and D quads may be lumped on either side and exact symmetry is not necessary.

Fig 2. Betatron functions (β_x, β_y, η) for a full SSC cell. The functions that appear in the sextupole tune shifts $(\beta_x\eta, \beta_y\eta)$ are also shown. Note the derivative discontinuity and the reflection symmetry about the center quadrupole.

Fig 3. Octupole tune-shift functions $(\beta_x^2, 2\beta_x\beta_y, \beta_y^2, \beta_x\eta^2, \beta_y\eta^2)$ on a half cell.

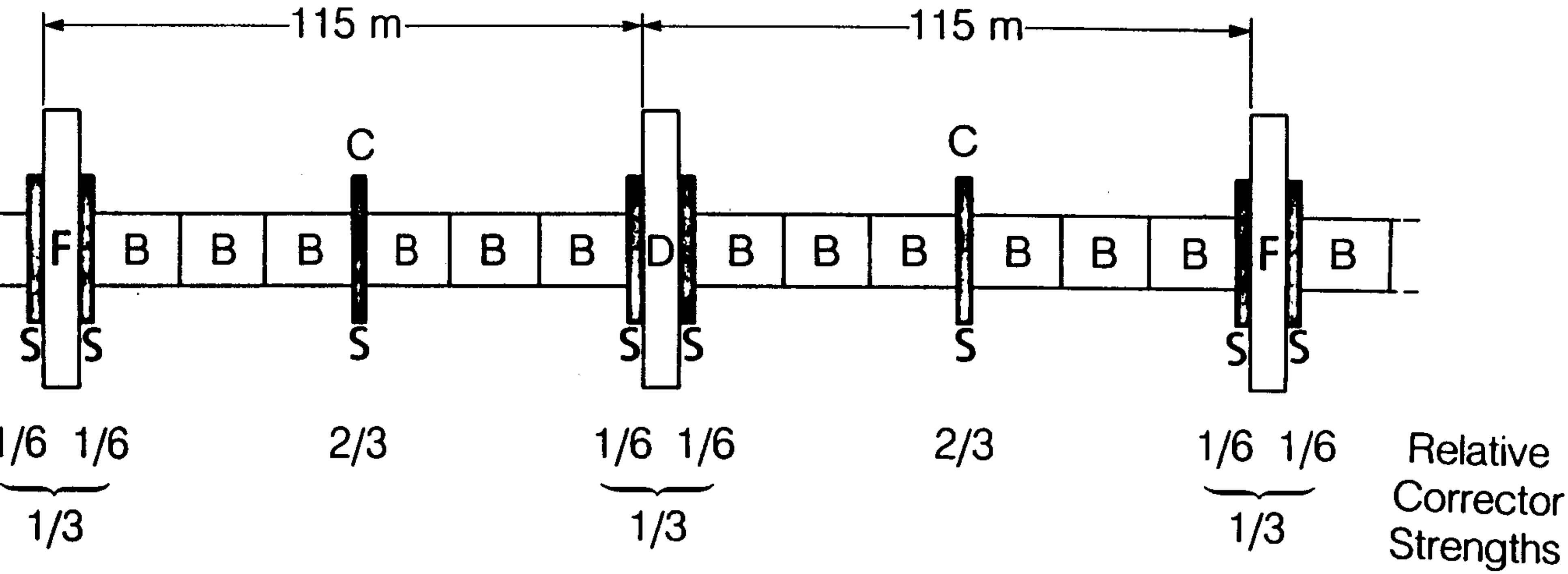
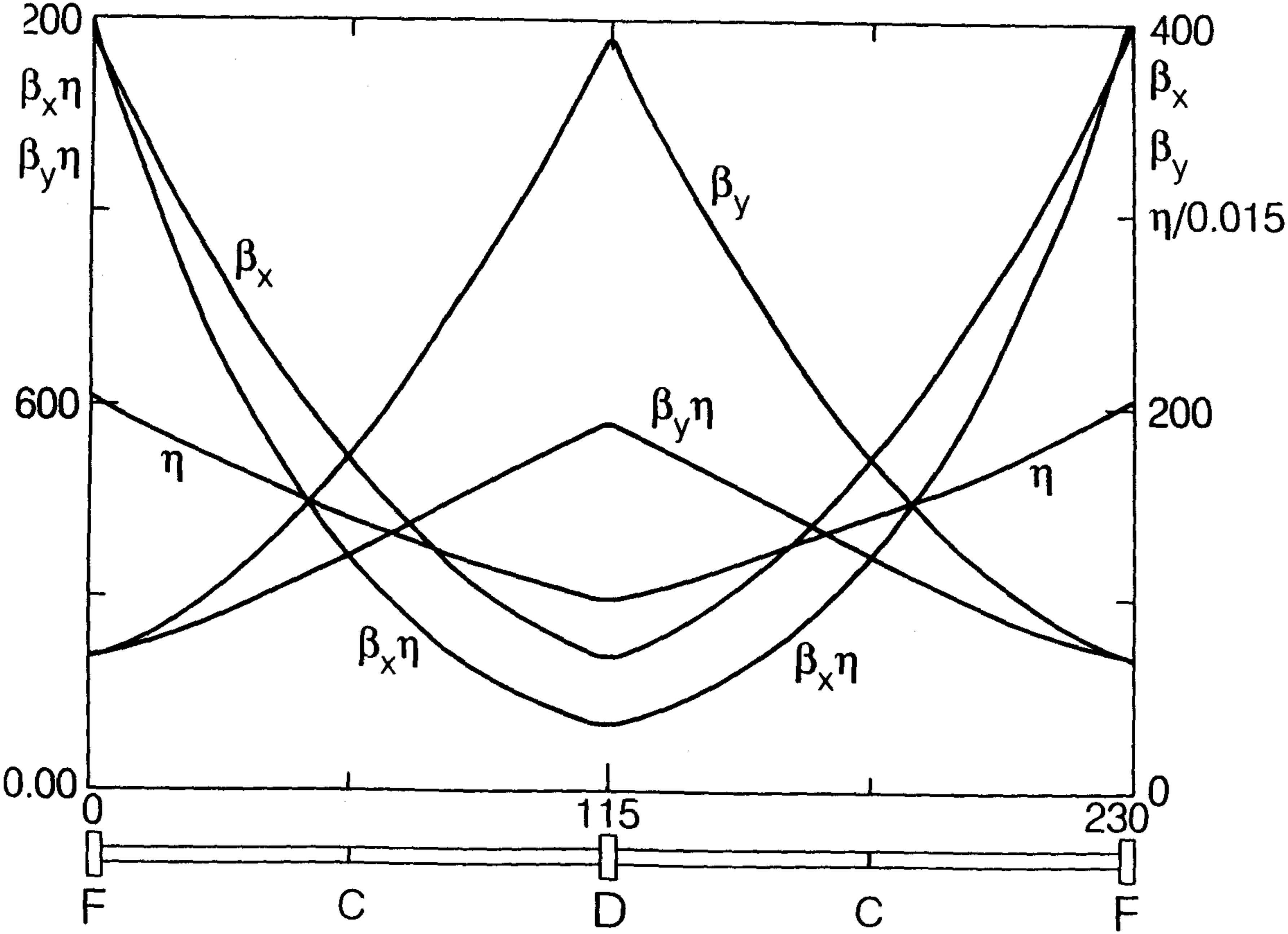


Fig. 1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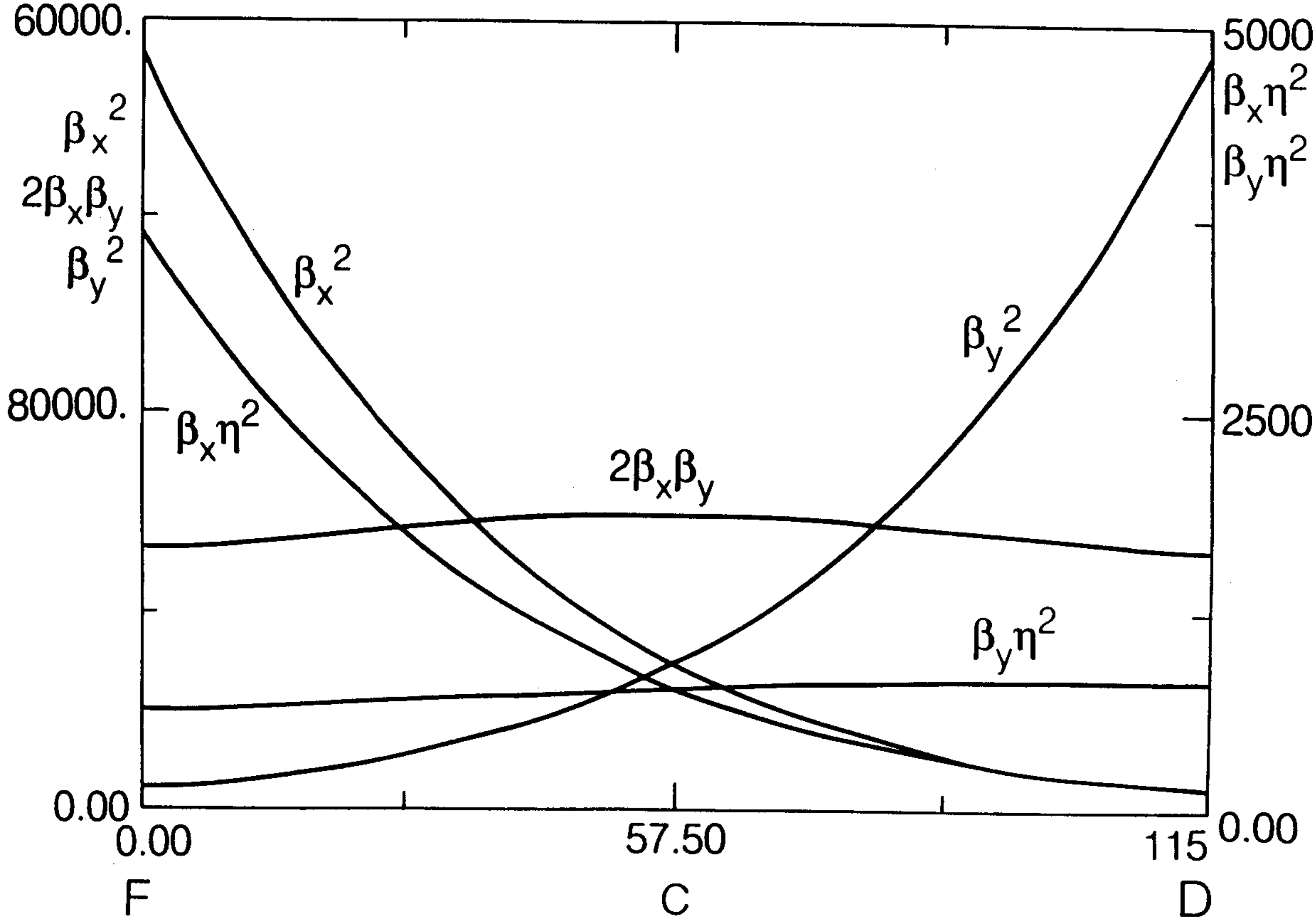
βxη
βyη
600
0.00
400
βx
βy
η/0.015
200
0
βx
βy
βyη
η
η
βxη
βxη
0
115
230
F
C
D
C
F

Fig. 2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$\beta_y{}^2$
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$\beta_y\eta^2$
2500
0.00
0.00
57.50
115
F
C
D

Fig. 3