Chromaticity Mug Shots

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Introduction

Chromaticity in a storage ring is observed as sidebands to the betatron frequencies which are always present. These sidebands are spaced at the synchrotron frequency, and appear because the synchrotron motion provides a frequency modulation of the betatron signal when the chromaticity is non- zero. In the case that the betatron signal is driven by Schottky noise or some other incoherent source, then the power distribution of the different sidebands can be calculated. These are plotted to give a spectrum analyzer- eye view of the effect different chromaticities have on the observed tune spectrum, with the intention that one could make a quick chromaticity estimate by comparing the plots with a displayed beam spectrum. If the beam motion is coherent, with correlated synchrotron and betatron motion (i.e. "head-tail modes") then these spectra do not apply.

Calculation

For a single particle with peak relative momentum deviation δ and chromaticity ξ, the maximum betatron tune deviation is

$$\Delta\nu = \xi\delta \ . \tag{1}$$

The betatron signal and its Fourier series are [1]

$$\begin{aligned} V(t) &= e^{i(\nu\omega t + (\Delta\nu/\nu_s)\sin(\nu_s\omega t))} \\ &= \sum_{m=-\infty}^{\infty} J_m(\xi\delta/\nu_s)\ e^{i(\nu+m\nu_s)\omega t} \ , \end{aligned} \tag{2}$$

where J_m is the ordinary Bessel function, ω is the revolution frequency, and ν and ν_s are the betatron and synchrotron tunes.

Assuming that the synchrotron and betatron motion are uncorrelated, the sideband power can be summed over all particles. For a Gaussian momentum distribution, the maximum momentum deviation is distributed as

$$\rho(\delta) \;=\; (2\pi\bar{\delta}^2)^{-1}\;\delta\;e^{-\delta^2/2\bar{\delta}^2}\;, \tag{3}$$

where $\bar{\delta}$ is the rms relative momentum deviation:

$$\bar{\delta} \;=\; \sigma_p/p\;. \tag{4}$$

The relative power in each sideband is given by the integral [2]

$$\begin{aligned} P_m \;&=\; 2\pi\int_0^\infty d\delta\;\rho(\delta)\;J_m^2(\xi\delta/\nu_s) \\ &=\; e^{-\alpha^2}\;I_m(\alpha^2)\;, \end{aligned} \tag{5}$$

where I_m is the modified Bessel function, and

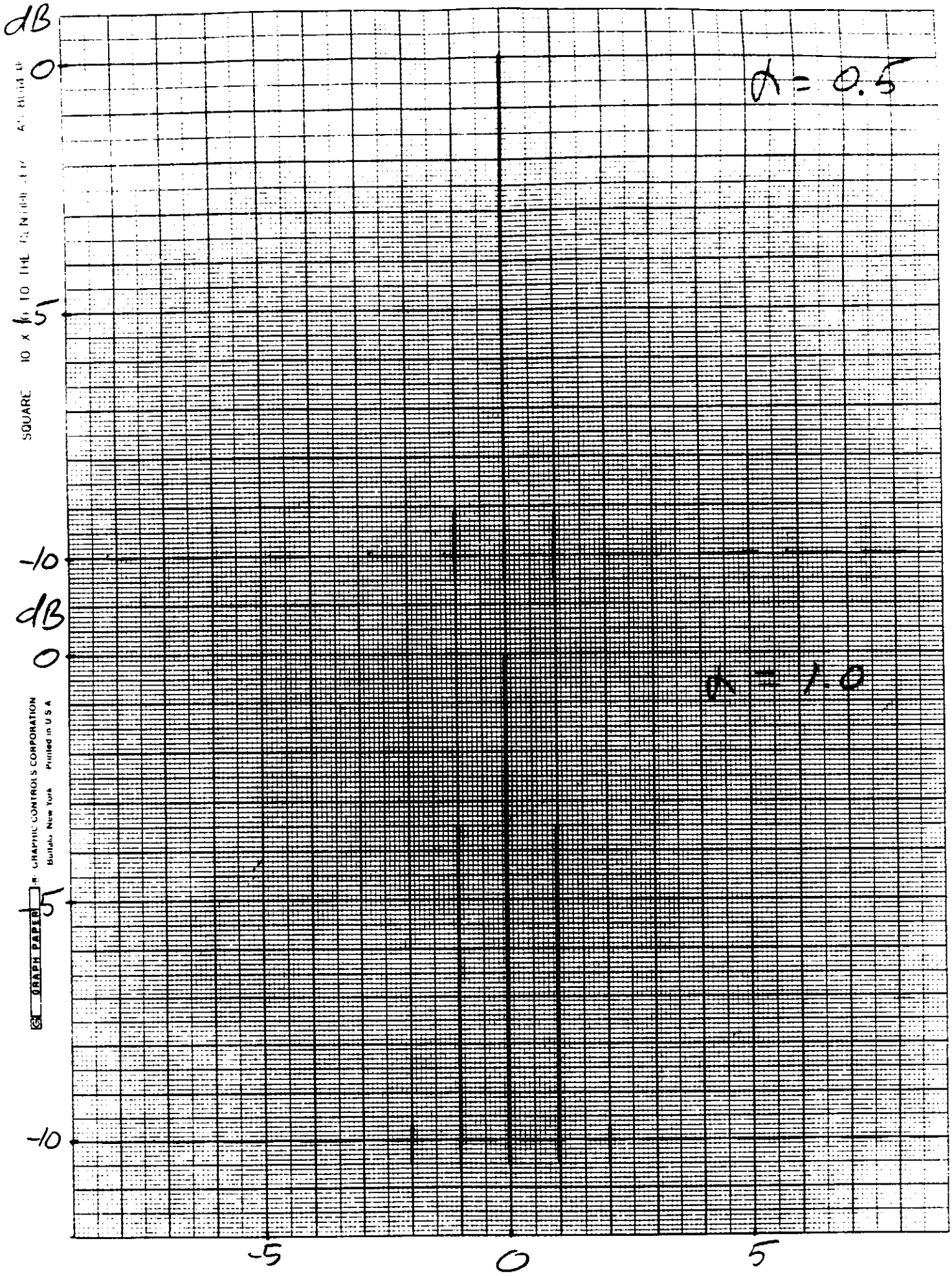
$$\alpha \;=\; \xi\bar{\delta}/\nu_s\;. \tag{6}$$

The function P_m versus m represents the spectrum of the transverse beam signal for given α. Plots of relative sideband amplitudes for values of α from 0.5 to 4.0 follow. These plots are in decibels relative to the $m = 0$ power. The chromaticity is given from α by

$$\xi \;=\; \alpha\nu_s\;p/\sigma_p\;. \tag{7}$$

References

1. Gradshteyn and Ryzhik, formula 8.511.3
2. Gradshteyn and Ryzhik, formula 6.633.2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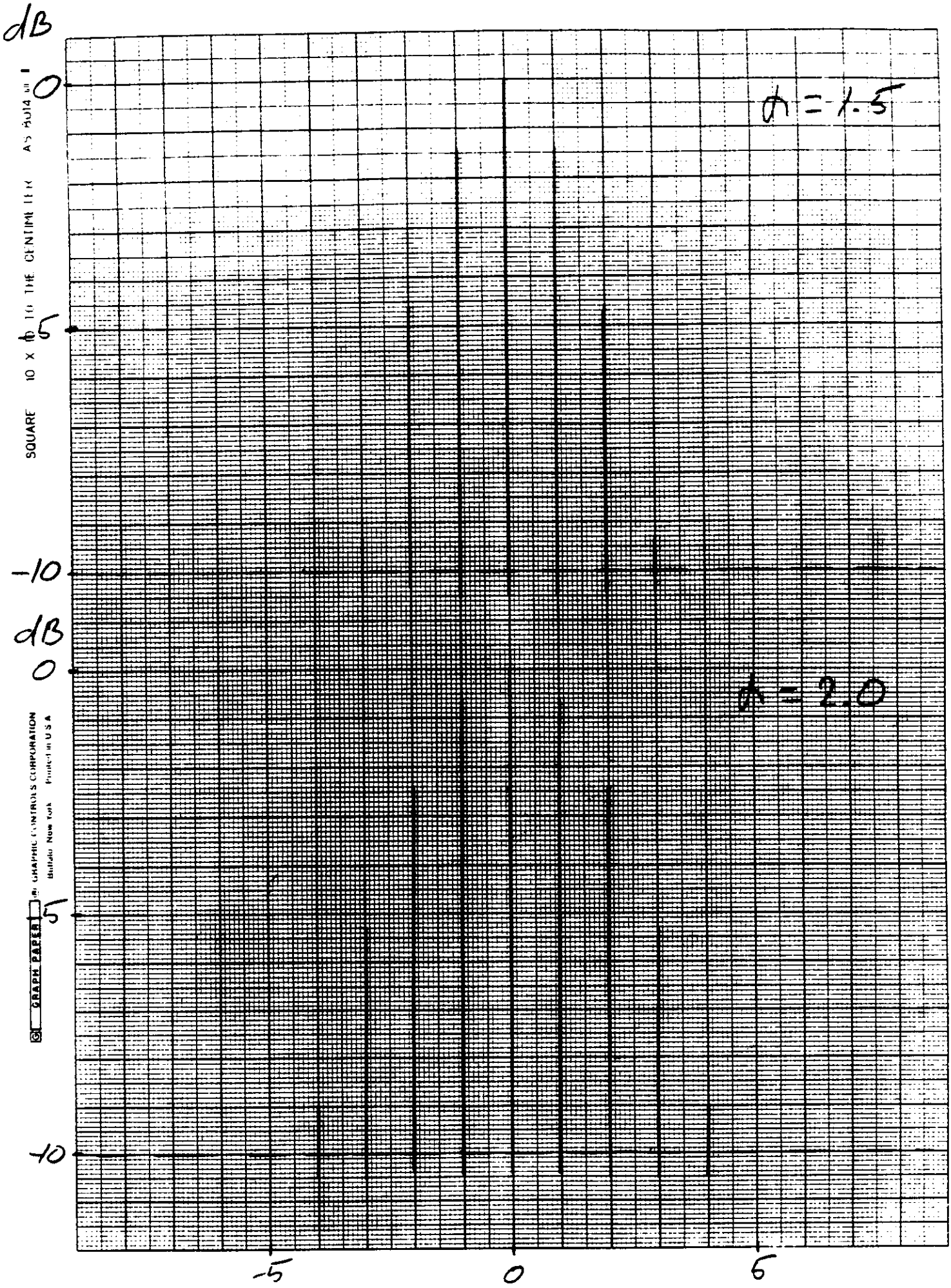
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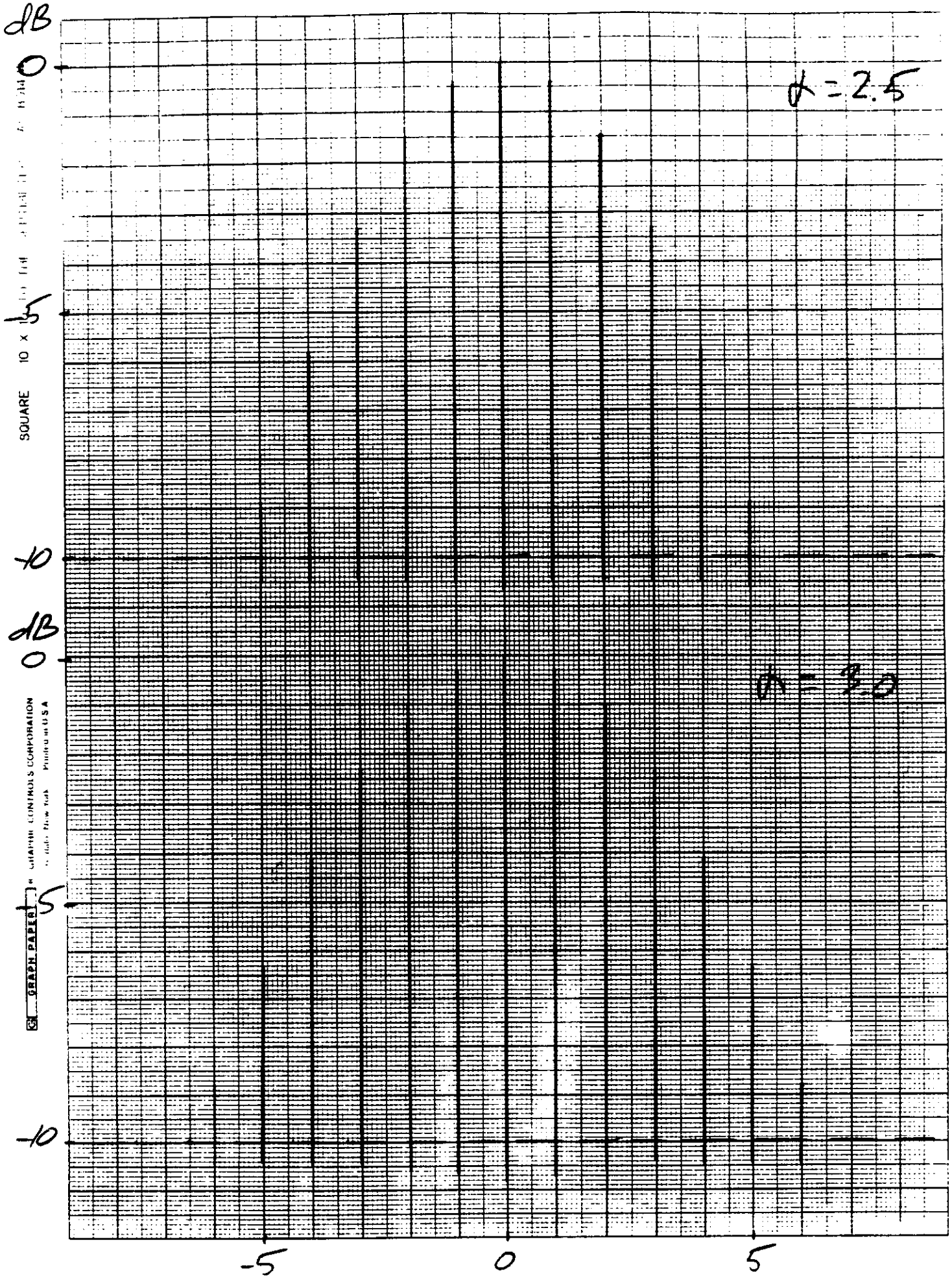
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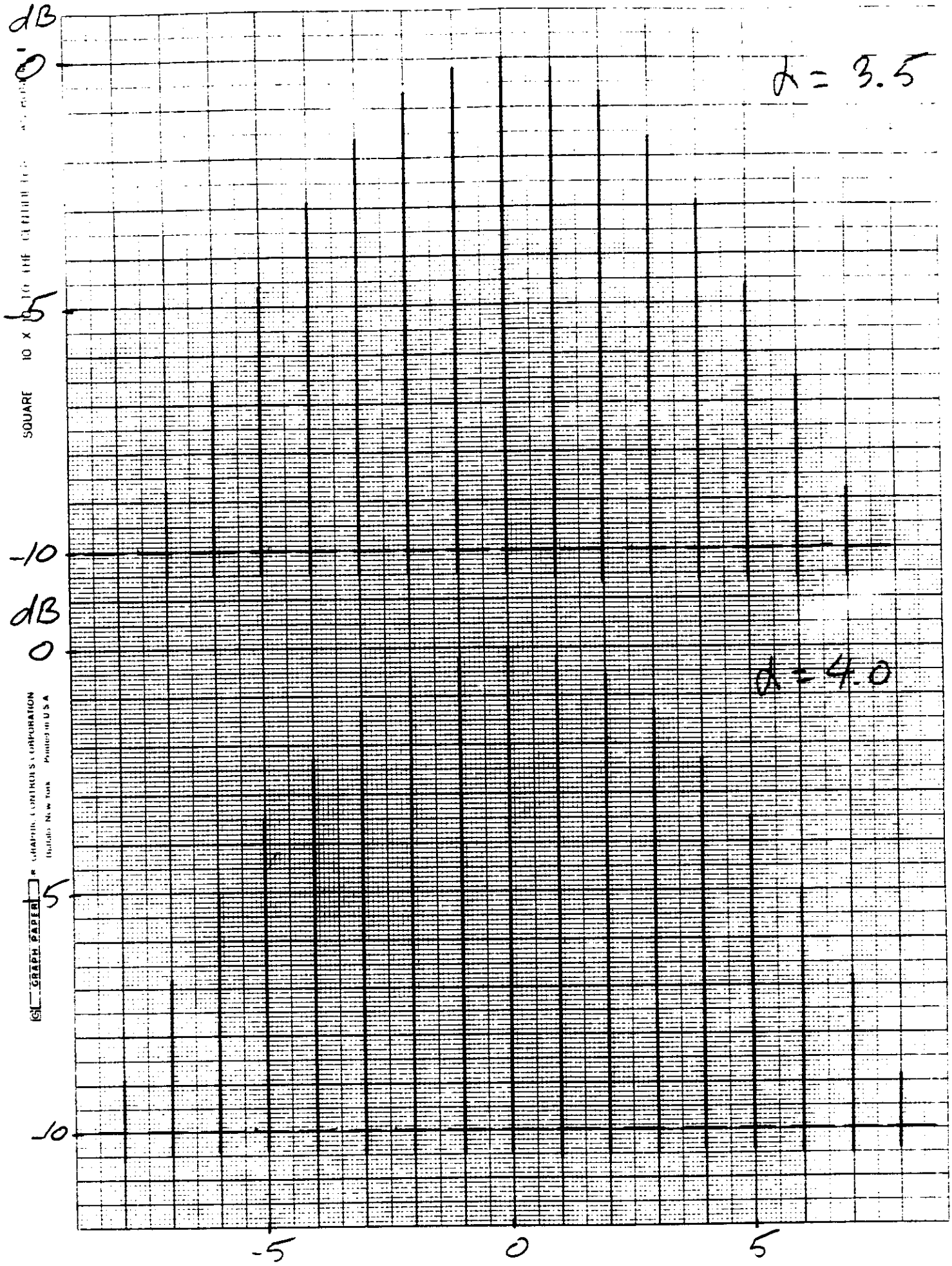
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