Resonances Due to Field Errors in the Dipole Magnet Ends

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Field error in the dipole magnet-ends, like other field errors, is a source of harmful nonlinearities. One criterion in setting its tolerance is that the strengths of the nonlinear resonances excited by magnet-ends are within a tolerable limit. In this note, we present a compilation of results of calculation on resonance strengths for various configurations of magnet-end field errors and running conditions. The hope is that these technical results can serve as part of the inputs to a determination of tolerances on the magnet-end field error.

The lattice model used in this study is a string of 320 cells, each slightly detuned to give total tunes of ν_x=80.265, ν_y=80.285. All dipoles are assumed to have b_2 of +1 "unit" in half of its body and -1 "unit" in the other half, simulating some degree of activation of a sextupole bore tube winding corrector, where "unit" is the usual 10^{-4} B_o at 1 cm radius. In all cases, chromaticities are corrected by the chromaticity sextupoles.

Results are given in tables 1 to 4 for $b_{2,3,4,5}$ type of magnet-end errors, respectively. The first column gives the integrated b_n over <u>each</u> magnet-end in "unit"-meters. Columns 2,3,4,5 are the sum of the strengths of all resonances (on-momentum) of orders 3,4,5,6 respectively. The strength of a particular resonance of order n is defined to be the amplitude of the corresponding coefficient in the Lie polynomial f_n when expanded in resonance basis. Terms giving rise to tune shifts with amplitudes are included as part of the sum. Resonance denomenators, which give rise to sensitive dependence on exact tune values, are not included in this strength definition. An estimate of the stopband widths can be obtained by multiplying the resonance strengths by $\varepsilon^{(n-2)/2}/\pi$ for the n-th order resonance, where ε is the emittance (in mm-mrad) under consideration.

For the b_3 and b_4 errors, we assume that a lumped correction has been applied. The lumped correctors are assumed to be located adjacent to each quadrupole and in the middle of all half cells. For the b_2 errors, the lumped correctors are just the chromaticity sextupoles. These corrections are crucial in supressing the resonances, as one would expect. We assume no corrections for the b_5 magnet-end errors, which explains the apparent sensitivity to the b_5 error in table 4.

One way to use the tables is to study the dependence of the resonance strengths as functions of the magnet-end field errors. Ideally, the magnet-ends are not going to make the resonances much stronger than the case with perfect magnet-ends. Tables 2,3,4 contain less number of rows than Table 1 because of the obvious scaling properties.

A second way is to compare these results with the resonance strengths due to the expected random errors in the magnet bodies (Tigner's suggestion). For this comparison, Table 5 gives the results for three random number seeds. In this context, the resonances due to magnet-ends are supposed to be much weaker than those due to random errors. The rms random multipoles used are the "expected" values (CDR): b_2(effective after sorting/binning) =0.4, a_2=0.61, b_3=0.35, a_3=0.69, b_4=0.59, a_4=0.14, b_5=0.059, a_5=0.16. Note that the 6-th order resonances in table 5 are relatively weak due to the fact that the assumed b_5 and a_5 errors are small. The last row of table 5 gives the rms total stopband widths for a particle executing 1 cm betatron oscillations.

The third way is to compute the stopband widths due to the resonances and consider certain tolerance on the stopbands. As an example, for a particle executing 1 cm oscillations, the resonance strengths corresponding to a 0.02 stopband width would be 0.12, 0.24, 0.48 and 0.94, respectively for 3-rd, 4-th, 5-th and 6-th order resonances.

Tables 6 to 9 are the results for the skew multipoles. No corrections are assumed, leading to the apparent larger resonance strengths (especially for a_3 and a_5) when compared with the corresponding normal multipoles. Splitting the x-and y-tunes by ±1 would help the situation. Tables 10 and 12 are the cases for b_5, a_3 and a_5 with ν_x=80.265, ν_y=81.285. One observes that splitting tunes does not help the b_5 case much, that it helps reducing the 4-th order resonances for the a_3 case, and that it helps the a_5 case substantially.

Some resonance effects are stronger off momentum than on-momentum. This is often the case for the even multipoles. Tables 13 to 15 are the results for b_4, a_2 and a_4 cases with $\delta=10^{-3}$ without tune spliting. Again, the b_4 case is lump corrected, while a_2 and a_4 cases are uncorrected.

Table 1.

integrated b2	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
-5.0	2.70	2.05	.0932	.0205
-9.9	3.59	2.38	.198	.0402
-16.5	4.85	5.83	.663	.170
-33.1	8.19	32.9	4.86	2.66
8.3	2.05	3.83	.151	.143
12.4	2.74	7.90	.385	.385
16.5	3.50	13.4	.808	.846

Table 2.

integrated b3	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
-6.6	1.91	30.8	1.26	2.97
-13.2	1.91	61.8	2.52	12.2
6.6	1.91	31.1	1.26	3.38
13.2	1.91	62.0	2.53	13.0
19.8	1.91	93.0	3.79	28.9

Table 3.

integrated b4	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
6.6	1.91	1.12	2.97	.904
13.2	1.91	1.12	5.93	1.81
-13.2	1.91	1.12	5.92	1.81

Table 4.

integrated b5	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
.165	1.91	1.12	.0351	115.6
.66	1.91	1.12	.0351	462.7

Table 5.

	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
seed # 1	26.1×10^{-2}	52.3×10^{-2}	87.5×10^{-2}	56.3×10^{-2}
seed # 2	22.6	41.9	83.1	49.5
seed # 3	20.0	39.6	101.	56.0
rms	23.0	44.9	90.9	54.0
stopband (ets.)	.037	.037	.038	.0115

Table 6.

integrated a2	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
1.65	2.86	3.57	.165	.0442
-1.65	2.86	3.57	.165	.0442
3.3	3.81	8.41	.554	.130
4.95	4.76	15.8	1.39	.397
6.6	5.71	25.3	2.86	1.03

Table 7.

integrated a3	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
1.65	1.91	292.	6.98	125.
3.3	1.91	582.	13.9	497.
-1.65	1.91	292.	6.98	125.
-3.3	1.91	582.	13.9	497.

Table 8.

integrated a4	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
1.65	1.91	1.12	3.77	6.98
3.3	1.91	1.12	7.50	13.9
-3.3	1.91	1.12	7.50	13.9

Table 9.

integrated a5	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0351 \times 10^{-2}$	$.0157 \times 10^{-2}$
.165	1.91	1.12	.0351	115.
.33	1.91	1.12	.0351	231.

Table 10.

integrated b5	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.93×10^{-2}	$.550 \times 10^{-2}$	$.0253 \times 10^{-2}$	$.0073 \times 10^{-2}$
.165	1.93	.550	.0253	78.6
.33	1.93	.550	.0253	157.
-.165	1.93	.550	.0253	78.6

Table 11.

integrated a3	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.93×10^{-2}	$.550 \times 10^{-2}$	$.0253 \times 10^{-2}$	$.0073 \times 10^{-2}$
.165	1.93	1.34	.0411	2.58
1.65	1.93	8.44	.183	248.
-1.65	1.93	8.44	.183	248.

Table 12.

integrated a5	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.93×10^{-2}	$.550 \times 10^{-2}$	$.0253 \times 10^{-2}$	$.0073 \times 10^{-2}$
.165	1.93	.550	.0253	6.00
.33	1.93	.550	.0253	12.0

Table 13.

integrated b4	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0350 \times 10^{-2}$	$.0157 \times 10^{-2}$
6.6	2.12	26.2	2.98	4.23
13.2	2.36	52.2	6.01	16.0

Table 14.

integrated a2	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0350 \times 10^{-2}$	$.0157 \times 10^{-2}$
1.65	2.92	3.37	.137	.0596
3.3	3.81	6.27	.332	.266
6.6	5.18	10.6	1.25	1.61

Table 15.

integrated a4	3rd order Σ resonances	4th order Σ resonances	5th order Σ resonances	6th order Σ resonances
0 "unit"-meters	1.91×10^{-2}	1.12×10^{-2}	$.0350 \times 10^{-2}$	$.0157 \times 10^{-2}$
1.65	2.66	318.	11.6	127.
3.3	2.74	621.	24.9	582.