SSC-N-401

A Design for SSC Multipole Correction Coils

Richard Talman
SSC Central Design Group
October 26, 1987

Abstract

Two coil designs are given for SSC correction magnets which generate dipole, sextupole and decupole magnetic fields within a single lumped element. The lengths of these magnets are determined by the dipole steering requirement. The increase in their length due to incorporating sufficient sextupole and decupole capability is negligible.

I. Introduction

In the SSC, multipole correction magnets are needed for various purposes. In the CDR it has been assumed that the correction of any particular multipole requires the corresponding pure single element. For example, b_2 compensation requires a sextupole magnet. Here a design will be given for a single-layer multi-coil magnet which can be used to correct all of b_0, b_2 and b_4. Symmetry makes it natural to correct these particular elements in the same magnet. It is also natural to correct b_1 and b_3 together in a magnet of opposite reflection symmetry, perhaps in the main focusing quads, but that case will not be considered here. Correction of the even skew elements a_{2r} can be performed by rotating the magnets considered here by 90°.

The designs considered here could perfectly well be applied to long bore-tube correction elements inserted into the main dipole magnets but the case of short lumped correctors will be emphasized. It may be that the fabrication techniques developed by Skaritka and others at BNL for fabricating correction coils can be used also for the coils discussed here.

The use for b_0 correction is to trim the closed orbit by local steering. It is assumed in the CDR that there will be one such, independently-powered, dipole correction element in each half cell. It has further been assumed in the CDR that the prototypical use for sextupoles and decupoles is to compensate for persistent current mutlipoles. For this purpose, families of b_2 elements are wired together in series, and similarly for b_4. We will give one design appropriate for such a correction scheme.

Another scheme, which has not previously been considered, and which would require twice as many power supplies, would permit the individual control of b_0 and b_2 elements within each half cell. It is conjectured here (but remains to be exhibited) that such a scheme can both correct the closed orbit and give a large improvement in dynamic aperture for the SSC. Anyway, a design appropriate for such a correction element will be given. This magnet will also contain a decupole coil not necessarily individually controllable. This case, for which the analysis is slightly simpler, will be described first.

II. Theory

Figure 1a shows a current distribution of the symmetry to be considered; symbols defining the geometry are shown there. Assume that the coil is thin and lies on a circle of radius R. To simplify the formulas we will set R = 1 and reintroduce R explicitly only at the end. By Ampere's law the y component of magnetic field at point P due to current in the range $d\theta$ is given by:

$$dBy(x,0) = C\frac{\cos\theta - x}{\sin^2\theta + (\cos\theta - x)^2}d\theta \qquad (1)$$

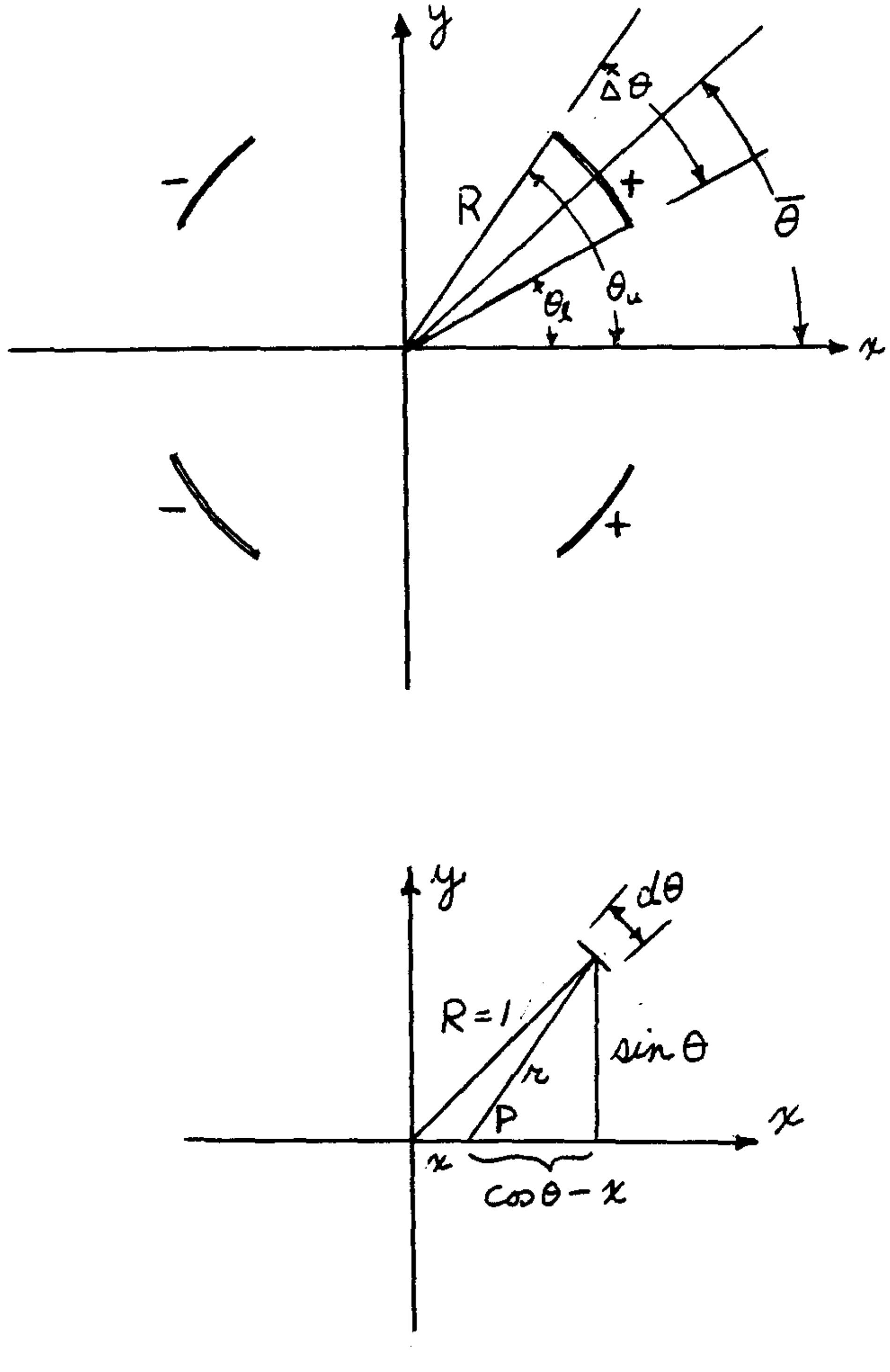

Figure 1. Geometry of coils having the same symmetry as a dipole.

where the constant C will be specified later. Summing and integrating over all four coils. We obtain

$$By(x,0) = 2C \int_{\theta_l}^{\theta_u} \left[\frac{\cos\theta - x}{\sin^2\theta + (\cos\theta - x)^2} + \frac{\cos\theta + x}{\sin^2\theta + (\cos\theta + x)^2}\right] d\theta \tag{2}$$

To study the multipole content we require an expansion in powers of x of the expression in square brackets.

$$2C[\,] = 4C\cos\theta \frac{1 - x^2}{1 - 2\cos2\theta x^2 + x^4} \tag{3}$$

$$= 4C\cos\theta\left[1 + (-1 + 2\cos2\theta)\, x^2 + (1 - 2\cos2\theta + 2\cos4\theta)\, x^4 + \ldots\right] \tag{4}$$

Finally we get

$$By(x,0) = b_0 + b_2x^2 + b_4x^4 + \ldots.$$

where

$$b_0 = 8C \sin\frac{\Delta\theta}{2}\cos\bar{\theta}$$
$$b_2 = \frac{8C}{3}\sin\frac{3\Delta\theta}{2}\cos 3\bar{\theta}$$
$$b_4 = \frac{8C}{5}\sin\frac{5\Delta\theta}{2}\cos 5\bar{\theta} \tag{5}$$

It can be seen that, when expressed in terms of the average coil angle $\bar{\theta}$ and the coil angular extent $\Delta\theta$, certain multipole attributes can be read directly from these analytic expressions for the b_n.

The constant C in (1) is given by

$$C = \frac{10^4}{B_0} 10^2 \frac{\mu_0}{2\pi} \frac{1}{R} \frac{NI}{\Delta\theta} \tag{6}$$

where the coil is assumed to have N turns and to carry current I. The units are MKS except that the factor of 10^2 permits distances to be measured in centimeters, as is coventional. The factor 10^4 is also part of the conventional "units" and B_0 is a reference dipole field; the b_n's are "fractional" errors referred to B_0. Finally in all previous formulas x should be replaced by x/R.

III. <u>A Dipole-Sextupole Coil With a Subsidiary Decupole Winding.</u>

Consider a coil design such as illustrated in Figure 2. There are three independent coils (only a single turn is shown but each coil can have mutliple turns.) For this magnet it is assumed that two currents I_{02} and I'_{02} are individually controlled while I_4 flows through a family of series-wired decupoles which compensates systematic b_4 terms. The currents I_{02} and I'_{02} are to be adjusted to give the desired dipole and sextupole (i.e., b_0 and b_2) fields. Since there is no independent b_4 control these coils must be (and have been) designed so that they give no b_4 contribution. By preference I_4 would not contribute to b_0 and b_2 but that is not really necessary (and is not achieved in the proposed design) because I_{02} and I'_{02} can be adjusted to compensate.

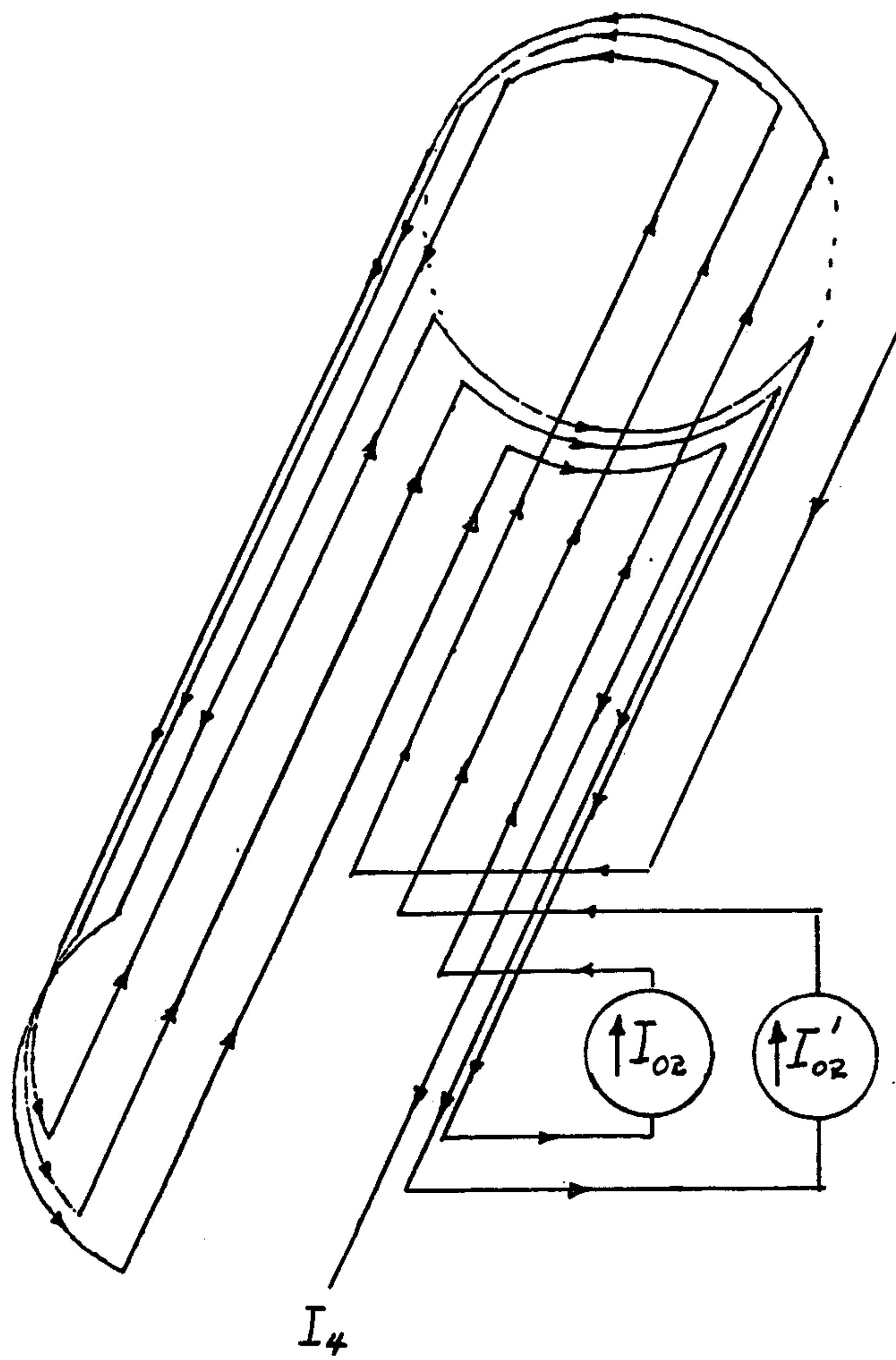

Figure 2. Wiring diagram (copied from M. Green) for a b_0, b_2, b_4 correction magnet with b_0 and b_2 individually controllable.

These conditions can be expressed analytically by introducing a 3×3 transfer matrix M which relates the vector $B = (b_0, b_2, b_4)^T$ to the vector $I = (I_{02}, I'_{02}, I_4)^T$ according to

$$B = MI$$
$$I = M^{-1} B \tag{7}$$

For the coil shown in Figure 3 the angles are

$$\theta_0 = 0$$
$$\theta_1 = 0.6284 = 36.00°$$
$$\theta_2 = 1.2565 = 71.99°$$
$$\theta_3 = 1.4 = 80.21° \tag{8}$$

and the matrix and its inverse are given by

$$M = \begin{pmatrix} 0.29392 & 0.18158 & 0.01722 \\ 0.15850 & -0.25641 & -0.04735 \\ 0 & 0 & 0.06577 \end{pmatrix} \tag{9}$$

$$M^{-1} = \begin{pmatrix} 2.4621 & 1.7435 & 0.61061 \\ 1.5219 & -2.822 & -2.4303 \\ 0 & 0 & 15.2045 \end{pmatrix} \tag{10}$$

From the structure of these matrices it can be seen that b_0 and b_2 can be adjusted arbitrarily without influencing b_4. On the other hand, to obtain a pure b_4 field the currents must be in the ratios 0.61:–2.43:15.2.

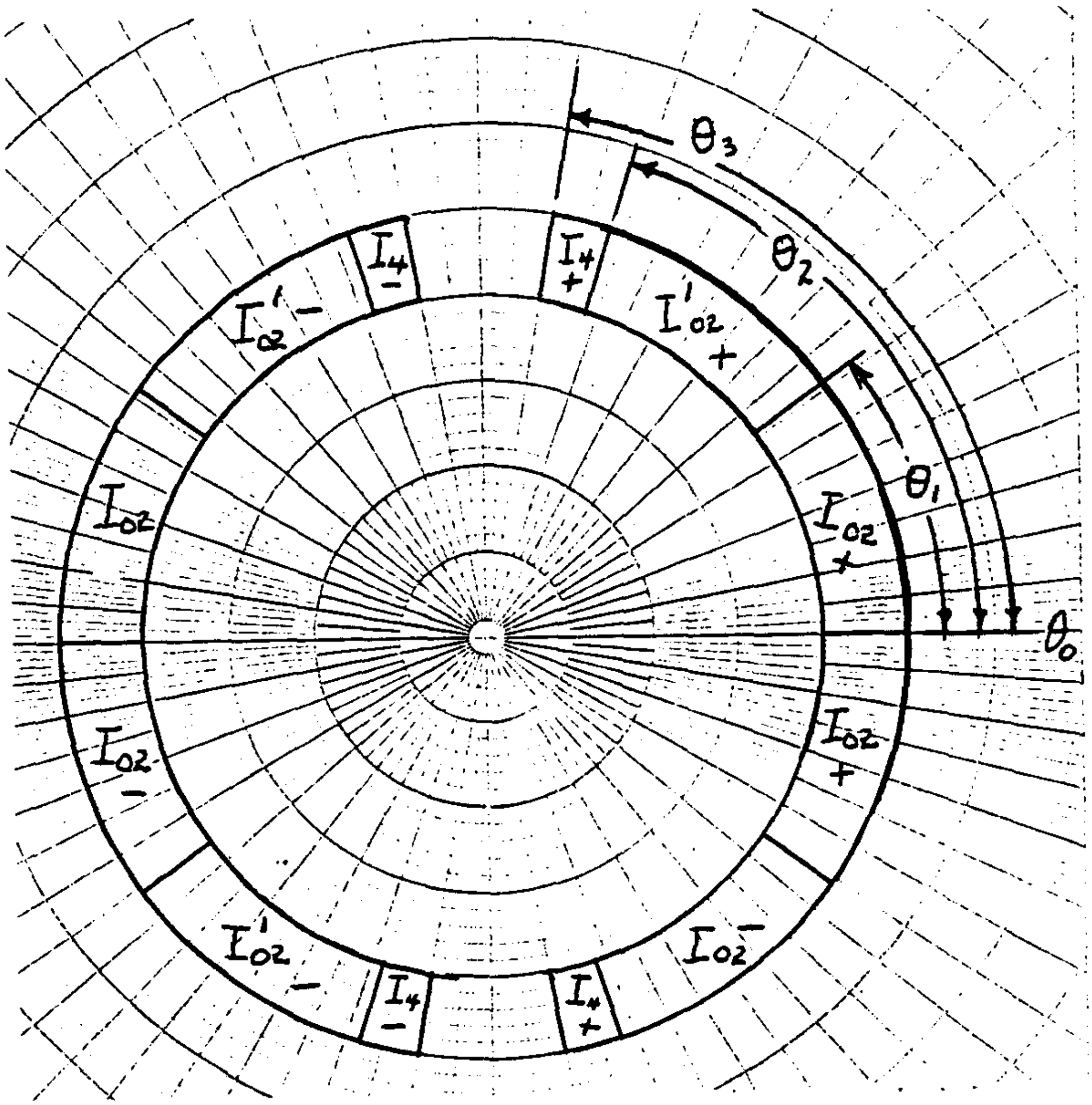

Figure 3. A dipole-sextupole coil with subsidiary decupole winding.

One can inquire whether this design is "inefficient" in the sense that the currents are "bucking" each other, thereby giving a smaller maximum field for a given maximum current than would otherwise be possible. For a pure dipole field the currents are in the ratio $I_{02}:I'_{02}$ = 2.46:1.52. Since the coils subtend equal arcs the current I'_{02} is less than maximum by some 38%, assuming that I_{02} is maximum. On the other hand a single pure dipole coil only subtends 60° instead of the 72° of these two coils. Combining factors the present arrangement is only 3% weaker than a 60° pure dipole coil with the same current limit.

Next we inquire about achievable sextupole strength. This time assume the maximum excitation is set by I'_{02} = 2.46, (the same curent limit but for the other coil). This is 0.87 times the value 2.82 (see formula (10)) and that is one factor by which the maximum value of b_2 will be less than the maximum value of b_0. Now in the CDR the maximum dipole correction field per half cell is 3.1 Tesla-m which can be compared to the full bend field per half cell which is 6 × 16 m × 6.6 T = 634 T-m. Using the conventional "units" (by which fractional fields are quoted in parts per 10^4) this required dipole field for the steering coil being designed can be quoted as $10^4 \times 3.1/634 = 50$ units. Since only horizontal bends can be modified this should be derated to 25 units. Taking the coil radius as 2.2 cm and working in the conventional cm units the maximum sextupole strength would be $b_2 = 0.87 \times 25/(2.2)^2 = 4.5$ units. This is a "full field" value, not an "injection" value. It greatly exceeds the value of 2 units presently specified for distributed bore tube sextupole correctors at full field.

The contents of the previous two paragraphs can be recapitulated as follows: the required integrated sextupole correction field strength per half cell can be achieved in the same elements as are used for steering. The lengths of these elements are determined by steering requirements; they do not need to be lengthened owing to the "piggyback-riding" sextupole coils. Furthermore, the decupole correction can be thrown in also at no cost in length along the beam line.

Before becoming too elated by this result one should be reminded that a lumped sextupole compensation scheme with just one lump per half cell is known to be inadequate. Also schemes which move horizontal steering away from horizontal focusing quads will force the steering correctors to be somewhat stronger. A scheme using two lumps per half cell is being worked on. It will be the subject of a subsequent report.

IV. A Dipole Coil With Subsidiary Sextupole and Decupole Windings.

Consider next the coil shown in Figure 4. This has four windings except that the first and third are wired in series to form a dipole coil which carries a current I_0. This coil has been designed to give zero contribution to both b_2 and b_4. The second winding, carrying current I_4 corrects decupole errors. It has a dipole component which must be compensated with I_0. The fourth winding carries current I_2 and corrects sextupole, with I_0 and I_4 being adjusted to compensate its dipole and decupole fields.

The angles defining the coil are

$$\begin{aligned} \theta_0 &= 0 \\ \theta_1 &= 24.40° \\ \theta_2 &= 36.10° \\ \theta_3 &= 60.16° \\ \theta_4 &= 74.48° \end{aligned} \tag{11}$$

and the matrix equation describing its performance is

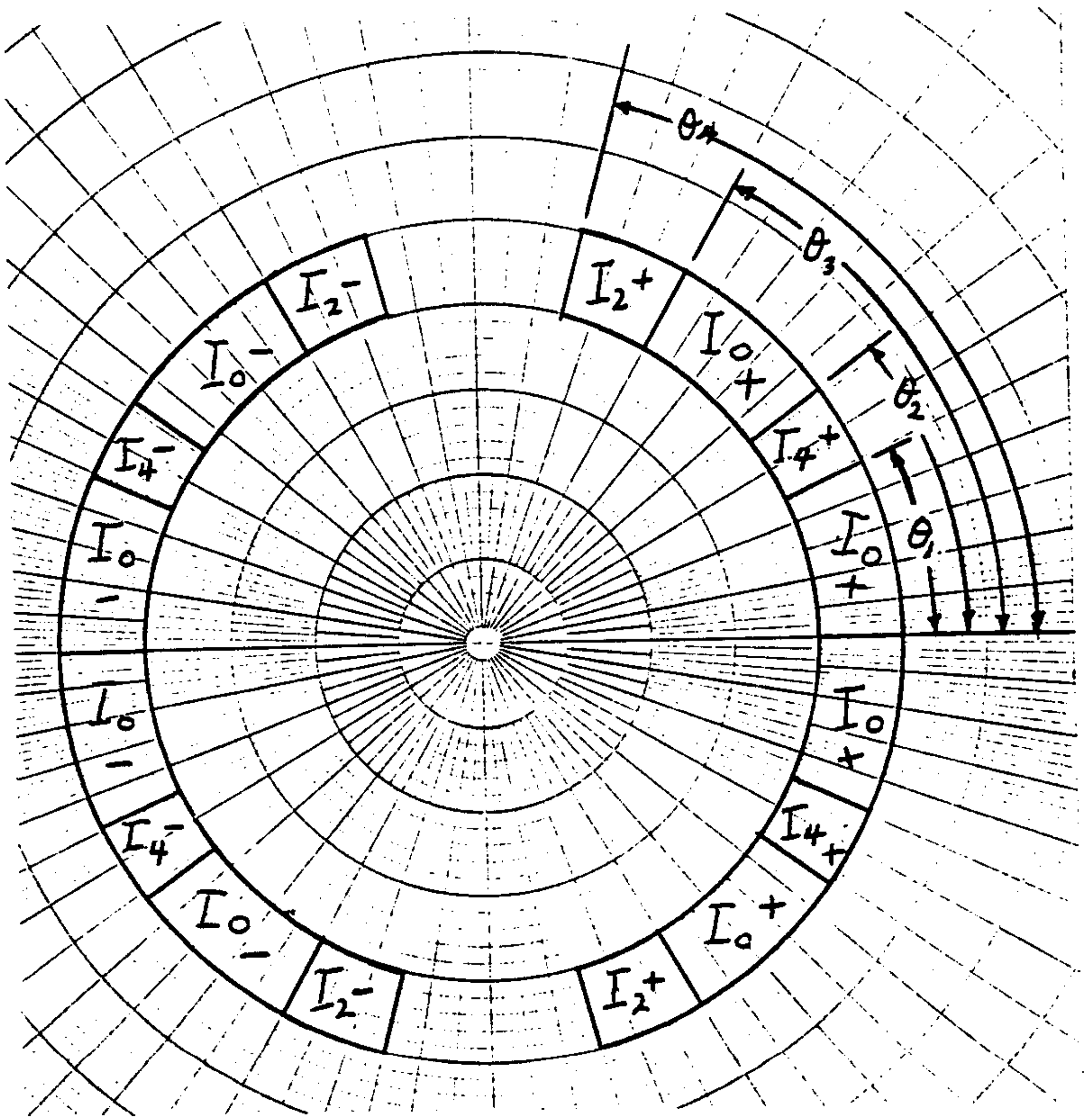

Figure 4. A dipole magnet with subsidiary sextupole and decupole windings.

$$\begin{pmatrix} b_0 \\ b_2 \\ b_4 \end{pmatrix} = \begin{pmatrix} 0.3456 & 0.0481 & 0.0880 \\ 0 & -0.1132 & 0 \\ 0 & 0.1074 & -0.0856 \end{pmatrix} \begin{pmatrix} I_0 \\ I_2 \\ I_4 \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} I_0 \\ I_2 \\ I_4 \end{pmatrix} = \begin{pmatrix} 2.893 & 4.051 & 2.974 \\ 0 & -8.834 & 0 \\ 0 & -11.084 & -11.682 \end{pmatrix} \begin{pmatrix} b_0 \\ b_2 \\ b_4 \end{pmatrix} \tag{13}$$

By introducing a more complicated arrangement with more coils it would be possible to improve the "orthogonality" of these correction coils. Also many other variants are possible.