SSC-N-384
September 1987

Correction of Residual Second-Order Sextupole Tune Shifts by the Simpson's Rule Octupoles

D. Neuffer
AT-6, Los Alamos National Laboratory, Los Alamos, NM 87545

Abstract

Residual second-order sextupole tune shifts can be corrected by an order of magnitude by the F, C, and D octupoles in the Simpson's Rule multipole correction scheme. Systematic sextupole (b_2) tolerances under the tune shift criteria can be increased to the thoroughly obscene level of ~ 20 SSC units. The octupole degree of freedom provides the scheme with a very large margin of safety in b_2 correction.

A multipole correction system for the SSC has been suggested[1] which consists of lumped elements located at the center (C) and ends (F and D) of the SSC half-cells (see Fig. 1). The relative strengths of the correctors per half cell are given to first order by Simpson's Rule (1:4:1). For the SSC, sextupole, octupole, and decupole corrections (b_2, b_3, b_4) are required. The proposed scheme corrects first-order nonlinearities due to systematic multipoles by two orders of magnitude, providing a large margin of safety for SSC magnets.

However, SSC dipoles may have a very large systematic sextupole component at injection due to persistent currents and second-order sextupole effects are important. The proposed scheme can correct second-order sextupole tune shifts to $b_2 \simeq 6$ units under the SSC linearity criteria, where a unit is 10^{-4} cm^{-2}. This is above the SSC Reference Design[2] value of 4–5 units, but does not provide a very large margin of safety.

The residual second-order sextupole tune shifts are proportional to b_2^2 and are of the form:

$$\Delta\nu_x = aA_x^2 + bA_y^2 + c\delta^2 \tag{1}$$

$$\Delta\nu_y = dA_y^2 + bA_x^2 + e\delta^2 \tag{2}$$

where A_x and A_y are the horizontal and vertical particle amplitudes and δ is the particle momentum deviation. In the numerical evaluations these are expressed in units of the standard SSC aperture (A_0 = 0.5cm at the maximum cell betatron value β_0 and $\delta = \Delta p/p$ =0.001). With Simpson's Rule and linear chromaticity correction of 10 units of b_2, a crude numerical evaluation in a simplified SSC lattice obtains $a \simeq 0.007, b \simeq 0.004, c \simeq 0.007, d \simeq 0.007, e \simeq 0.004$. As discussed in Ref. 1, these terms all have the same sign and are similar in magnitude.

First-order octupole tune shift expressions contain the same terms. The tune shifts due to lumped octupole elements may be written as:

$$\Delta\nu_x = -\frac{1}{2\pi B\rho}\sum B_{3,i} l_i \left(\frac{3\beta_{x,i}^2}{8\beta_0}A_x^2 - \frac{3\beta_{x,i}\beta_{y,i}}{4\beta_0}A_y^2 + \frac{3}{2}\beta_{x,i}\eta_i^2\delta^2\right) \qquad (3)$$

$$\Delta\nu_y = -\frac{1}{2\pi B\rho}\sum B_{3,i} l_i \left(\frac{3\beta_{y,i}^2}{8\beta_0}A_y^2 - \frac{3\beta_{x,i}\beta_{y,i}}{4\beta_0}A_x^2 - \frac{3}{2}\beta_{y,i}\eta_i^2\delta^2\right) \qquad (4)$$

where the sum is taken over the correction elements labelled by i and the betatron functions are evaluated at the element locations. $B_{3,i}$ and l_i are the corrector strength and length. The dependences along the SSC half-cell of the various combinations of the betatron functions which appear in these expressions are shown in Fig. 2. Unlike the second-order sextupole terms, the octupole terms have opposing signs. Also there are five independent terms and F, C, and D octupoles only provide three independent correctors. Fortuitously, the terms inside the parentheses in Eqs. 3 and 4 with negative signs have similar dependence, with enhanced values at the center correctors. Also the two positive terms in Eq. 3 have similar dependences.

The correction strategy is to use the center (C) octupoles to correct the b and e terms of Eqs. 1 and 2, and the end (F and D) octupoles to correct the a, c, and d terms using opposite sign fields. The ratios of the F, C, and D correctors per half cell are roughly (1:–3:1).

A crude empirical fit with b_2 = 10 units and $B_{3,i}l_i/B\rho$ = (+0.03, –0.075, +0.03) m^{-3} reduces the terms in Eqs. 1 and 2 by an order of magnitude; the largest surviving term is less than 0.0005. Extrapolation and reevaluation find that the SSC tune shift criteria can be met for b_2 less than $\sim$ 20 units.

The above discussion must be reevaluated, explored, amplified, and re-optimized by more extensive calculations and analyses, possibly using the ample resources provided by the new SUN workstations and the excellent

SSC accelerator physics staff. A particular concern is the possibility of relatively large orbit distortion in the tune- shift corrected orbits.

A few further comments:

- The octupole strengths required are quite small; the maximum octupole strength required to supercorrect 10 units of b_2 at SSC injection is only 300 T/m^2. Since the octupole tune shifts are linear, there is no interference between the second-order b_2 and first-order b_3 correction roles of the octupoles. Independent control of the F, C, and D octupoles is required.

- The initial evaluations reported in this note apply to second-order correction of sextupoles which have Simpson's Rule first-order sextupole correction (F, C, and D). F, C, and D octupoles should also provide a similar degree of relative improvement in second-order correction if only end correctors (F and D) are used in first-order sextupole correction.

- The possibility of correction should not be used to encourage the design and production of SSC dipoles with large nonlinear fields. It is still highly desirable to design the dipoles to minimize the maximum value of b_2, as proposed by Limon and others.

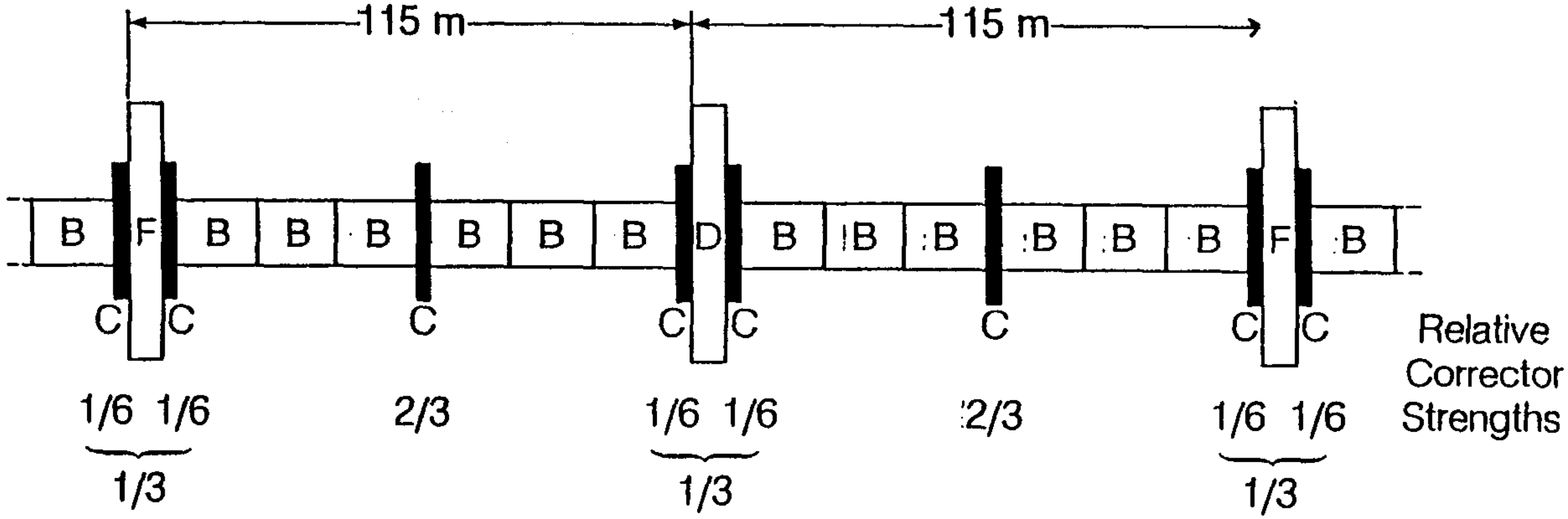
- The possibility of using F, C, and D octupoles to correct second-order sextupole effects does add an additional dimension of operational flexibility, conceptually similar to the use of F and D sextupoles to correct quadrupole-induced chromaticity. The margin of safety in b_2 correction is greatly enlarged.

I acknowledge extremely helpful communications from A. Dragt, E. Forest, and J. Peterson.

References

1. D. Neuffer, SSC-132, June 1987.
2. SSC Conceptual Design, SSC-SR-2020, March 1986.

3-Lump Correction for SSC Systematic Multipoles



B – Dipoles (16 m SSC dipoles)

F,D – Quadrupoles

C – Slots for correctors (correctors on opposite sides of quads can be combined in units on either side)

Figure 1.

XBL 876-10208

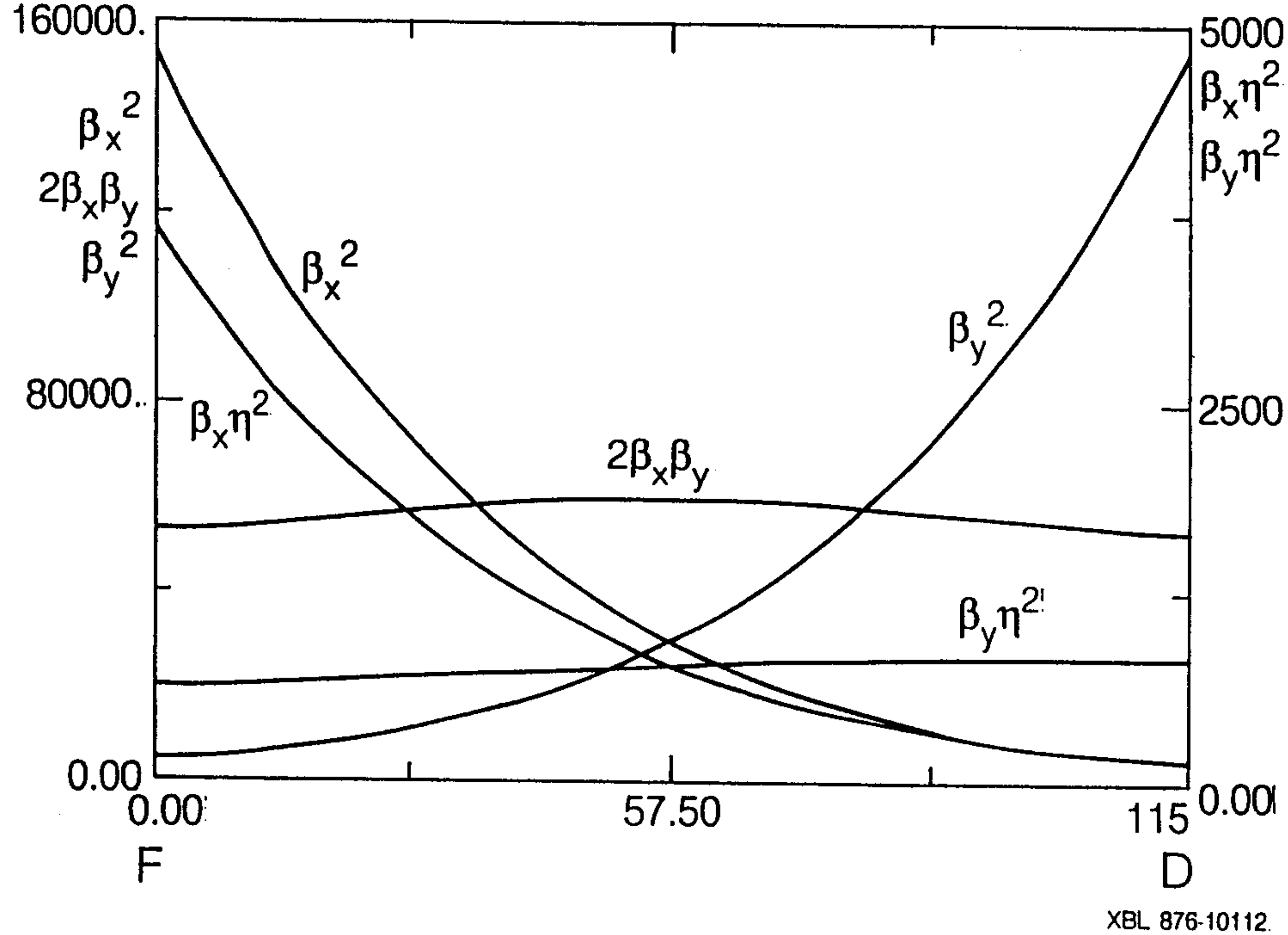

160000.
$\beta_x{}^2$
$2\beta_x\beta_y$
$\beta_y{}^2$
80000.
0.00
0.00
F
57.50
115
D
5000
$\beta_x\eta^2$
$\beta_y\eta^2$
2500
0.00
$\beta_x{}^2$
$\beta_x\eta^2$
$2\beta_x\beta_y$
$\beta_y{}^2$
$\beta_y\eta^2$
XBL 876-10112

Figure 2.