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Systematic Compensation of the SSC With Two Lumped Correctors Per Half-Cell

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ABSTRACT

Lumped systematic field compensation of the SSC is analysed using **teapot** for particle tracking followed by Fourier analysis. The smallest number of correctors per half cell yielding acceptable compensation is at most three, and is probably two; the latter case is studied here. The best scheme found meets the requirement of constancy of the off-momentum, large-amplitude tunes, is sufficiently fine-grained to yield satisfactory improvement of the linear aperture by means of the "binning" compensation of random errors, and has been checked to be satisfactory for chromaticity adjustment and orbit flattening. Only a single point in the tune plane has been studied and the minimum set of multipoles needed in the lumped correctors remains to be determined. Also sensitivity to errors which are partly random, partly systematic has not been studied.

1. Introduction.

The desirability of lumped compensation schemes has been emphasized by Neuffer.[1] He considers schemes using one lumped corrector at the present spool-piece location (next to the quadrupole at the end of the half-cell) plus one more at the center of each half-cell. Since there are six dipole magnet units within each half-cell it is possible to contemplate placing the same number of correctors in other locations; for example a possible layout (to be labelled 1*4*1 in this report, where * represents a corrector and the integer represents the number of dipoles) would have the same number of correction elements but placed just inside the outermost dipoles. The extra flexibility of such arrangements, with little impact on the cost, might be expected to give improved compensation, and that is what is found here.

In the CDR every dipole has distributed b_2, b_3, and b_4 bore tube correction windings. This is at least as good as having six lumped correctors per half-cell and, from an accelerator physics point-of-view, that is certainly adequate for systematic compensation. These coils can also be used to correct the random parts of these particular multipoles using compensation by "binning."[2] On the other hand, from an engineering point-of-view, bore-tube coils may be undesireable and the purpose of the present report is to consider lumped alternatives. In making a final choice amongst the various possibilities, considerations of cost and of "practicality" will be important. These will not be considered here, other than to make two small points which show that the presence of lumped correctors will not lengthen the cells appreciably and, as a result, will not necessarily increase the cost.

(*i*) Though there is a certain superficial economy in stringing compensation coils along the full dipole length, the strength per unit length of such a coil cannot be more than a few percent of the strength per unit length of a dedicated corrector. For one thing the available radial thickness is at most one millimeter compared to say one centimeter for a dedicated coil.

For another thing, the critical current for a given superconducting wire in a regular dipole excited to 6.6 Tesla will be much less than the critical current for the same wire in a field of, say, 2 Tesla in a lumped element. Data of Wanderer[3] shows that the ratio of peak currents over this range is 3.5.

(*ii*) It is possible to add extra windings to the lumped dipole correctors (already needed in every half cell for steering) to obtain the required multipole fields without much impact on the length.[4]

2. Assumed Errors and Specification of Performance.

In this report magnetic multipole errors in the dipoles are studied, assuming that there are no field errors in other magnetic elements and no survey errors. The errors used are listed in Table 1 which is copied from Neuffer[1] which collects values from the CDR. The main systematic errors studied are $b_2 = -4.7, b_4 = 0.30$, and $b_6 = 0.07$ coming from persistent currents at the injection energy. These are in the usual units of parts per 10^4 at one centimeter. They are larger than the saturation multipoles though the latter may demand more powerful correctors since they appear at full field. The main random errors have standard deviations given by $\sigma_{b2} = 2.0, \sigma_{b3} = 0.35$, and $\sigma_{b4} = 0.60$.

A lattice consisting only of 320 simple 90^o FODO cells, with parameters identical to those in the regular arcs of the SSC, is assumed.

The degradation of accelerator performance due to field errors will, as usual, be discussed by quantifying the tune variation and the smear. The former tends to be dominated by systematic errors and the latter by random errors though this separation is not perfectly clean. The independent variables are x, the horizontal, and y, the vertical transverse amplitude and the momentum offset δ. For this report calculations are performed with these variables set to all combinations of the following extremes:

(*i*) $\delta = \pm 0.001$.

(*ii*) $x = 5\text{mm}$ at the point in the lattice where β_x is maximum.

(*iii*) $y = 5\text{mm}$ at the point in the lattice where β_y is maximum.

as well as near the origin for reference.

Table 1. Copied from Neuffer[1]

Tolerances and estimated strengths of systematic multipole context in the SSC dipoles. All multipole strengths are in units of 10^{-4}cm^{-n}. The tolerances are obtained from the linearity criteria. Estimated strengths are extrapolated from Tevatron data on calculated from the magnet properties.

Multipole	Tolerance (Conceptual Design)	Tolerance (Current Lattice)	Estimated Random Error (Tevatron)	Estimated Systematic Strength (Tevatron)	Estimated Persistent Current Multipole Strength	Estimated Saturation Multipole Strength
b_2	0.0072	0.0097	2.0	0.45	-4.7	1.2
b_3	0.011	0.017	0.35	-0.14	--	--
b_4	0.016	0.031	0.60	-0.33	0.30	-0.05
b_5	0.024	0.054	0.06	-0.024	--	--
b_6	0.035	0.096	0.08	1.57$^+$	0.07	-0.01
b_7	0.051	0.17	0.16	0.009	--	--
b_8	0.074	0.29	0.02	-2.1$^+$	<0.02	0.02

$^+$ **Tevatron magnet design; to be reduced in SSC magnet design.**

In subsequent tables values of the tune difference

$$\Delta Q_x(x, y, \delta) = Q_x(x, y, \delta) - Q_x(0, 0, 0)$$

at these extreme points will be presented in the following format, in units of 0.001:

$$\begin{pmatrix} \Delta Q_x(0,5,+0.001) & \Delta Q_x(5,5,+0.001) \\ \Delta Q_x(0,5,0.000) & \Delta Q_x(5,5,0.000) \\ \Delta Q_x(0,5,-0.001) & \Delta Q_x(5,5,-0.001) \\ & \\ \Delta Q_x(0,0,+0.001) & \Delta Q_x(5,0,+0.001) \\ \Delta Q_x(0,0,0.000) & \Delta Q_x(5,0,0.000) \\ \Delta Q_x(0,0,-0.001) & \Delta Q_x(5,0,-0.001) \end{pmatrix}$$

and $\Delta Q_y(x, y, \delta)$ will be exhibited similarly. Smear values in percent $S(x, y, \delta)$ will be arrayed in a similar format. This array is intended to suggest a coordinate system in which x points right, y points up, and δ points into the page.

The following performance specifications have been set for values of the variables in the interior of the region defined by the above extremes:

(i) The maximum tune variation should remain in the range ± 0.005.

(ii) The smear should remain less than 0.10. This specification is not entirely uncontraversial and is subject to continuing study, for example experimentally in the Tevatron experiment E778. Even the definition of smear is not universally established, especially in cases where the x and y invariant amplitudes are unequal. In this paper the smear is taken to be the bigger of an x-smear and a y-smear defined as follows. The normalising amplitude, $\bar{a}$, for both is taken to be $\sqrt{a_x^2 + a_y^2}$ where $a_x = \sqrt{x^2/\beta_x}$ and $a_y = \sqrt{y^2/\beta_y}$ are invariant amplitudes averaged over the motion.. The x-smear is $\sqrt{2}/3 \times (a_{x,max} - a_{x,min})/\bar{a}$ and the y-smear is $\sqrt{2}/3 \times (a_{y,max} - a_{y,min})/\bar{a}$. The off-momentum smear is calculated using the same formulas except that the correct off-momentum lattice functions are used and the transverse amplitudes are measured relative to the appropriate off-momentum closed orbit.

The units have been chosen in the tables of both tune variation and smear such that 10 marks the boundary between acceptable and unacceptable performance according to these specifications.

3. Comparison of Various Schemes on the Basis of Off-Momentum Tune Variation.

In this section it is assumed that the only errors present are systematic (with the values given above.) These errors are large enough, if uncompensated, to give unnaceptably large smear, but after compensation by any of the following schemes the smear is less than 1 percent and will not be exhibited. The five arrangements of lumped correctors indicated in Table 2 were studied with the results shown in Figures 1 to 5. The order of quadrupoles, dipoles and correctors is indicated at the side of the figures, and notations like 1*4*1 have already been defined. Except when it is explicitly indicated, for example when Simpson's rule is used, the most simple-minded rule is used to set the strengths of the compensation; the strength of the lumped corrector being the negative of the error multipole of one dipole times the number of dipoles per corrector. That number is written beside the * on the figures and in Table 2. All of $b2, b4$, and $b6$ are compensated. In every case there is a further adjustment of the sextupoles to bring both the horizontal and vertical chromaticities to zero. (For these first five datasets, sextupoles in the CDR locations next to the quads were used for this purpose. For some of the configurations this would require extra spool pieces, which would be extravagant. For the cases analysed after these five figures chromaticity correction uses only the same sextupoles as are used for error compensation, and the indication is that the strengths needed in meeting this need can be neglected in comparing different configurations.)

Table 2. Lumped correction schemes investigated.

Notation	Magnet array	Symbol for array	Safety factor	Extra spools per half-cell
1*2*2*1	2 2 2 2 2 2 ()=*==*==*=)(=*==*==*=()	13	5.3	2
1*4*1,3*3	3 3 6 ()=*====*=)(===*===()	121c	1.0	0.5
1*4*1,2*2*2	3 3 3 3 ()=*====*=)(==*==*==()	1212p	1.47	1
*3*3*	1 4 1 1 4 1 ()*===*===*)(*===*===*()	simp2	0.78	2
*3*3	2 4 2 4 ()*===*===)(*===*===()	simp1	0.80	1

() focusing quad

)(defocusing quad

= 16m dipole

* lumped corrector

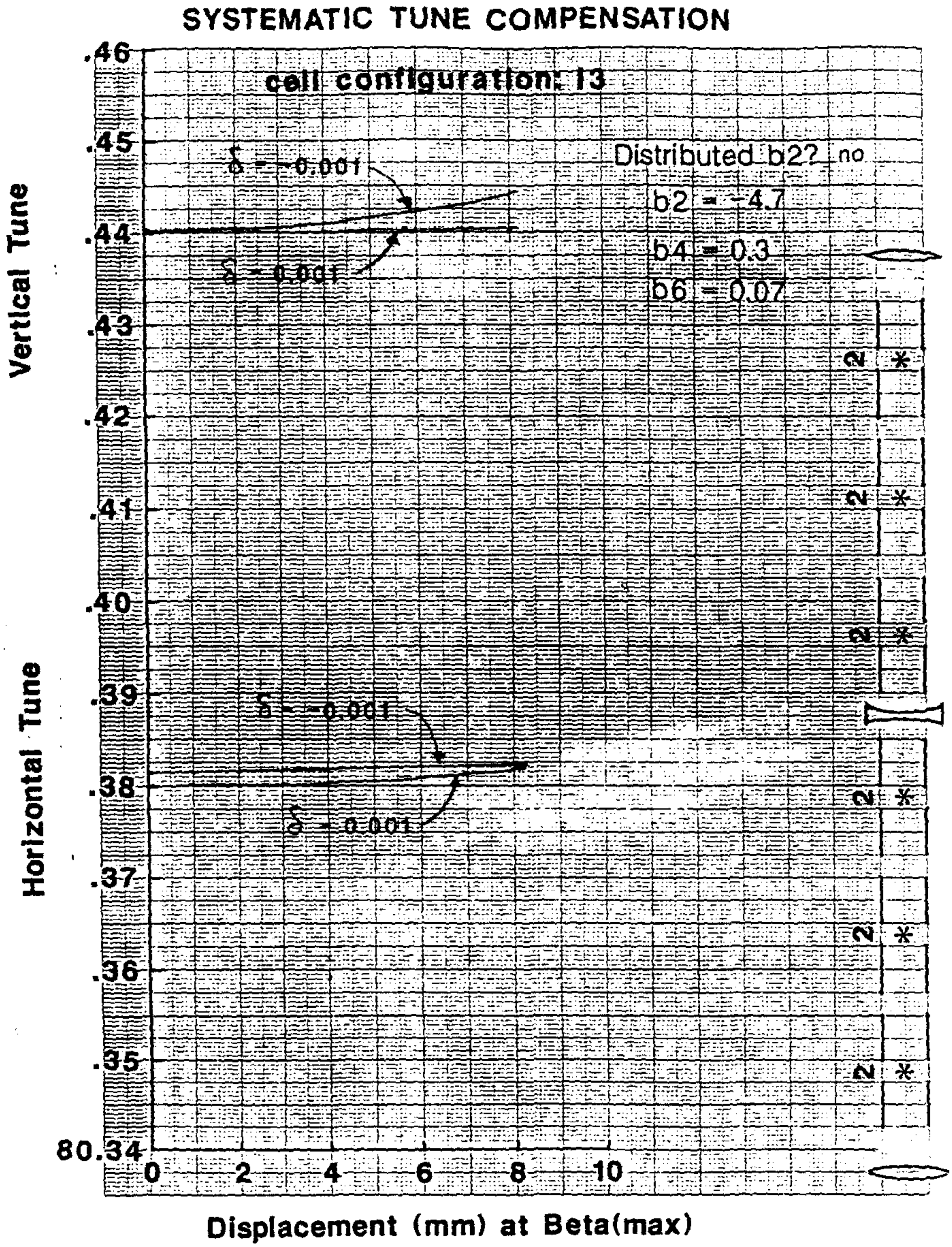

SYSTEMATIC TUNE COMPENSATION
cell configuration: 13
Distributed b2? no
b2 = -4.7
b4 = 0.3
b6 = 0.07
δ = -0.001
δ = 0.001
δ = -0.001
δ = 0.001
Vertical Tune
Horizontal Tune
.46
.45
.44
.43
.42
.41
.40
.39
.38
.37
.36
.35
80.34
0
2
4
6
8
10
Displacement (mm) at Beta(max)

Figure 1

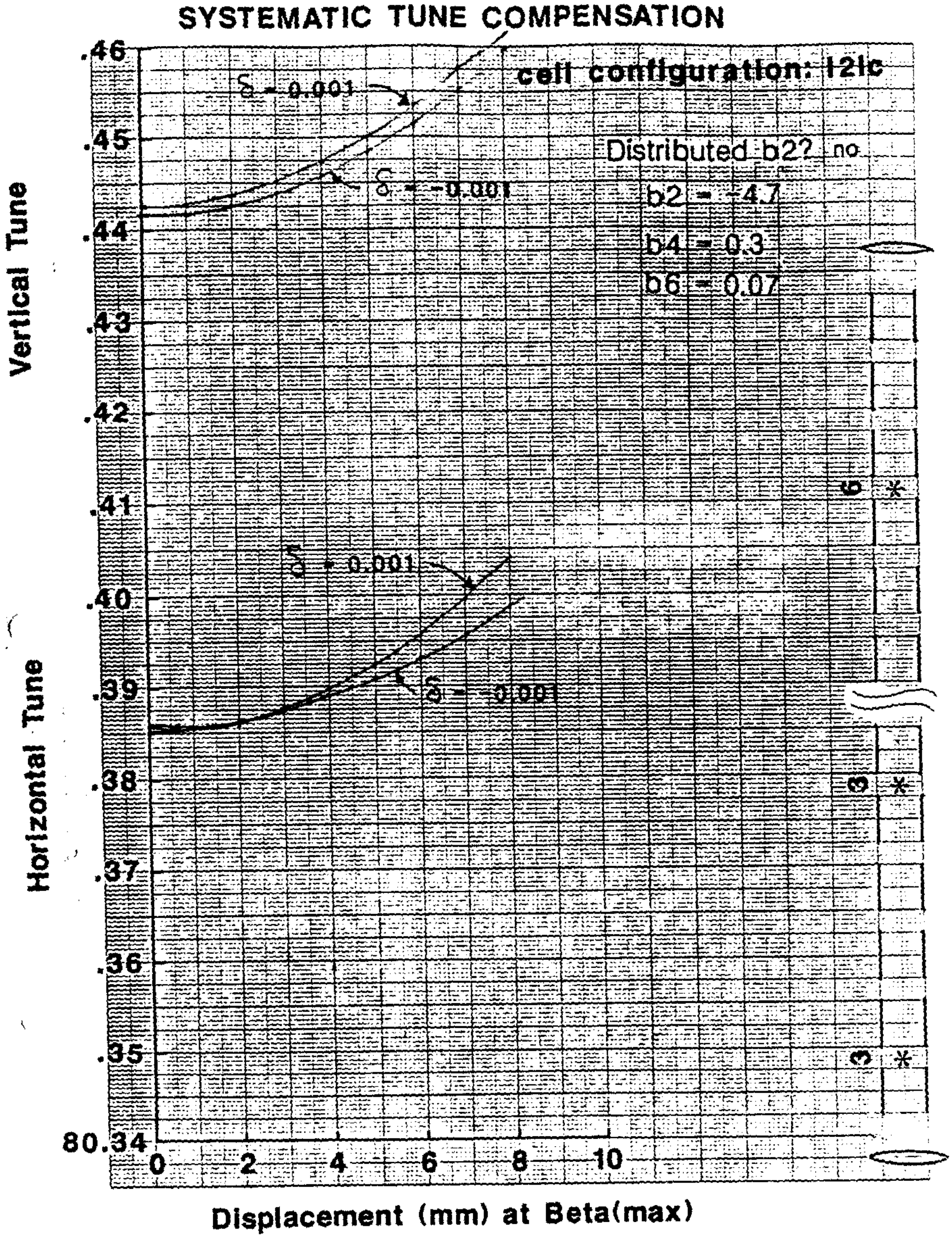

SYSTEMATIC TUNE COMPENSATION
cell configuration: I2lc
Distributed b2? no
b2 = -4.7
b4 = 0.3
b6 = 0.07
δ = 0.001
δ = -0.001
δ = 0.001
δ = -0.001
Vertical Tune
Horizontal Tune
.46
.45
.44
.43
.42
.41
.40
.39
.38
.37
.36
.35
80.34
0
2
4
6
8
10
Displacement (mm) at Beta(max)

Figure 2

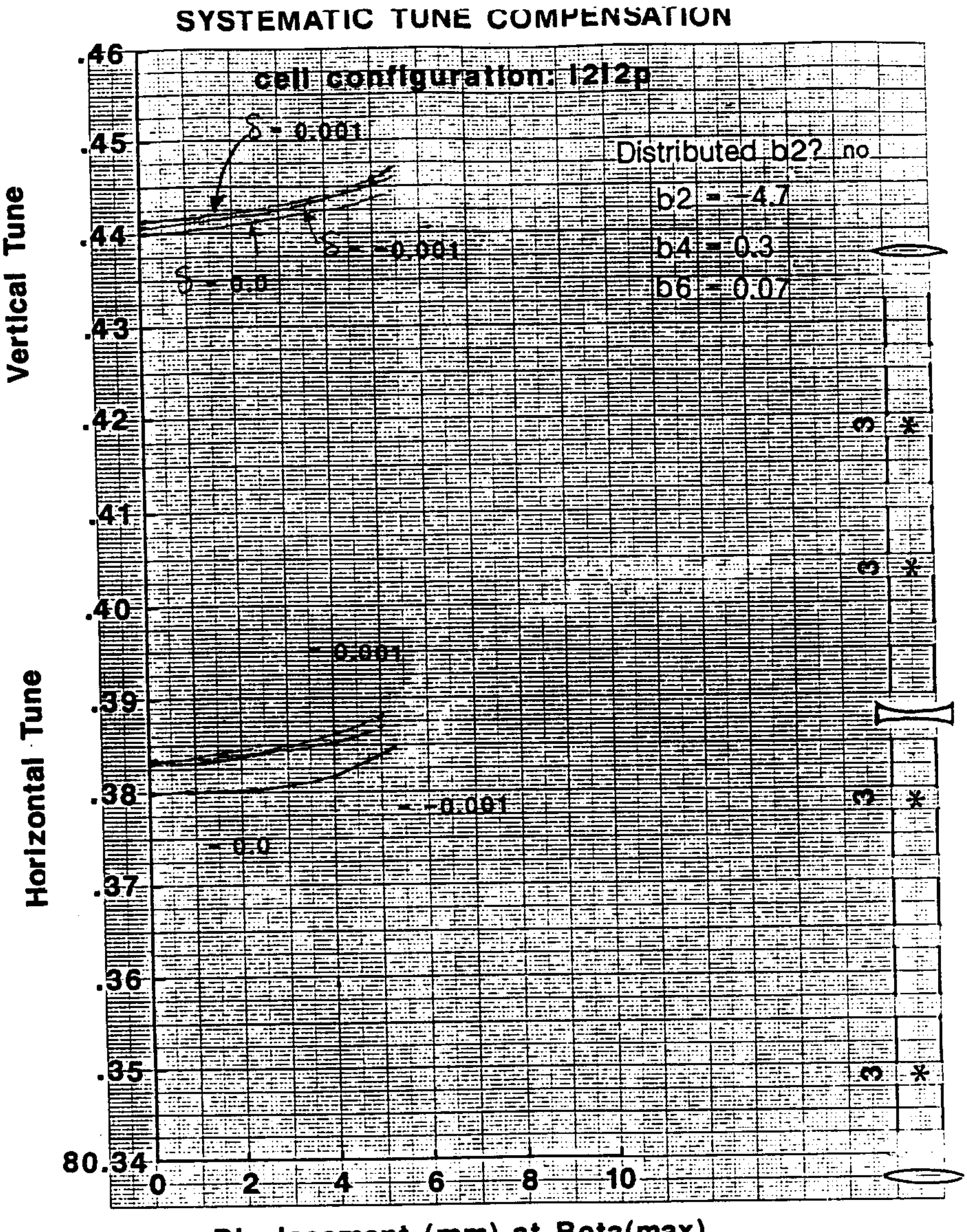

SYSTEMATIC TUNE COMPENSATION
cell configuration: I2I2p
Distributed b2? no
b2 = -4.7
b4 = 0.3
b6 = 0.07
δ = 0.001
δ = -0.001
δ = 0.0
- 0.001
- -0.001
- 0.0
Vertical Tune
Horizontal Tune
.46
.45
.44
.43
.42
.41
.40
.39
.38
.37
.36
.35
80.34
0
2
4
6
8
10
Displacement (mm) at Beta(max)

Figure 3

SYSTEMATIC TUNE COMPENSATION

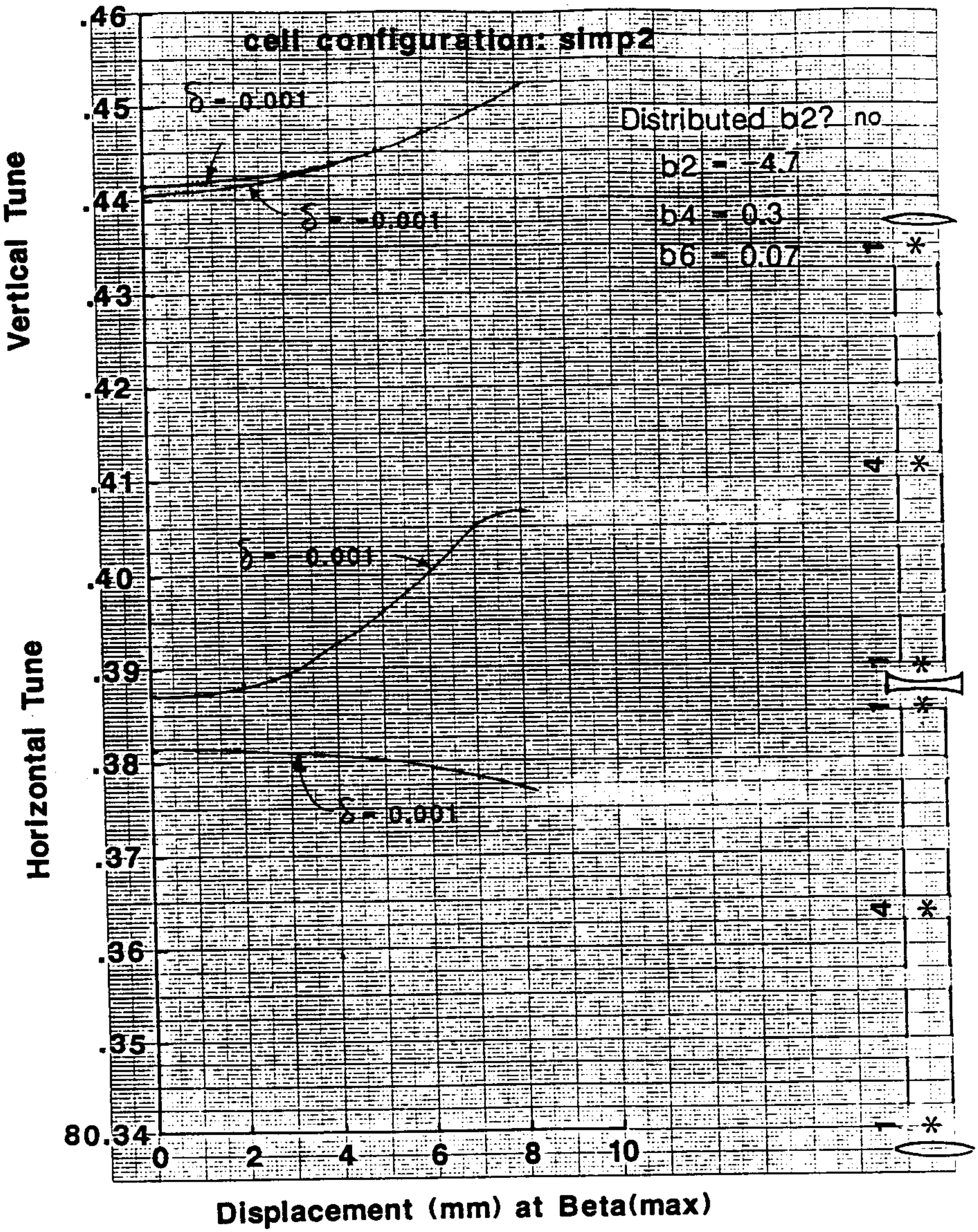


Figure 4

SYSTEMATIC TUNE COMPENSATION

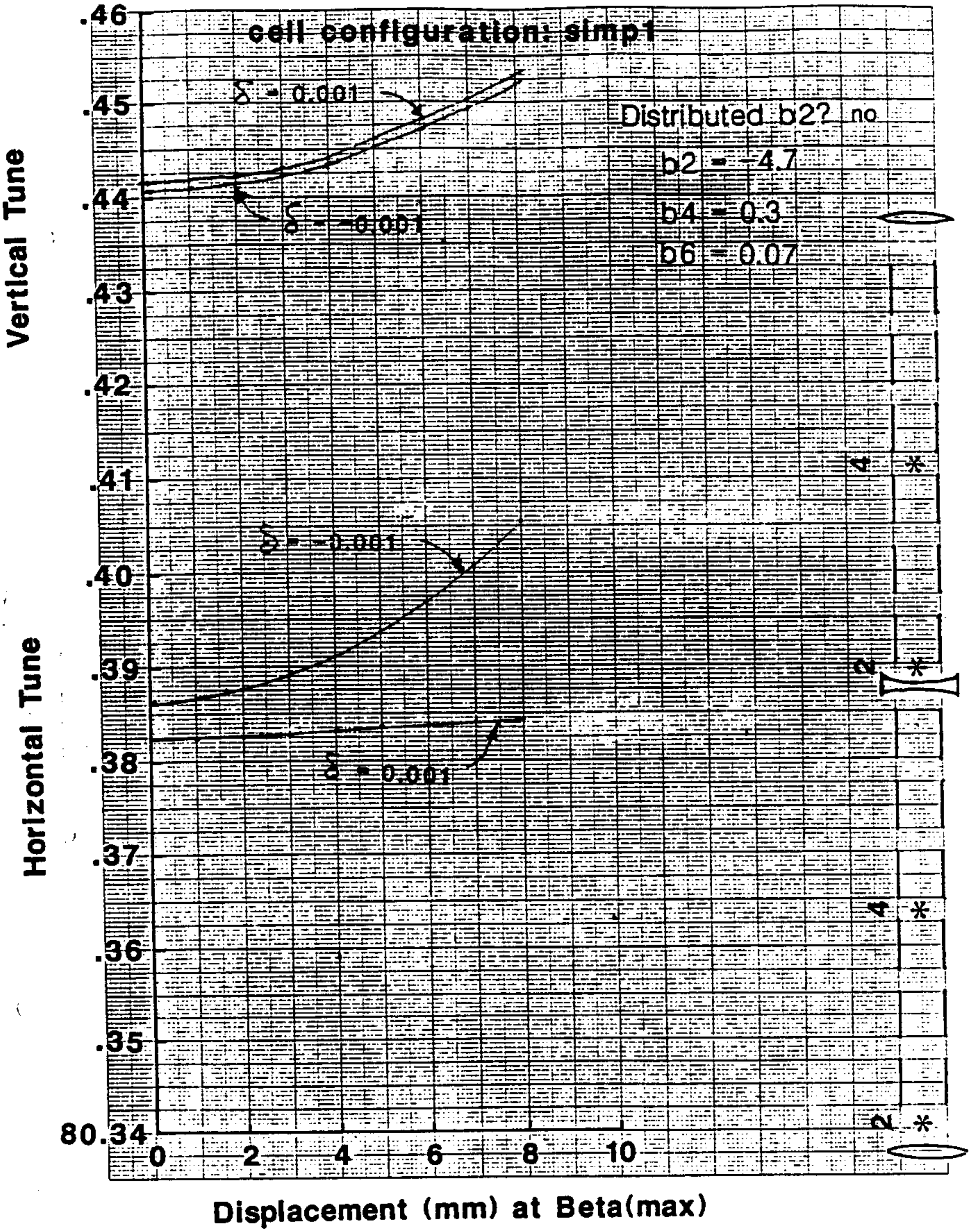


Figure 5

In these figures Q_x and Q_y are plotted as a function of amplitude, with $a_x = a_y$. The different possibilities can be assigned a figure-of-merit which is the factor by which the tune variation is superior to the specification. The values are listed in Table 2. The same information could be expressed as the factor by which the magnet error could be increased for the particular configuration, which would be expected to vary somewhere between linearly and as the square root of the figure-of-merit. Also shown in Table 2 is the number of spool-pieces needed per half-cell beyond those needed for steering; this may give an indication of the incremental cost.

From these data the following conclusions can be drawn:

(*i*) The scheme **l3** with three lumped correctors per half cell meets the requirements by a big factor. It is on this basis that the statement was made in the abstract that at most three lumps per half-cell are required. For the anticipated level of errors this almost certainly provides compensation as good as can realistically be expected with fully distributed correction.

(*ii*) The scheme labeled **l2lc** meets the required specifications with only one extra spool-piece for every two half-cells. This is the cheapest of the possibly satisfactory schemes.

(*iii*) The scheme labeled **l2l2p** is almost twice as good as the other schemes having one extra spool-piece per half-cell. Although I can see no *a priori* intuitively persuasive reason why this is a good configuration, I will stress it as being the most promising in what follows. It has the potential defect that the steering elements are not coincident with the beam position monitors (which presumably stay at the quadrupole locations,) but this has been shown not to cause any difficulty for the orbit-closing algorithm of **teapot**.

(*iv*) The "Simpson's Rule" scheme labeled **simp2** almost meets the specification. This should be regarded as being in reasonable agreement with the calculation of Neuffer[1] since the present calculation differs from his in minor ways.

(v) Also in agreement with Neuffer is the result that the correctors on either side of the quadrupole in the Simpson's Rule scheme can be combined into a single element on one side. (See **simp1**.)

To this point only the case of equal transverse amplitudes has been considered. This has reasonably been expected to be the "worst case" in most CDR investigations, but it seems sensible to analyse also cases where x and y are separately large to guard against being fooled by a conspiracy between different multipoles which just happens to look good with $a_x = a_y$. The systematic behavior of various configurations is shown in Tables 3 to 8, using a data format which has been described previously. Remember that the units are such that the numbers (or rather, in the case of ΔQ, the difference between the most positive and the most negative entry) must not exceed 10. The following comments can be made:

(i) Table 3 shows the behavior of the pure lattice with no errors. As is true for all data in this report, both chromaticities have been set to zero, in this case using the two sextupoles closest to the quadrupoles in the **l2l2p** scheme. The largest entry is 0.7 which is the basis for the statement made above that the sextupole strengths needed for this can be neglected in these considerations. (Irreverent aside: this may suggest, since stronger chromaticity sextupoles can be tolerated, that the machine integer tunes should be higher, thereby reducing the sensitivity to magnet errors.)

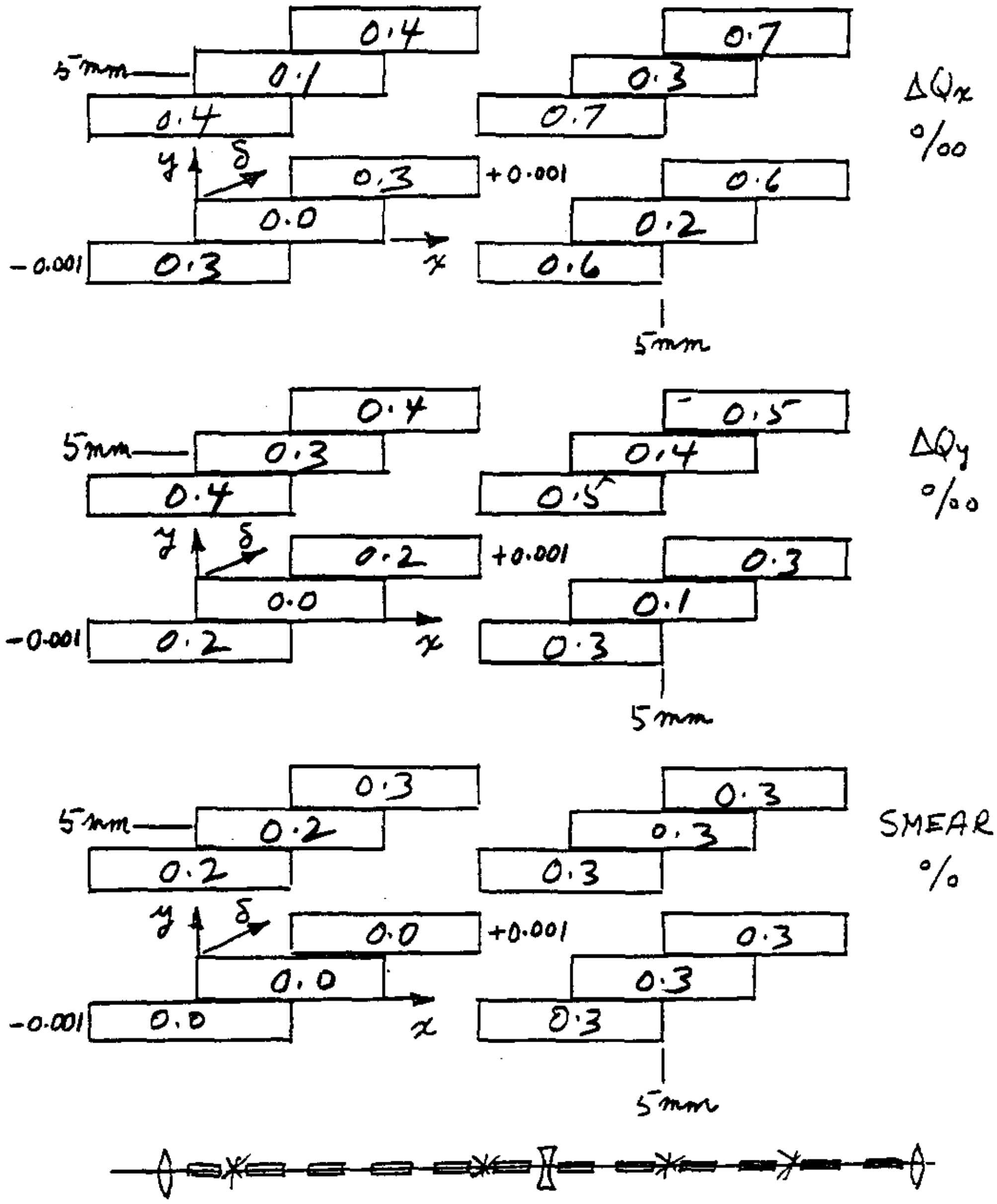
(ii) Table 4 shows the behaviour after compensation in the **l2l2p** scheme with all of b_2, b_4, and b_6 systematic errors present and Table 5 shows the same thing with only b_2 errors assumed. Since the numbers are noticeably different at least b_4 cannot be neglected, though b_2 is dominant.

(iii) Table 6 shows the behavior of the **l2l2p** scheme after the strengths of two octupoles, situated at the two quadrupole locations, have been optimized (by minimizing the maximum entry in tables such as these.) The largest entry is 5.6 which may not be a big enough improvement over 7.9, the biggest entry with no octupoles, to justify the complication of incorporating such octupoles.

(*iv*) Table 7 shows the same data for the **simp1** Simpson's Rule scheme. It fails by a factor 1.37 to meet the specification.

(*v*) A variant of this has the lumped correctors in the same locations as for **simp1** but uses uniform (3,3) weighting for the b_2 multipole. The Simpson's Rule (2,4) weighting is used for the other multipoles. The results don't differ noticeably from those for **simp1**.

PURE LATTICE	SYSTEMATIC	COMP?	RANDOM	COMP?
NO ERRORS	$b_2 = 0$		$\sigma_{b2} = 0$	
CHROMATICITIES	$b_4 = 0$		$\sigma_{b3} = 0$	
SET TO ZERO	$b_6 = 0$		$\sigma_{b4} = 0$	



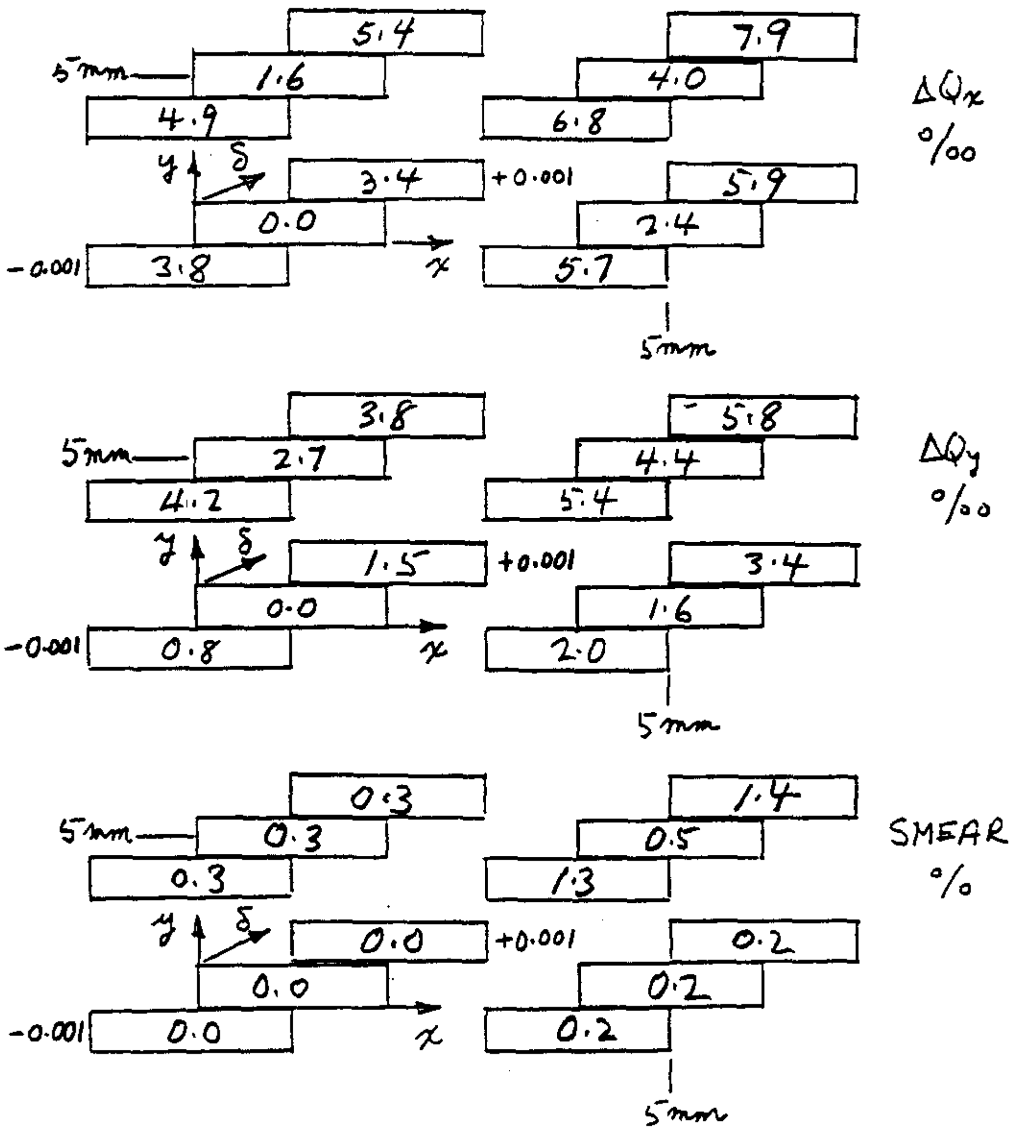
cell configuration: I2I2p but no errors

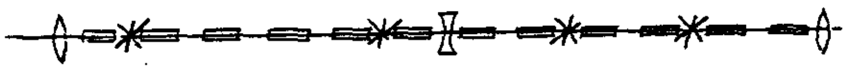
Table 3

$\boxed{1*4*1,\ 2*2*2}$

SYSTEMATIC

SYSTEMATIC	COMP?	RANDOM	COMP?
$b_2 = -4.7$	✓	$\sigma_{b2} = 0$	
$b_4 = 0.3$	✓	$\sigma_{b3} = 0$	
$b_6 = 0.07$	✓	$\sigma_{b4} = 0$	





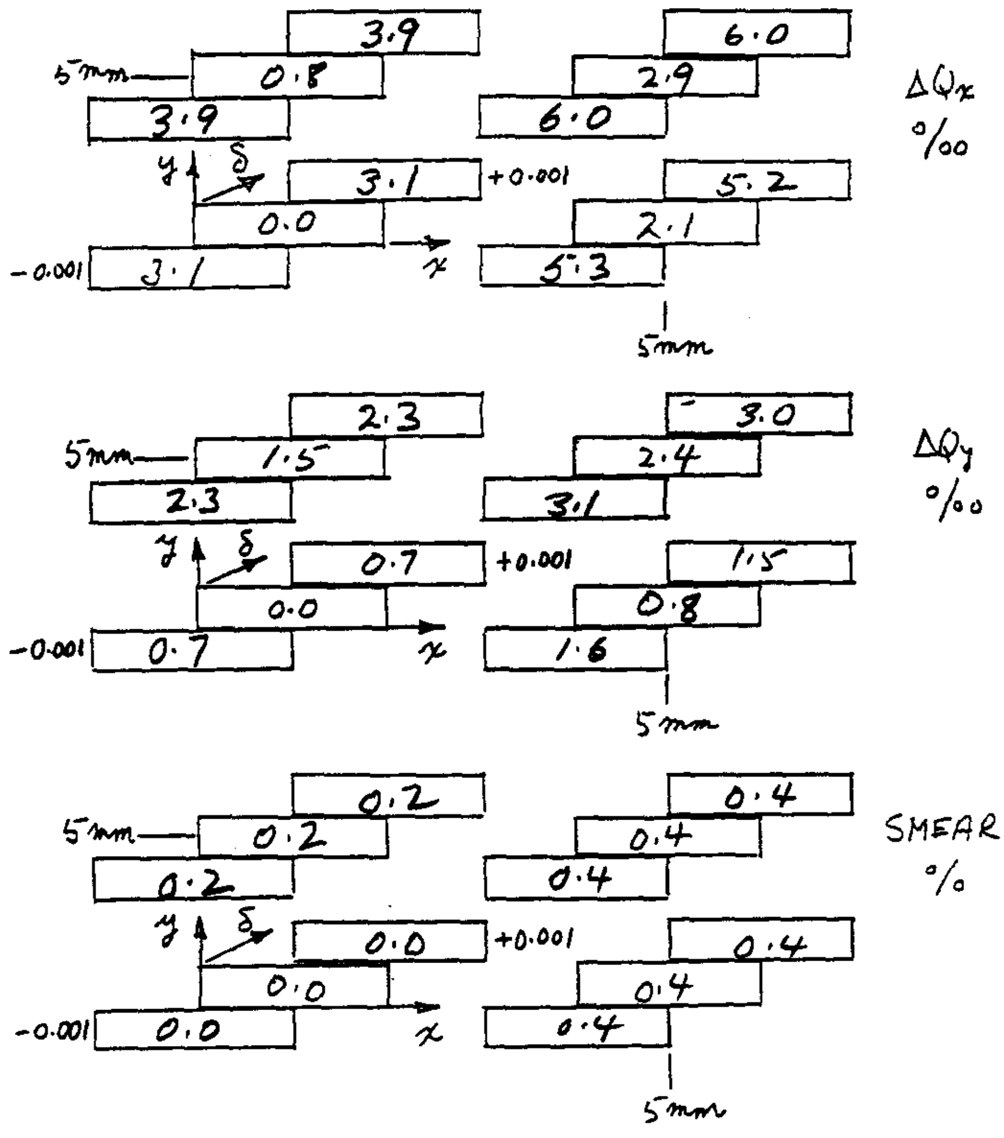
cell configuration: l2l2p

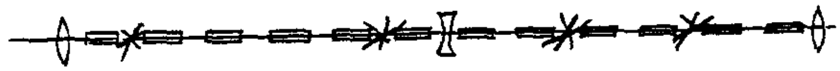
Table 4

$\boxed{1*4*1,\ 2*2*2}$

SYSTEMATIC
b_2 ONLY

SYSTEMATIC	COMP?	RANDOM	COMP?
$b_2 = -4.7$	✓	$\sigma_{b2} = 0$	
$b_4 = 0$		$\sigma_{b3} = 0$	
$b_6 = 0$		$\sigma_{b4} = 0$	





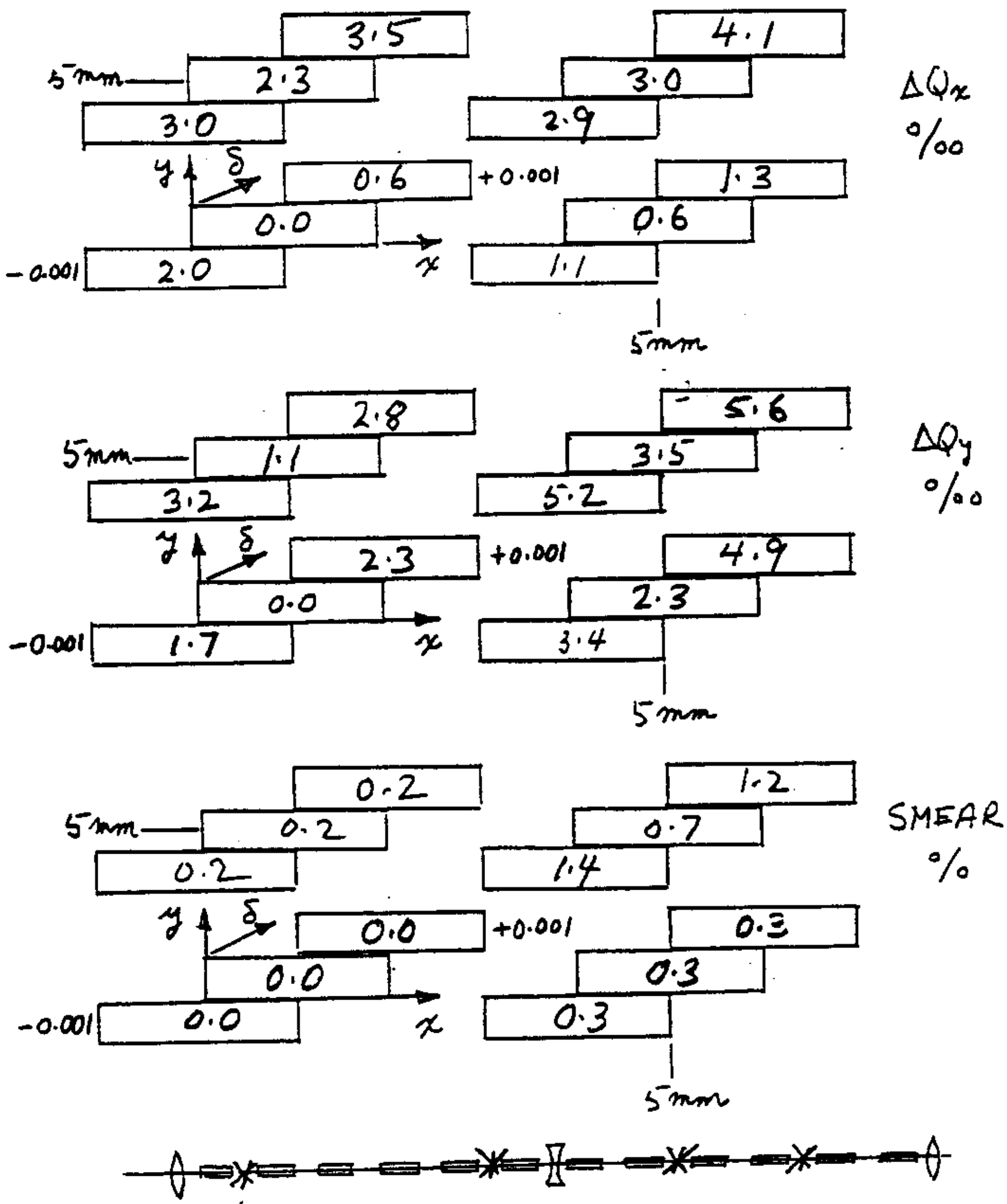
cell configuration: l2l2p with b2 only

Table 5

$\boxed{1*4*1, 2*2*2}$

SYSTEMATIC WITH OCTUPOLE

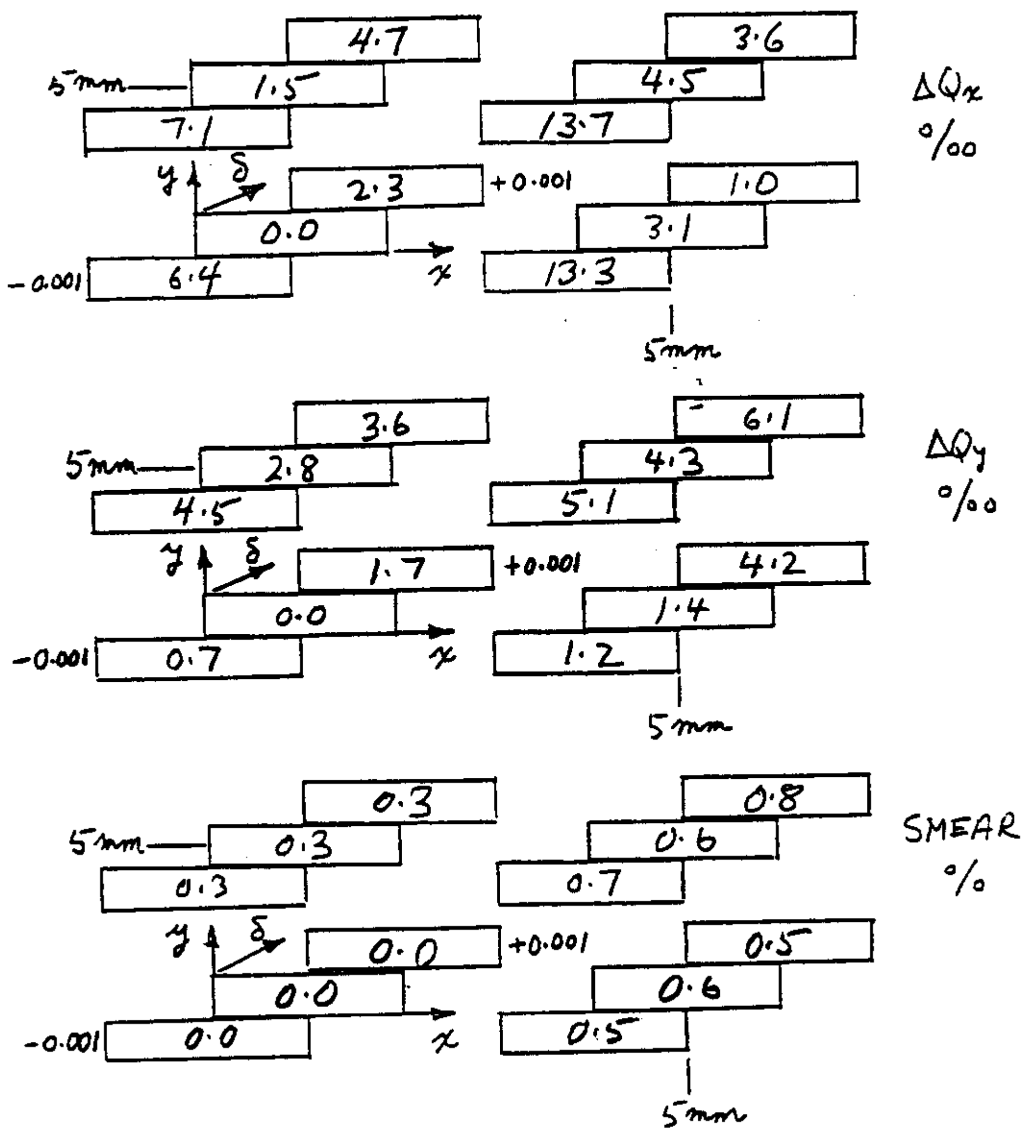
SYSTEMATIC	COMP?	RANDOM	COMP?
$b_2 = -4.7$	✓	$\sigma_{b2} = 0$	
$b_4 = 0.3$	✓	$\sigma_{b3} = 0$	
$b_6 = 0.07$	✓	$\sigma_{b4} = 0$	
$b_3 =$	optimal		

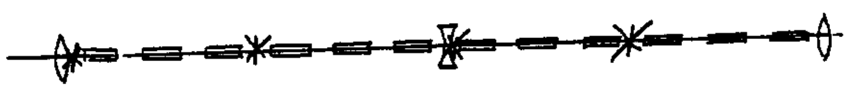


cell configuration: l2l2p with optimal octupole

Table 6

*3*3, *3*3	SYSTEMATIC	COMP?	RANDOM	COMP?
SIMPSON RULE	$b_2 = -4.7$	✓	$\sigma_{b2} = 0$	
ONE END	$b_4 = 0.3$	✓	$\sigma_{b3} = 0$	
SYSTEMATIC	$b_6 = 0.07$	✓	$\sigma_{b4} = 0$	





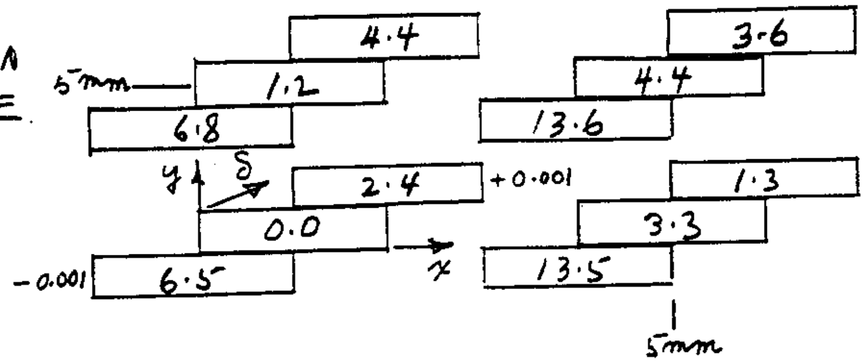
cell configuration: simp1

Table 7

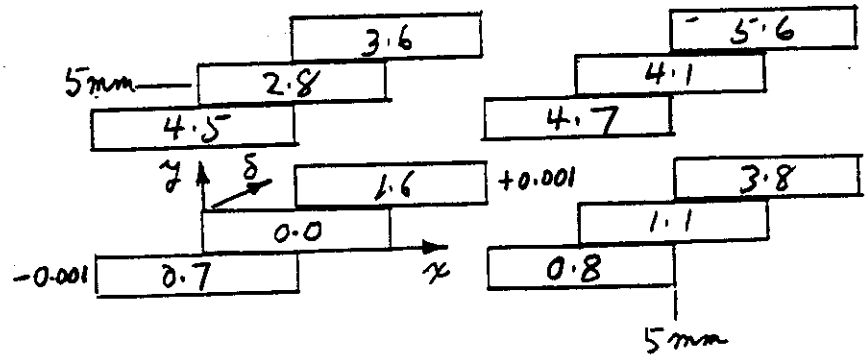
*3*3, *3*3	SYSTEMATIC	COMP?	RANDOM	COMP?
SIMPSON RULE	$b_2 = -4.7$	UNIF	$\sigma_{b2} =$	
ONE END	$b_4 = 0.3$	SIMP	$\sigma_{b3} =$	
SYSTEMATIC	$b_6 = 0.07$	SIMP	$\sigma_{b4} =$	

b_2 COMPENSATION UNIFORM.

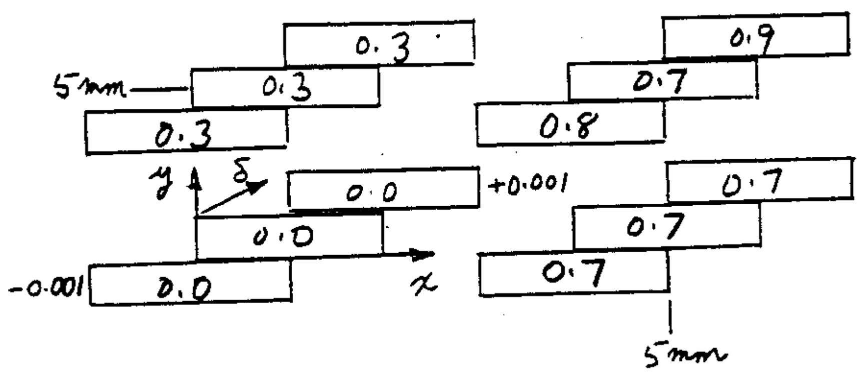
$b_4 + b_6$ SIMPSON RULE.



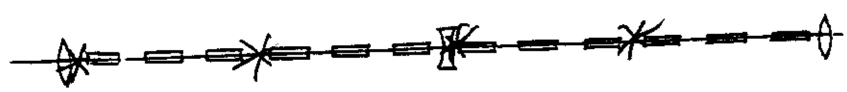
ΔQ_x ‰



ΔQ_y ‰



SMEAR %



cell configuration: simp1 but b2 uniform

Table 8

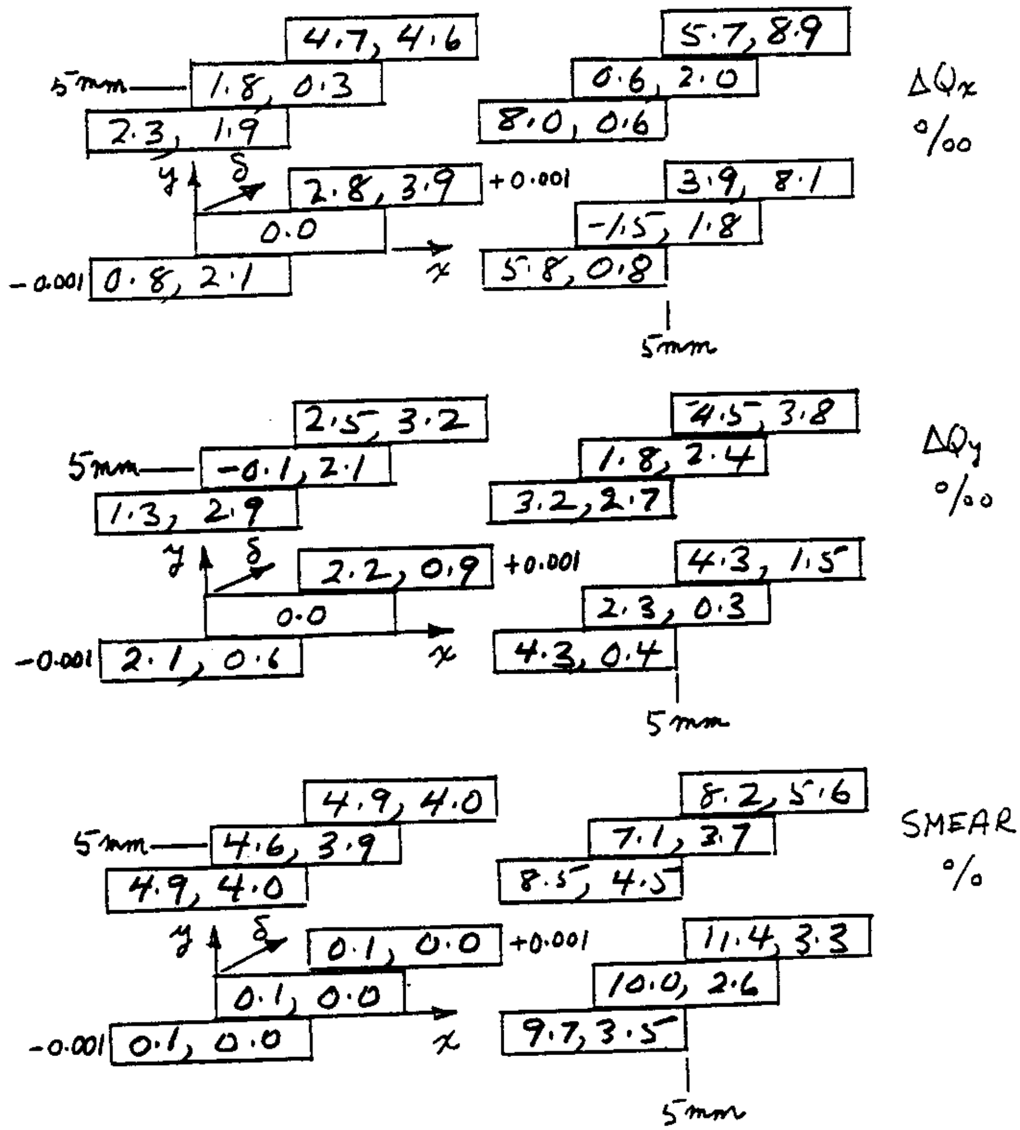
4. Use of the Lumped Correctors to Compensate for Random Dipole Errors.

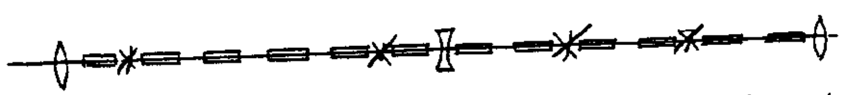
One remaining question is the degree to which the lumped correctors, which have so far been assumed to be present only to compensate systematic errors, can also be used to compensate random errors. Table 9 shows, for two different random seeds, the result of adding random errors to the dipoles. The rms errors used have been given above. Table 10 shows the results after compensation of all the random multipoles, for the same two random seeds. Again the most simple-minded scheme is used, with each lumped corrector being set to the negative of the sum of the multipole errors of the three neighboring dipoles in the same half-cell. In the worst case the smear is reduced below 3 percent which has to be regarded as highly satisfactory. With the binning scheme of compensation[2] the lumped correctors would not be set with quite the accuracy which has been assumed here, so the smear improvement would be almost, bit not quite as great as has been obtained here. A first-class and possibly not too extravagant improvement would be to use lumped correctors of the design given in[4] with each sextupole powered individually.

$\boxed{1*4*1,\ 2*2*2}$

RANDOM b_2, b_3, b_4
UNCOMPENSATED
2 SEEDS

SYSTEMATIC	COMP?	RANDOM	COMP?
$b_2 = -4.7$	✓	$\sigma_{b2} = 2.0$	No
$b_4 = 0.3$	✓	$\sigma_{b3} = 0.35$	NO
$b_6 = 0.07$	✓	$\sigma_{b4} = 0.59$	NO



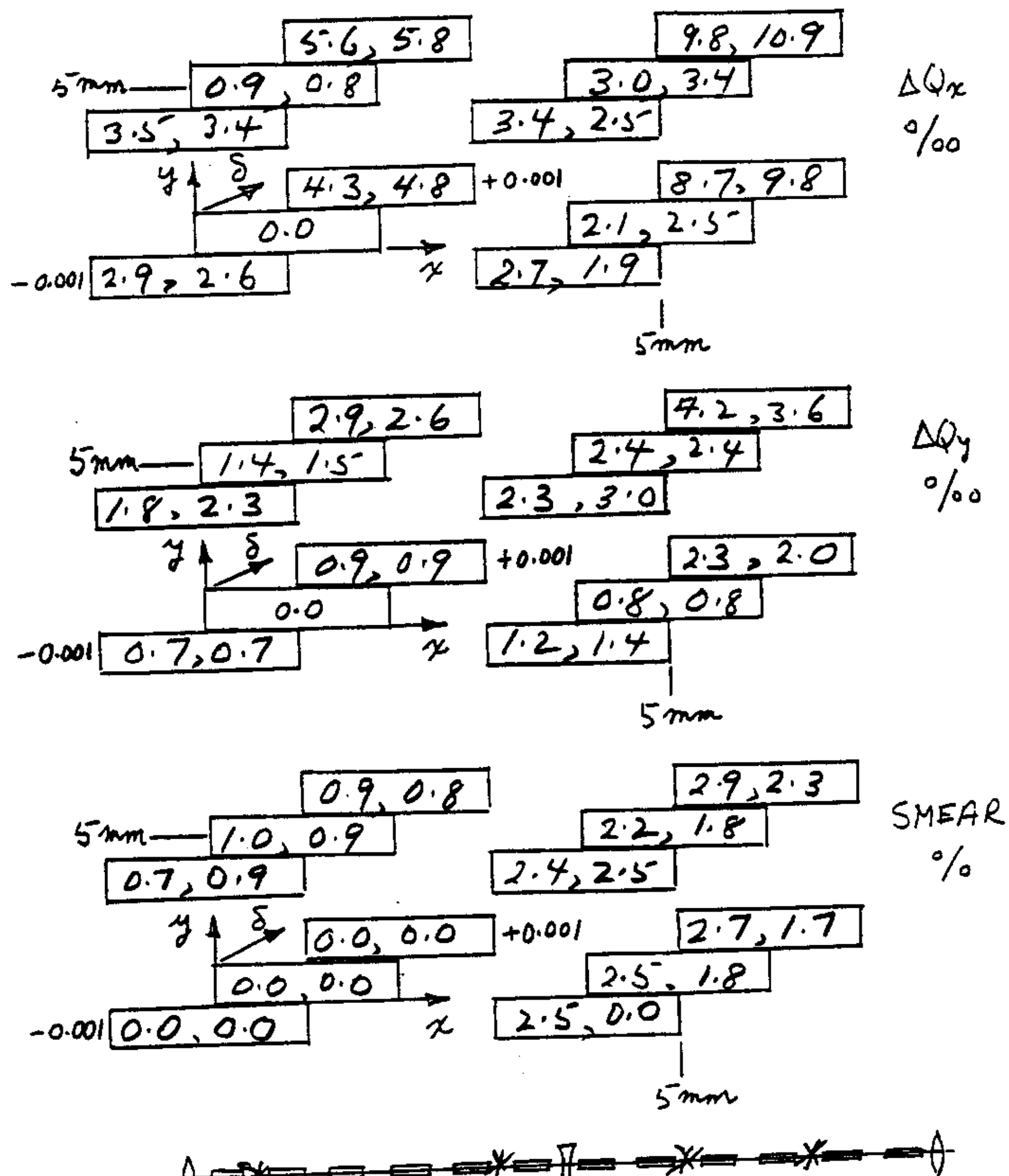


cell configuration: 1212p with uncompensated randoms
Table 9

$\boxed{1*4*1, 2*2*2}$

RANDOM $b_2 b_3 b_4$
COMPENSATED
2 SEEDS

SYSTEMATIC	COMP?	RANDOM	COMP?
$b_2 = -4.7$	✓	$\sigma_{b2} = 2.0$	✓
$b_4 = 0.3$	✓	$\sigma_{b3} = 0.35$	✓
$b_6 = 0.07$	✓	$\sigma_{b4} = 0.59$	✓



cell configuration: 1212p with compensated randoms

Table 10