SSC-N-412

A POSSIBLE MODIFIED LATTICE FOR THE LOW ENERGY BOOSTER

December 1987

L. K. Chen

Shenzhen University, Peoples Republic of China
and
SSC Central Design Group*
c/o Lawrence Berkeley Laboratory, Berkeley, California 94720

---

* Operated by Universities Research Association for the U.S. Department of Energy

INTRODUCTION

The lattice for the Low Energy Booster (LEB), described in the CDR[1] and shown in Fig. 1, has an outstanding advantage in that it has a high transition gamma. Because of this, we can avoid passing through transition in order to minimize beam loss and dilution of longitudinal emittance. But the price paid for raising the transition energy is a high maximum dispersion of 10 meters. In particular, the dispersion values in the long straight sections where the rf cavities, injection, and extraction would be arranged are too large — 5–10 meters — instead of the vanishing dispersion conventionally accepted as optimal.

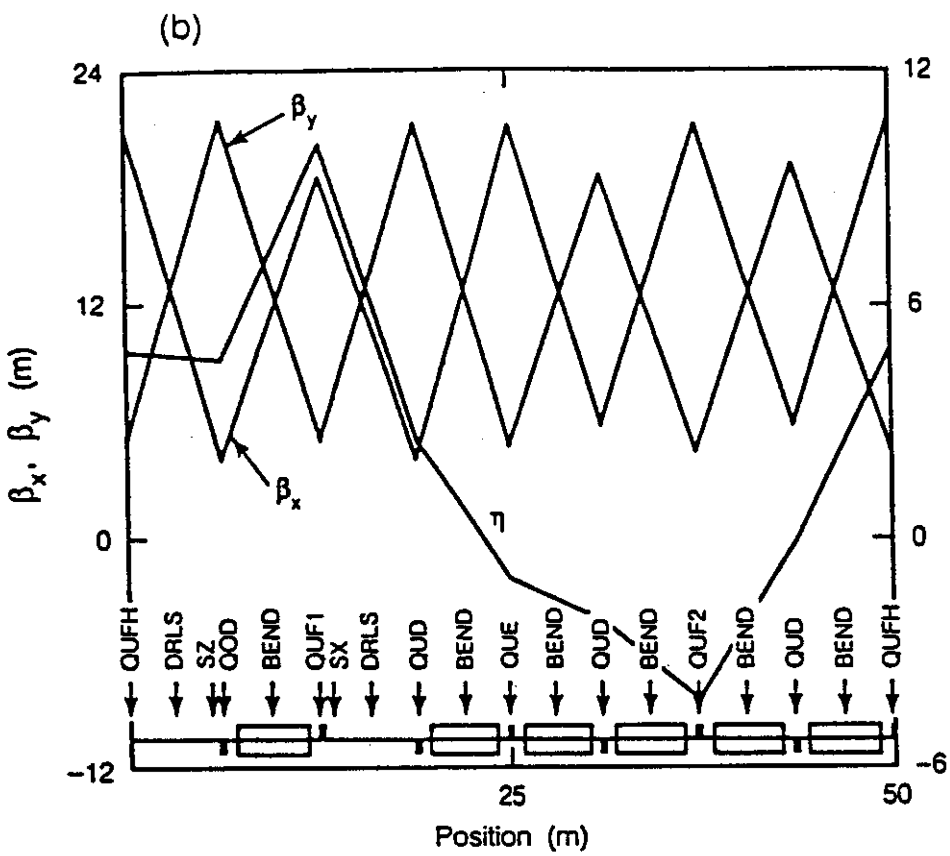

FIG. 1. The CDR lattice for the LEB.

Can we find a lattice for the LEB with both high gamma and small dispersion, including a zero-dispersion in the long straight section? Such lattices are possible. A typical lattice is shown in Fig. 2; the parameters are listed in Table I.

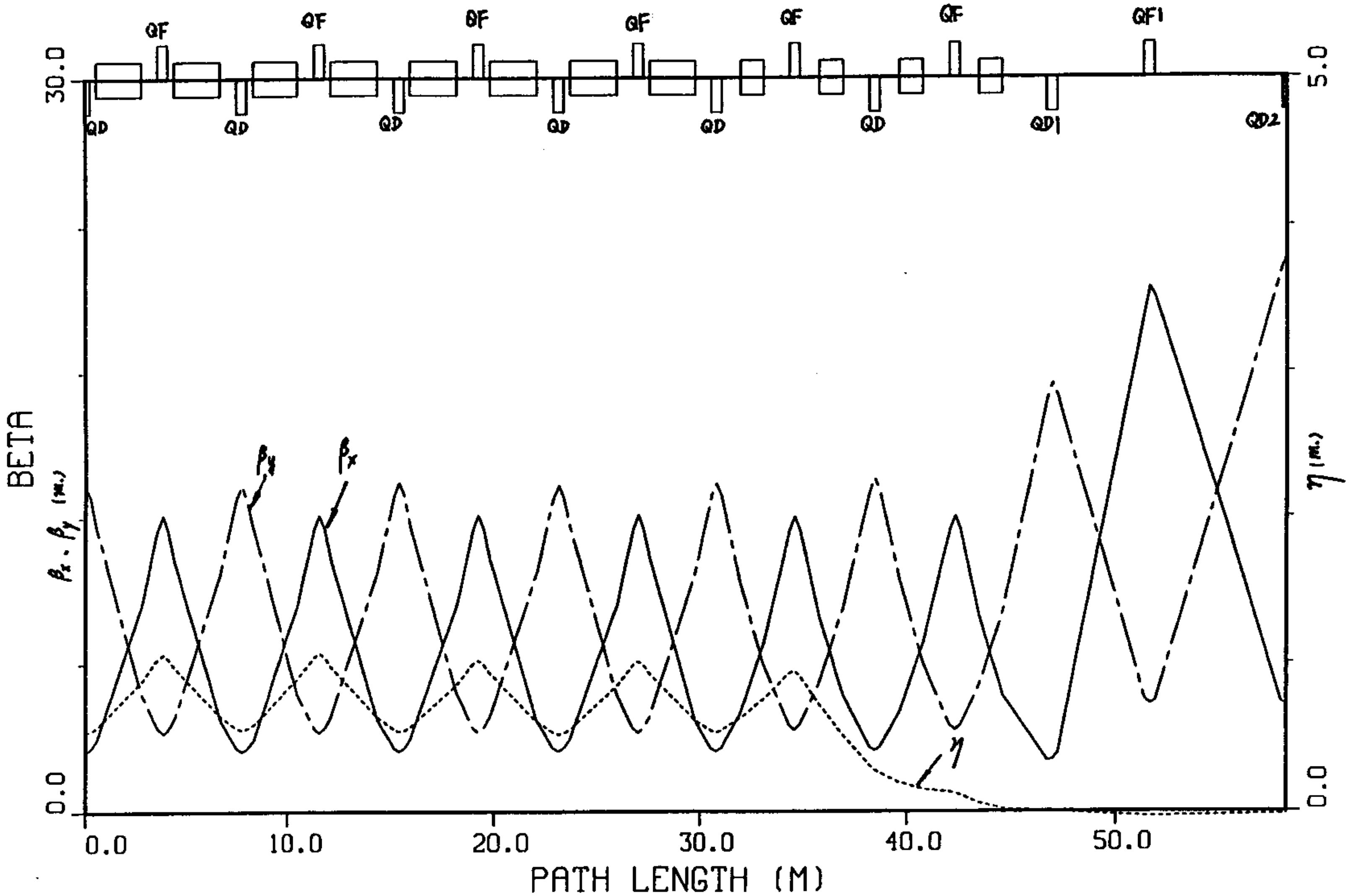

FIG. 2. The PML lattice for the LEB.

TABLE I. Low Energy Booster Parameters

	CDR	PML (Possible Modified Lattice)	
Injection momentum	1.22 GeV/c	1.22 GeV/c	
Extraction momentum	8.0 GeV/c	8.0 GeV/c	
Circumference	249.6 m	341.8 m	
Number of bunches	52	71	
Protons/bunch	1.0×10^{10}	1.0×10^{10}	
Circulating current	99 mA	99 mA	
Normalized transverse emittance (rms)	0.75 mm-mrad	0.75 mm-mrad	
Longitudinal emittance (rms area/π)	0.0016 eV-s	0.0016 eV-s	
Horizontal tune	4.39	10.83	
Vertical tune	4.41	8.42	
Transition gamma	10.5	9.17	
Natural chromaticity			
(H)	–5.2	–12.6	
(V)	–4.9	–10.8	
Lattice type	FODO	FODO	
Superperiodicity	5	3	
Maximum beta (arcs)	21.5 m	24.0 m	
maximum dispersion	10.1 m	1.0 m	
Number of dipoles	30	48 (B)	24 (BB)
Dipole length	4.5 m	2.25 m	1.125 m
Dipole field (max)	1.24 T	1.24 T	1.24 T
Number of standard quadrupoles	40	84	
Standard quad length	0.3 m	0.5 m	
Standard quad strength (max)	18.4 T/m	20.5 T/m	
rf frequency (injection)	49.5 MHz	49.5 MHz	
rf frequency (extraction)	62.0 MHz	62.0 MHz	
rf voltage	350 kV	350 kV	

ESTIMATE OF THE THRESHOLDS OF SINGLE-BUNCH INSTABILITIES

Among the single-bunch instabilities, the dangerous ones are the transverse mode-coupling, transverse microwave, longitudinal microwave, and longitudinal coupling instabilities. We can calculate the thresholds of the instabilities.

1. Transverse Mode-Coupling Instability

This instability arises when the real frequency shift of any mode becomes equal to the synchrotron frequency. Assuming that the largest shift is due to mode $\mu = 0$, the limit on $Z_\perp$ is[2]

$$(\overline{Z_\perp})_{th} \leq \frac{4\sqrt{\pi}\,\eta\,(\frac{E}{e})\,(\frac{\sigma_E}{E})}{I\bar{\beta}} \tag{1}$$

where

$$(\overline{Z_\perp}) = \frac{\sigma_z}{c\sqrt{\pi}} \int_{-\infty}^{\infty} I_m Z_\perp(\omega)\, e^{-(\omega\frac{\sigma_z}{c})^2}\, d\omega$$

for a Gaussian bunch. In Eq. (1), η is the frequency-slip factor, $\eta = \gamma^{-2} - \gamma_t^{-2}$, $\frac{\sigma_E}{E}$ is the rms energy spread of the beam, $\bar{\beta}$ is the average betatron function, and I is the average single-bunch current.

For our case of LEB, when $\gamma \to \gamma_t$, η decreases rapidly, so $(Z_\perp)_{th}$ reaches its minimum at extraction. Using $E = 8.06$ GeV, $\bar{\beta} = 10$ m, $I = 2$ mA,

$$\frac{\sigma_E}{E} = \begin{cases} 4.57 \times 10^{-4} & \text{(CDR)} \\ 5.39 \times 10^{-4} & \text{(PML)} \end{cases} \quad \text{and} \quad \eta = \begin{cases} 4.48 \times 10^{-3} & \text{(CDR)} \\ 1.66 \times 10^{-3} & \text{(PML)} \end{cases}$$

in Eq. (1), we get for the threshold $(Z_\perp)_{th} = \begin{cases} 5.84\ \text{M}\Omega/\text{m} & \text{(CDR)} \\ 2.55\ \text{M}\Omega/\text{m} & \text{(PML)} \end{cases}$

Assuming $Z_\perp = \frac{2R}{b^2}\frac{Z_{||}}{n}$, $b = 0.04$ m, $R = 40$ m (CDR), and $R = 55$ m (PML), we get the threshold of $\frac{Z_{||}}{n}$,

$$(\frac{Z_{||}}{n})_{th} = \begin{cases} 117\ \Omega & \text{(CDR)} \\ 37\ \Omega & \text{(PML)} \end{cases}$$

2. Transverse Microwave Instability

This instability has a rise time faster than the synchrotron period and is driven by disturbances of wavelengths much shorter than the bunch length. According to Bane and Ruth,[3] the impedance limit is

$$\left|Z_\perp\right| \le \frac{4\eta\,(\frac{E}{e})\,(\frac{\sigma_p}{p})\,\sigma_\tau\,\omega_\gamma}{I\,\bar{\beta}} \tag{2}$$

Noting $\sigma_\tau = \sigma_{z/c}$, and $\sigma_p\,\sigma_z = \epsilon_L$, we get

$$\left|Z_\perp\right| \le \frac{4\eta\,\epsilon_L\,\omega_\gamma}{e\beta\,\bar{\beta}I} \tag{3}$$

Assuming ω_γ is given by the cut-off frequency of the beam tube

$$\omega_\gamma = \frac{3.83c}{b} = 28.7\ \mathrm{GHz},$$

$\epsilon_L = 1.6\times10^{-3}$ eV-s, $\beta = .993$, $\bar{\beta} = 10$ m, we get the threshold of the transverse microwave instability

$$\left|Z_\perp(\omega_\gamma)\right|_{th} = \begin{cases} 41\ \mathrm{M\Omega/m} & \text{(CDR)} \\ 15\ \mathrm{M\Omega/m} & \text{(PML)} \end{cases} .$$

Using the relational formula $Z_\perp = \frac{2R}{b^2}\frac{Z_{||}}{n}$, we get the threshold for transverse microwave instability

$$\left|\frac{Z_\perp}{n}\right|_{th} = \begin{cases} 820\ \Omega & \text{(CDR)} \\ 218\ \Omega & \text{(PML)} \end{cases} .$$

3. Longitudinal Microwave Instability

According to Keil and Schnell[4,5]

$$\left|\frac{Z_{||}}{n}\right| \le F\,\frac{E\eta}{e}\,\frac{(\frac{\sigma_E}{E})^2}{I_p} = F\,\frac{\sqrt{2\pi}\ \eta\ (\frac{E}{e})\ (\frac{\sigma_z}{R})\ (\frac{\sigma_E}{E})^2}{2\pi I} , \tag{4}$$

where F is a form factor, for a Gaussian bunch $F = 2\pi$, I_p the peak current, and I the average current. Thus

$$\left|\frac{Z_{||}}{n}\right| \le \frac{\sqrt{2\pi}\ \eta\ (\frac{E}{e})\ (\frac{\sigma_z}{R})\ (\frac{\sigma_E}{E})^2}{I} \quad . \tag{5}$$

The results are

$$\left|\frac{Z_{||}}{n}\right|_{th} = \begin{cases} 32\ \Omega & \text{(CDR)} \\ 14\ \Omega & \text{(PML)} \end{cases} \quad (\text{for } eV_0 = 80\ \text{KeV})$$

$$= \begin{cases} 46\ \Omega & \text{(CDR)} \\ 20\ \Omega & \text{(PML)} \end{cases} \quad (\text{for } eV_0 = 350\ \text{KeV}) \quad .$$

4. Longitudinal Mode-Coupling Instability

The longitudinal mode-coupling instability occurs when two modes meet as the real frequencies shift. The stability limit for $\frac{Z_{||}}{n}$ is given by[2]

$$I_m\ \frac{Z_{||}}{n} \le \frac{8\sqrt{\pi}\ \eta\ (\frac{E}{e})\ (\frac{\sigma_z}{R})\ (\frac{\sigma_E}{E})^2}{I} = \begin{cases} 180\ \Omega & \text{(CDR)} \\ 78\ \Omega & \text{(PML)} \end{cases}$$

ESTIMATION OF LINEAR SPACE-CHARGE TUNE SHIFT

The linear space-charge tune shifts at injection for the CDR have been calculated by Furman and Peterson.[6] We can now estimate the linear space-charge tune shifts for PML. According to Furman and Peterson,

$$\Delta\nu_x = -\frac{r_0\, N_B\ C\ \bar{\beta}}{2\pi\, \beta^2\, \gamma^3\, \overline{\sigma_x}\, (\overline{\sigma_x} + \overline{\sigma_y})\, \sqrt{2\pi}\, \sigma_z} \quad . \tag{6}$$

Where β and γ are normal relativisitic factors, $r_0 = 1.536 \times 10^{-18}$ m the classical radius of the proton, $N_B = 1 \times 10^{10}$ the number of particles per bunch, C the circumference of the ring, $\bar{\beta}$ the average beta-function, $\overline{\sigma_x}$ and $\overline{\sigma_y}$ the transverse rms bunch widths averaged over the ring, and σ_z the rms bunch length in the lab frame.

The formula for $\Delta\nu_y$ is similar to Eq. (6) (with $\overline{\sigma_x} \longleftrightarrow \overline{\sigma_y}$). The rms widths are given by

$$\overline{\sigma_x} = \sqrt{\bar{\beta}\epsilon + \overline{\eta^2}\,(\frac{\sigma_p}{p})^2}\ ,\ \overline{\sigma_y} = \sqrt{\bar{\beta}\epsilon}\quad . \tag{7}$$

where $\overline{\eta^2}$ is the averaged square of the dispersion function and $\epsilon = \frac{\epsilon_N}{\beta\gamma}$ is the emittance (we assume $\epsilon_x = \epsilon_y = \epsilon$).

For PML of the LEB, $\overline{\eta^2}$ is very small, about 0.2 m^2, so $\overline{\eta^2}\,(\frac{\sigma_p}{p})^2 << \bar{\beta}\epsilon$. Therefore,

$$\overline{\sigma_x} = \sqrt{\bar{\beta}\epsilon} = \overline{\sigma_y}\ ,$$

and the tune shifts become

$$\Delta\nu_x = \Delta\nu_y = \frac{r_0\, N_B\, C}{4\pi\sqrt{2\pi}\,\beta\,\gamma^2\,\epsilon_N\,\sigma_z} \tag{8}$$

where

$$\frac{\sigma_z}{C} = \sqrt{\frac{1}{\beta}\,(\frac{\epsilon_L}{ET})\sqrt{\frac{|\gamma^{-2} - \gamma_t^{-2}|\,E}{2\pi h\ eV_0\ \cos\varphi_s}}} \tag{9}$$

For PML, we hope to keep eV_0 the same as the CDR in order to maintain the number of rf cavities. If we assume $eV_0 = 350$ KeV at injection, and $\varphi_s = 0$, we get $\Delta\nu_x = \Delta\nu_y = -0.25$.

LONGITUDINAL ACCEPTANCE AT INJECTION FOR LEB

According to Bovet,[7] the bucket half height is

$$\Delta E = \sqrt{\frac{eV_0\,E}{\pi h\eta}}\ Y(\varphi_s)\,\beta \tag{10}$$

where V_0 is the total accelerating voltage around the ring, h the harmonic number, η the frequency split factor , and $Y(\varphi_s)$ the special function shown in Ref. 7.

For the LEB, when $\gamma \rightarrow \gamma_t$, η decreases rapidly, so ΔE reaches its minimum at injection. The momentum spread of the beam, σ_p, is given by[6]

$$\sigma_p = \frac{\epsilon_L}{\sigma_z} = \frac{\epsilon_L}{C}\sqrt{\frac{\beta ET}{\epsilon_L}}\ \sqrt[4]{\frac{2\pi heV_0}{E\eta}}\quad . \tag{11}$$

Here C is the circumference of the ring. The energy spread of the beam, σ_E, is given by $\sigma_E = \beta c \sigma_p$, with c the velocity of light.

The data are as given below

Case		Longitudinal Acceptance ΔE (MeV)	Energy Spread σ_E (MeV)	$\frac{\Delta E}{\sigma_E}$
CDR	$V_0 = 200$ kV	2.53	0.80	3.16
	$V_0 = 350$ kV	3.35	0.92	3.64
PML	$V_0 = 200$ kV	2.14	0.73	2.93
	$V_0 = 350$ kV	2.83	0.84	3.37

DISCUSSION

1. An alternative LEB lattice has been designed. It has a circumference of 342 m instead of the 250 m in the CDR. The linear optics, however, are believed to be much improved from the CDR.
2. It appears that the thresholds of single bunch instabilities are high. It is not very difficult to keep the $|\frac{Z_{||}}{n}|$ of the LEB at about 10 Ω. If $|\frac{Z_{||}}{n}|$ could be no more than 14 Ω, the single bunch instabilities are not of serious concern.
3. The linear space-charge tune shift is larger than that of the CDR. This effect will have to be studied further by particle tracking.
4. We suggest that the accelerating voltage around the ring be kept at 350 kV from injection to extraction in order to increase both the longitudinal acceptance at injection and the thresholds of beam instabilities at extraction. The particle ramping time then increases according to the ratio $\frac{C}{C_0}$, where C_0 is the circumference of the CDR LEB and C is the circumference of the PML LEB.

ACKNOWLEDGMENTS

I would like to thank A. Chao and D. Johnson for their helpful discussions with me. I would also like to thank A. Garren for his help, especially his suggestions relating to the configuration of the dispersion suppressor.

REFERENCES

[1] *Conceptual Design of the Superconducting Super Collider,* SSC-SR-2020, edited by J. D. Jackson (March 1986).

[2] K. Y. Ng, "An Estimate of the Contributions of Bellows to the Impedances and Beam Instabilities of the SSC," in *Report of the SSC Impedance Workshop,* SSC-SR-1017, edited by J. Bisognano (October 1985).

[3] K. Bane and R. D. Ruth, "Bellows Wake Fields and Transverse Single Bunch Instabilities in the SSC," in *Report of the SSC Impedance Workshop,* SSC-SR-1017, edited by J. Bisognano (October 1985).

[4] E. Keil and W. Schnell, CERN Report ISR-TH-RF/69-48 (1969).

[5] A. Hofmann, "Beam Stability in the VUV Ring, the X-ray Ring and the Booster of the NSLS, BNL," BNL-25465 (October 1978).

[6] M. Furman and J. Peterson, "Linear Space-Charge Tune Shifts for the SSC and its Injectors," SSC-N-403 (November 1987).

[7] C. Bovet, et al, "A Selection of Formulae and Data Useful for the Design of A. G. Synchrotrons," CERN/MPS-SI/Int. DL/70/4 (April 1970).