Tolerances for SSC Quadrupoles

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Introduction

The Superconducting Super Collider (SSC) requires an adequate linear aperture for reliable operation. Linear motion is required over a working region in amplitude and momentum space sufficient to include the beam size and momentum spread, closed-orbit deviations and injection errors. For the SSC, this linearity requirement has been translated into a limit on orbit distortion or "smear" of 10% and a limit on momentum and amplitude dependent tune shifts of $\Delta\nu \lesssim 0.005$, for $A_x, A_y \lesssim 0.005$ m at $\beta = \beta_F$ in the SSC arcs, and for $\Delta p/p \lesssim \pm 0.001$. These linearity requirements have severely restricted the allowable multipole content of SSC dipoles, necessitating the introduction of sextupole, octupole, and decapole correction coils. In this note, we explore the constraints which the tune shift linearity criteria place on the multipole content of SSC quadrupoles. These constraints are compared with estimated SSC quadrupole errors, as obtained by extrapolation from Tevatron quadrupole data or from displacement error analysis of the SSC quadrupole current elements.

Arc Quadrupole Tolerances

The tolerances for SSC arc quadrupoles from the tune shift criterion are not as strict as those on the dipoles, since the quads occupy only a small fraction of the SSC circumference. The 1–D horizontal tune shifts due to quadrupoles may be found from:

$$\Delta\nu = \frac{1}{2\pi} \int \frac{B_0\, b_n(s)}{B\rho}\, \frac{\beta \cos\phi}{A} \left(A\cos\phi + \eta\,\frac{\Delta p}{p}\right)^n ds \tag{1}$$

where the integration is taken over the quadrupoles. The effects of the focusing (F) quads are dominant with

$$\Delta\nu \cong \frac{B_0\, \beta_{max}}{2\pi\, B\rho} N_Q\, l_Q\, \overline{b_n} \overline{\left[\frac{\cos\phi}{A_{max}} \left(A_{max}\cos\phi + \eta_{max}\,\frac{\Delta p}{p}\right)^n\right]} \tag{2}$$

where N_Q is the number of F quads, l_Q their length, B_0 is the quad field at 1 cm, and $\bar{b}_n$ gives the average fractional multipole component of order n at the normalizing radius of 1 cm. β_{max}, A_{max}, η_{max} are evaluated at the F quads. The contributions of the D quads to the horizontal $\Delta\nu$ should be of opposite sign and much smaller in magnitude; their effects are small in these initial estimates, but have been included in Table I. Equation (2) may be evaluated using SSC parameters, and the resultant tolerances on systematic quadrupole components are displayed in Table I. Typically, $<b_n> \lesssim 10^{-4}$ cm^{-n} is required. Tolerances on random multipoles are a factor of $\sqrt{N_Q}$ larger, where N_Q is the number of statistically independent quadrupoles. If we optimistically take $N_Q \cong 400$, the random tolerances are an order of magnitude larger [$<b_n^2>^{1/2} \leq 10^{-3}$ cm$^{-\eta}$].

Table I. Tolerances for Arc Quadrupole Systematic Multipole Components

Multipole	CD Lattice (60°, 192 m) 10^{-4} cm^{-n}	Current Lattice (90°, 230 m) 10^{-4} cm^{-n}
b_2	0.75	0.87
b_3	0.82	1.08
b_4	0.94	1.44
b_5	1.10	1.90
b_6	1.28	2.51
b_7	1.51	3.29
b_8	1.76	4.30

Tolerances for IR Quads

High luminosity operation of the SSC requires focusing of the beam to very small sizes at the IR centers, and this focusing is obtained by strong IR quads. The strengths of the IR quads and the enlarged size of the beam passing through them magnify the nonlinear effects of the quad multipoles. The enlarged size of the beam for high luminosity optics is in fact much too large for the injection lattice aperture. It is therefore necessary to detune the IR optics at injection.

The linearity requirements ($\Delta\nu \lesssim 0.005$ for A_x, $A_y \lesssim .007$ at $\beta = \beta_{max} = 393$ m) at injection still place significant constraints on the field linearity in IR quads at injection optics ($\beta^* = 6.0$ m, β_{max} in the IR quads = 600 m). These constraints may be estimated using Eq. (2) with IR quad parameters ($N_Q = 16$, $l_Q = 15$ m, $B'/B\rho = .00345$, $\beta_F = 600$ m). The dispersion is zero at the IR quads, therefore $\eta\delta = 0$. However, the fact that the beams cross at an angle introduces a displacement Δ from the center line in the IR quads with $\Delta = \alpha L/2$. Here α is the separation angle of the beams, and L is the distance form the IR crossing point. For typical SSC parameters, $\Delta \cong 0.2$ cm. This displacement leads to multipole feed-down similar to that from dispersion. An important difference is that Δ changes sign in crossing the IR, and tune shifts proportional to odd powers of Δ change sign. In fact, if the crossings are paired with alternating vertical or horizontal crossings, the tune shifts due to systematic (b_2, b_4, b_6, . . .) cancel.

In Table II, we display the tolerances on the systematic multipoles in the IR quads with the injection lattice. No sign cancellation of even b_n is assumed in these tolerances. Random multipole tolerances would be a factor of $\sqrt{N_Q}$ larger or ~4. Tolerances are on the order of 10^{-4} cm^{-n}, similar to the tolerances for arc quadrupoles.

Low-β optics can only be imposed at full energy after a stable, small-amplitude closed orbit has been established. With $\beta^* = 0.5$ m, $\beta_{max} = 8000$ m, the beam size in the IR quads is ~5 times larger than in the arcs. Nonlinearities in the IR quads can dominate the nonlinearities in the remainder of the machine. The rms beam size in the IR quads is only ~0.63 mm, however. Requiring linearity for orbits up to 12σ (.76 cm in the IR quads; .17 cm in the arcs) obtains the linearity constraints displayed in Table III. These constraints are systematic constraints; random multipole constraints are ~4 times larger. Linearity approaching 10^{-5} cm^{-n} is desirable; this is an order of magnitude lower than the constraint on arc quad nonlinearities.

Table II. IR Quad Tolerances at Injection Optics

[$\Delta\nu_x \lesssim \pm .005$ for $A_x \lesssim 0.7$ cm at β_{max}]

Multipole	Constraint (10^{-4} cm^{-n})
b_1	1.3
b_2	3.1
b_3	1.8
b_4	2.6
b_5	2.2
b_6	2.5
b_7	2.3
b_8	2.4

Table III. Constraints on IR Quad Nonlinearities with low-β Optics

[$\Delta\nu \lesssim \pm 0.005$ for $A_x \lesssim 0.76$ cm at $\beta^* = 8000$]

Multipole	Limit (10^{-4} cm^{-n})
b_1	.094
b_2	.24
b_3	.17
b_4	.25
b_5	.24
b_6	.29
b_7	.31
b_8	.35

Estimated Errors for SSC Quadrupoles

The multipole content of SSC quadrupoles has been estimated by two separate techniques: 1) extrapolation[1] from measured multipole content in previously constructed superconducting magnets,[2] or 2) an error analysis based on estimated construction errors expected in quadrupole construction.[3]

In Tevatron construction, approximately 250 quadrupoles were constructed, and their multipole content was accurately measured. The SSC would follow similar construction techniques and follow similar error patterns. The major difference in SSC quadrupoles is the choice of a smaller coil radius [$\bar{r}$ = 5.28 cm (Tevatron) $\rightarrow \bar{r}$ = 3.03 cm (SSC)]. If the fractional coil placement errors were the same, the multipoles would scale as:

$$a_n \, , b_n \propto \bar{r}^{\,-(n-1)} \tag{3}$$

Fractional errors are expected to increase as $\bar{r}^{-1/2}$, so that the resulting multipoles scale as $\bar{r}^{-(n-1/2)}$. In Table IV we display the resulting systematic and random SSC multipoles as scaled from the Tevatron data. The systematic multipoles are typically only factors of one to four times smaller than the rms random multipoles. This implies that the systematic multipoles are due to repeated construction errors rather than the statistical residue of random errors. Barring significant changes in construction methods, similar results would be expected for SSC magnets.

J. Herrera et al. have estimated SSC quadrupole multipoles based on an error analysis of the effects of estimated coil errors. They assume coil placement errors in radius and azimuth of 2 mils (.005 cm) and in coil width and thickness of 1 mil (.0025 cm). The resulting scaling would then be

$$a_n, b_n \sim \bar{r}^{\,n} \tag{4}$$

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1 "SSC Quadrupole Errors Scaled from Tevatron," J.M. Peterson, unpublished.

2 E.E. Schmidt et al., IEEE Trans. MS-30, 3384 (1983).

3 "Random Errors in the Magnetic Field Coefficients of Superconducting Quadrupole Magnets," J. Herrera, R. Hogue, A. Prodelly, P. Thompson, P. Wanderer, E. Willen, 1987 PAC.

a somewhat steeper dependence than obtained by Peterson. Their resulting estimated random errors are included in Table IV; they are somewhat larger than the Tevatron-scaled parameters. This indicates that coil placement accuracy was somewhat better in Tevatron construction than the quoted estimates.

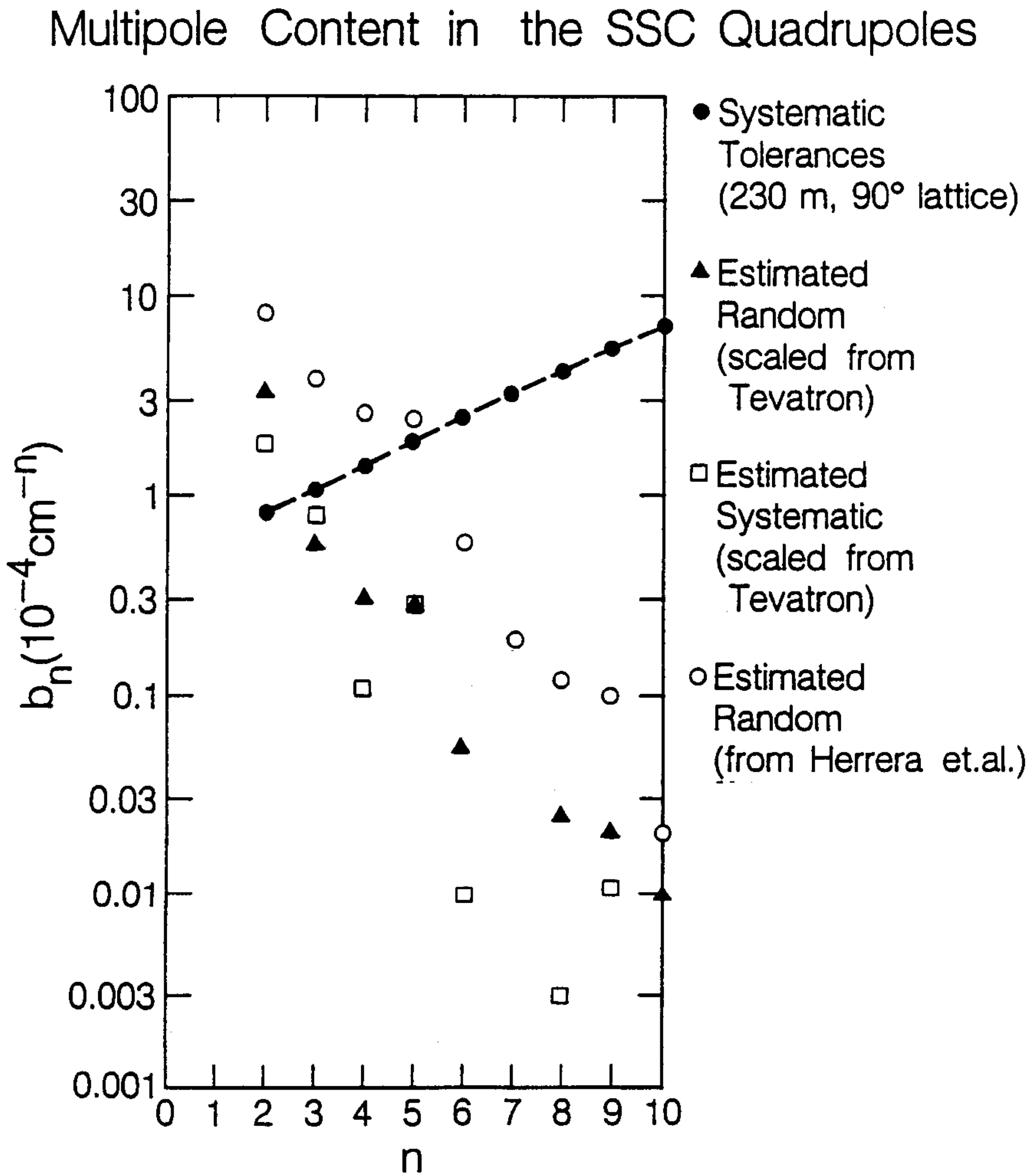
Table IV. Estimated Multipole Content of SSC Quadrupoles

Multipole	Random Magnitude Scaled from Tevatron (Peterson)[1] 10^{-4} cm^{-n}	Systematic Magnitude Scaled from Tevatron (Peterson)[1] 10^{-4} cm^{-n}	Random Magnitude Calculated from Error Analysis (Herrera et al.)[3] 10^{-4} cm^{-n}
b_2	3.35	1.80	8.5
b_3	0.58	0.78	3.9
b_4	0.32	-0.11	2.6
b_5	0.29	0.29	2.4
b_6	0.056	0.010	0.58
b_7	—	—	0.19
b_8	0.025	-0.003	0.12
b_9	.021	-0.11	0.10
b_{10}	.01	0.0004	0.02

Comparisons with Tolerances

The calculated quadrupole tolerances may be compared with the estimated multipole content and that comparison is displayed in Fig. 1. The systematic tolerances for arc quads are significantly greater than the estimated random multipoles for $n \gtrsim 5$. The $n = 2$ (sextupole) component is locally correctable by the spool pieces. The systematic b_3 and b_4 components should also be within tolerances, provided that they are somewhat less than the larger rms random multipole estimates. The SSC linear aperture is not seriously endangered by the arc quadrupoles, provided that their multipole content has not been greatly underestimated.

Similarly, the injection lattice linear aperture is not seriously endangered by the IR quads. However, the low-β lattice used at collider energies does place strict constraints on IR quad multipoles. These quads should be constructed to higher quality specifications and may need local ($b_2 - b_5$) correction.



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