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MICHAEL A. GREEN	MECHANICAL	BERKELEY	APRIL 1987	
PROGRAM - PROJECT - JOB				
MECHANICAL ENGINEERING DEPARTMENT - GENERAL				
SSC - SUPERCONDUCTING DIPOLE MAGNET				
TITLE				
USING THE WEDGES TO CARRY FIELD TRIM COILS				

A Revision - April 30, 1987

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R. Talman, in his report of July 25, 1986, suggested that the wedges of the SSC dipole magnet could be continuous correction windings for the SSC in place of the bore tube correction coils postulated in the present design. R. Talman's report showed that the wedges could be used to produce a pure sextupole of 1 unit (0.00066 T at a radius of 1 centimeter from the magnet center) even when they are used in the resistive state. The problem with the correction method, as he calculated it, was heat generation in the wedges of 0.47 W m^{-1} . This is high compared to the synchrotron radiation at full energy (0.14 W m^{-1}). As a result, a superconducting wedge would have to be used as R. Talman suggested in his note.

R. Talman remarked that the beam could be steered using the wedges as correction elements and that other multipoles, such as decapole, could also be corrected. Depending on the number of leads that one wants to bring out of the coil, one can correct all of the terms both normal and skew from $N=1$ (dipole) up to $N=6$ (12 pole) or $N=8$ (16 pole) depending on whether one uses the three inner wedges or all four coil wedges.

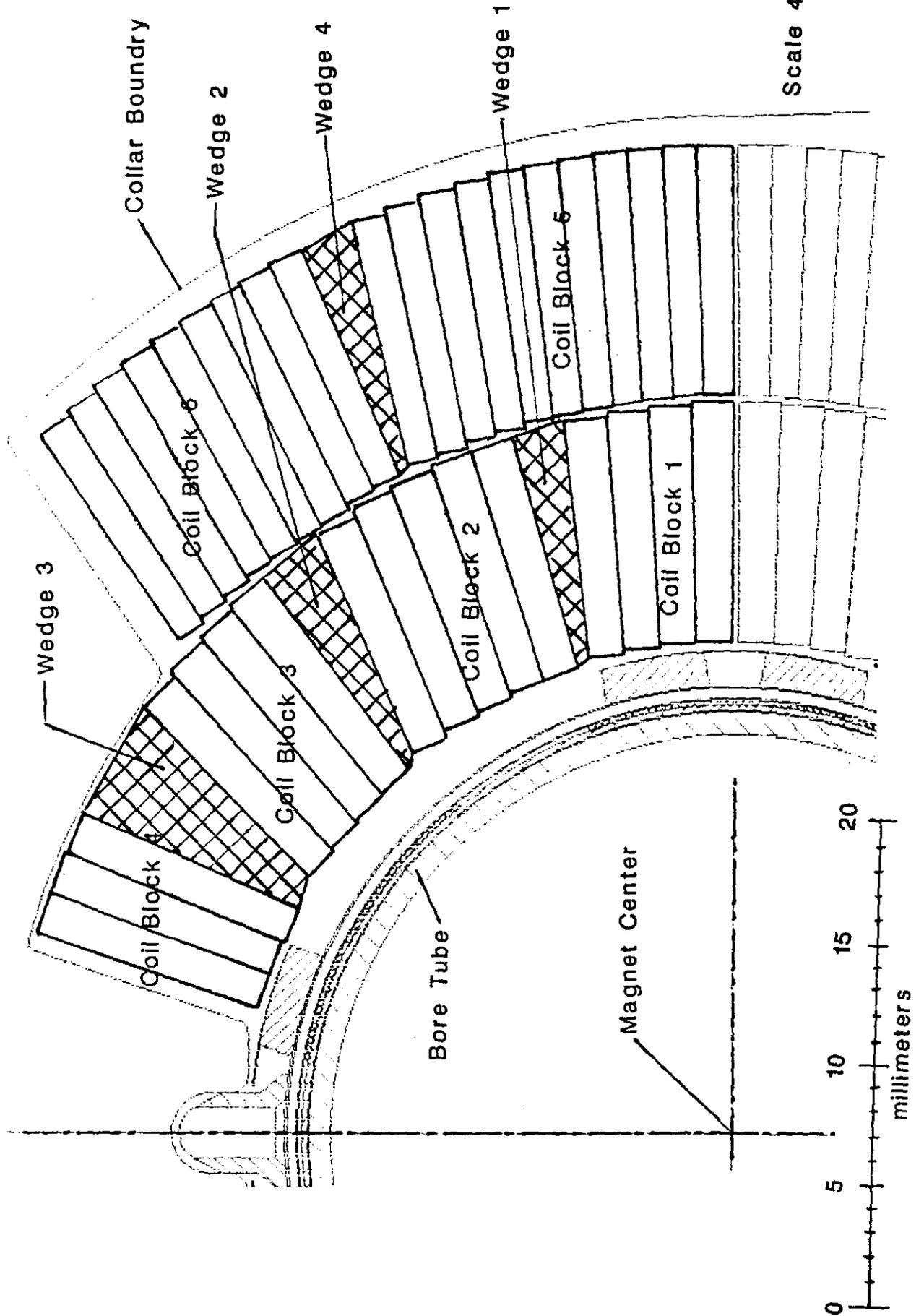
1. Calculation of Field Correction Using the Wedges

Figure 1 shows the SSC coil cross-section BNL C385A. This coil has 4 blocks made with 32 turns of 23 strand cable with strands which are 0.808 mm in diameter, and 2 blocks made with 40 turns of 30 strand cable with strands which are 0.648 mm in diameter. Wedges 1 through 3 are between coil blocks 1 through 4, and wedge 4 is between coil blocks 5 and 6. The wedges are the cross-hatched pieces shown in Figure 1.

*This work was supported by the U.S. Department of Energy, Office of Basic Energy Science, under Contract No. DE AC02-76SF00009

SSC DIPOLE CROSS-SECTION BNL C358A COIL

Figure 7



Scale 4.38

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A series of 29 computer runs were made using the SCMAG01 computer code to see how pieces of the superconductor attached to the wedges could be used to correct normal dipole, sextupole and decapole in the field. (These are the dominant terms which must be corrected using wedge correctors.) In one case, the normal $N=7$ (14 pole) was calculated using all four of the wedges. The algorithm for the SCMAG01 code which is used for the computer calculations is given in the Appendix.

The four wedge corrections shown in Figure 2 could produce 3 units of normal dipole (at full field) with a maximum current in the wedge of 35.9 A in wedge 4. The four-wedge corrector could produce 1 unit of normal sextupole (at full field) with a maximum current of 264.4 A in wedge 4. (Note: The $N=9$ is less than 0.00001 T at a radius of 1 centimeter.) The four-wedge corrector can produce 0.5 units of normal decapole (at full field) with a maximum current of 480.3 A in wedge 4. (The $N=9$ in this case is 0.00001 T at a radius of 1 centimeter, and as in the previous case $N=7$ is zero.) Currents of the order of 3000 A are required to generate even 0.15 units of $N=7$ (at full field). It does not appear too attractive to use all four wedges for correction, and it is probably not attractive to correct the $N=7$ term either.

If one uses only the three inner wedges, the currents for correction will go down, but the first allowable higher multipole will be larger. Since the wedge configuration does not make it attractive to correct for $N=7$, using only the three inner wedges for carrying the superconductor appears to be very attractive.

LOCATION OF SUPERCONDUCTOR ON WEDGES OUTSIDE OF ALL FOUR WEDGES EXCEPT WEDGE 3

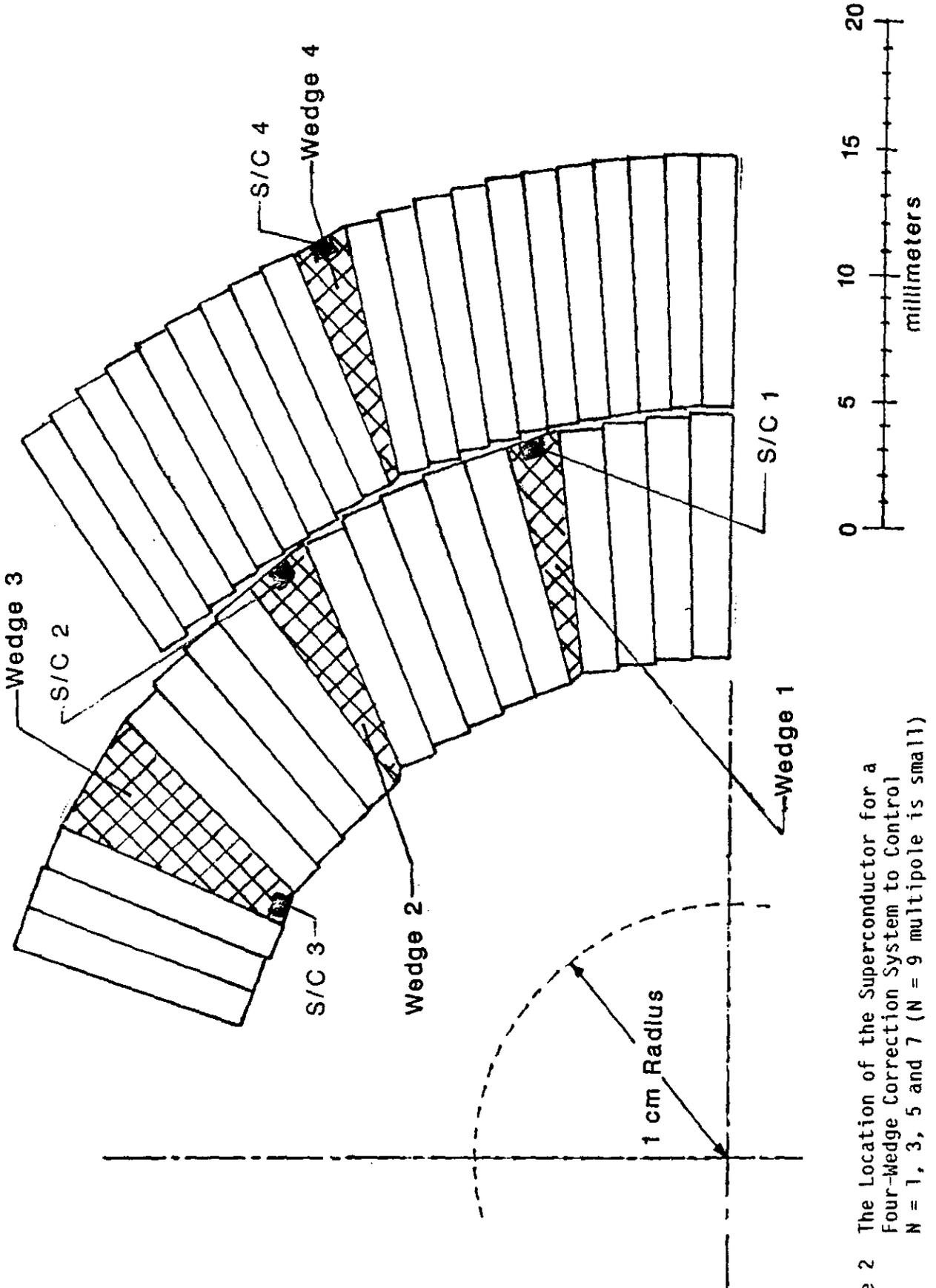


Figure 2 The Location of the Superconductor for a Four-Wedge Correction System to Control $N = 1, 3, 5$ and 7 ($N = 9$ multipole is small)

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Figure 3 shows the location of the superconductor for correcting multipoles up to normal, $N=6$ and skew $N=6$. Since SCMAG01 is a program which uses symmetry, only the normal $N=1, 3$ and 5 were calculated. In order to create 3 units of normal dipole ($N=1$) at full field, one needs a maximum current of 46.17 A in wedge 1. If one unit of normal sextupole ($N=3$) at full field at 1 centimeter radius is desired, the maximum current would be 129.4 A in wedge 2. If one desires 0.5 units of decapole ($N=5$) at full field at a radius of 1 centimeter, the maximum current is 404.5 A in wedge 2. The worst case higher multipole is 0.00001 T $N=7$ and 0.00002 T $N=9$ at a 1 centimeter radius.

The highest currents are to be found in wedges 1 and 2, which have the superconductor on the outside surface. One wants to reduce the currents to the lowest possible values for any given correction. The correction currents are more effective if they are brought closer to the magnet center. Figure 4 shows a configuration of the superconductor on the wedges which will yield the lowest possible current in the wedges when a set amount of normal dipole ($N=1$), normal sextupole ($N=3$) and normal decapole ($N=5$) are produced. Table 1 shows the values of various normal symmetrical multipoles when the currents are set to produce a dipole of 0.002 T (3 units at full field), a sextupole of 0.00066 T at a radius of 1 centimeter (1 unit at full field) and a decapole of 0.00033 T at a radius of 1 centimeter (0.5 units at full field). Table 2 shows the values of the currents in the superconductor put at the inside of wedges 1, 2 and 3.

LOCATION OF SUPERCONDUCTOR ON WEDGES OUTSIDE OF THE THREE INNER WEDGES EXCEPT WEDGE 3

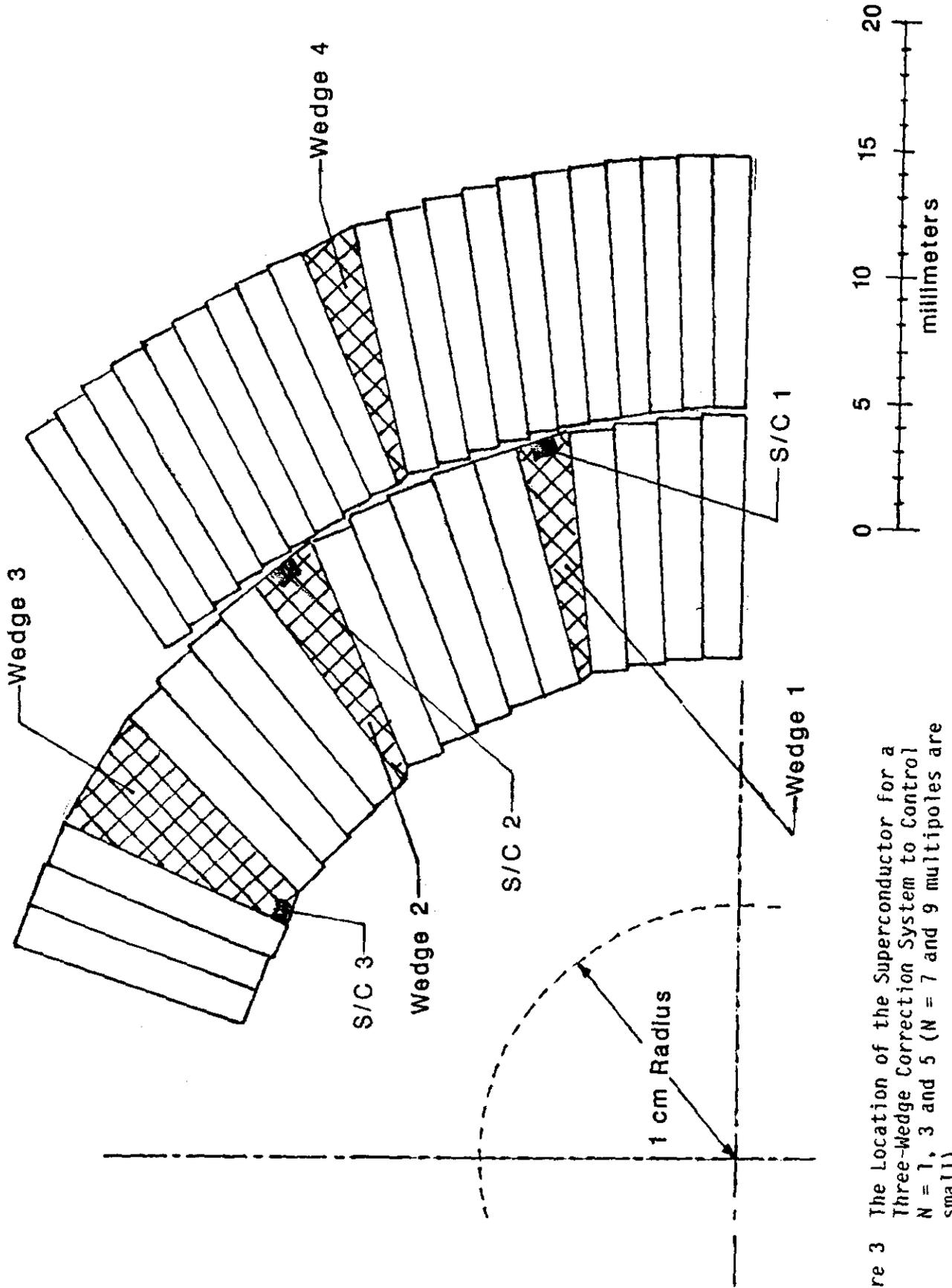


Figure 3 The Location of the Superconductor for a Three-Wedge Correction System to Control $N = 1, 3$ and 5 ($N = 7$ and 9 multipoles are small)

LOCATION OF SUPERCONDUCTOR ON WEDGES LOW CURRENT ON THE INSIDE OF THE THREE INNER WEDGES

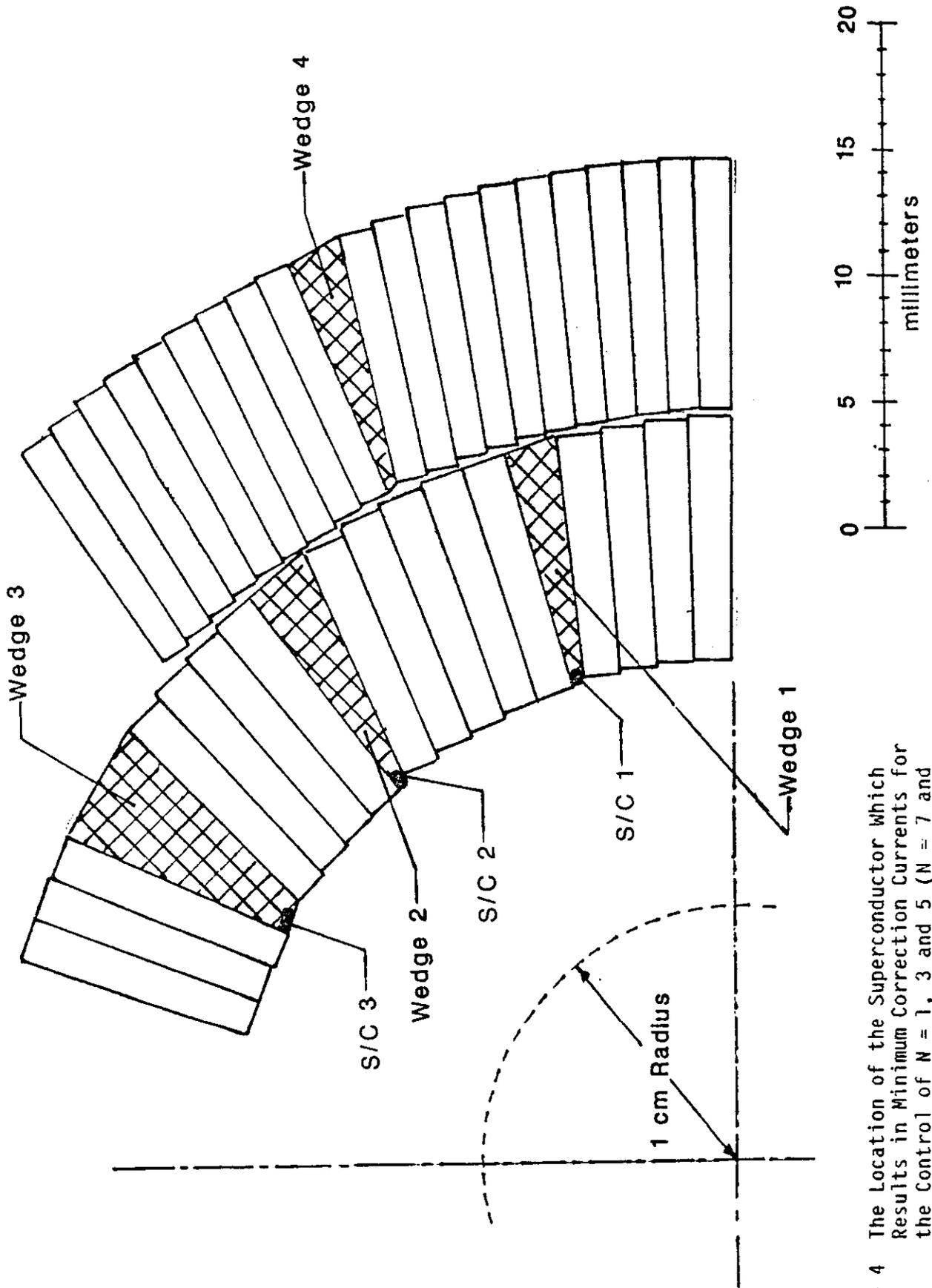


Figure 4 The Location of the Superconductor Which Results in Minimum Correction Currents for the Control of $N = 1, 3$ and 5 ($N = 7$ and 9 multipoles will be larger)

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Table 1. The Value of Various Multipoles for Various Dipole, Sextupole and Decapole Corrections at a Full Central Induction of 6.6 T

Multipole Number	Multipole value*		
	Dipole Correction 3 Units	Sextupole Correction 1 Unit	Decapole Correction 0.5 Units
1	0.00200	0.00000	0.00000
3	0.00000	0.00066	0.00000
5	0.00000	0.00000	0.00033
7	-0.00001	-0.00002	-0.00001
9	0.00000	0.00000	-0.00003

*at a radius of 1 centimeter

Table 2. The Current on the Inside of the Various Wedges to Generate the Multipoles Shown in Table 1*
(See Figure 4 for Location of the Superconductor)

Wedge Number	Superconductor Current (A)		
	Dipole Correction 3 Units	Sextupole Correction 1 Unit	Decapole Correction 0.5 Units
1	-32.75	-37.27	-46.62
2	- 8.65	21.80	111.18
3	-14.97	38.10	-83.01

*at full induction of 6.6 T, the peak induction at the wire is 7.0 T

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The largest superconductor which can be attached to wedge 1 is about 0.4 mm in diameter; wedge 2 might allow a 0.5 mm diameter superconductor to be attached; wedge 3 would allow a much larger piece of the superconductor to be attached to it. At all three locations, a peak field of about 7.0 T will occur. If one limits the peak current in each wedge of the superconductor to 90 percent of the superconductor critical current at 7 T (based on a superconductor specification of 2750 A mm^{-2} 5.0 T 4.2 K), the maximum correction which can be expected for the superconductor at the inside of the wedges (see Figure 4) is shown in Table 3. (The limit for the current in wedge 1 of Figure 4 is 89.6 A; the limit for wedge 2 of Figure 4 is 140 A.)

If the corrections shown in Table 3 are not high enough (at full field), one has to move the superconductor radially outward on wedges 1 and 2 so that one can increase the current within that superconductor by increased conductor area and reduced field. A configuration which will produce more correction than is given in Table 3 is shown in Figure 5. (For a given correction, the currents will be increased when one moves the superconductor out on wedges 1 and 2 as shown in Figure 5.)

TABLE 3. Maximum Correction at Full Field for Various Multipoles for the Cross-Section Shown in Figure 4#

Multipole	Correction Available (Parts in 10,000)
1	8.21
2*	~4.5
3	2.40
4*	~1.1
5	0.63

*Quadrupole and octupole correction is a rough estimate.

#The superconductor has a copper to superconductor ratio of 1, and it

operates at 7.0 T and 4.2 K

LOCATION OF SUPERCONDUCTOR ON WEDGES HIGH CURRENT TOWARD THE INSIDE OF THE INNER WEDGES

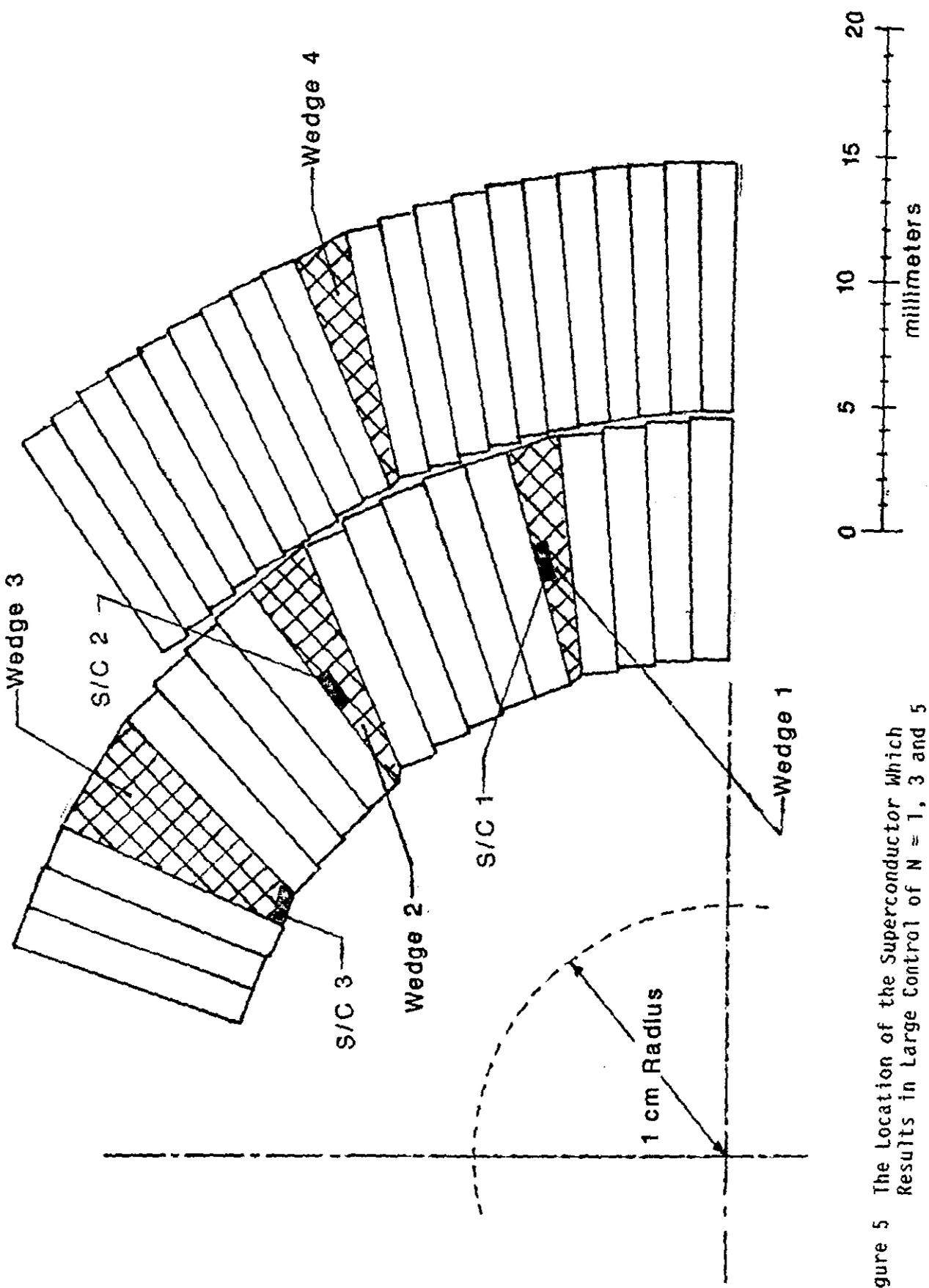


Figure 5 The Location of the Superconductor Which Results in Large Control of $N = 1, 3$ and 5

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2. The Pros and Cons of a Wedge Correction System.

Before going on, it is useful to look at the pros and cons of a wedge correction system as compared to bore tube windings or passive coils. The best of all worlds is to have no continuous correction coil whether it is in the wedges or on the bore tube. Before looking at the pros and cons of the wedge corrector, it is useful to know what the correction coils are used to correct. A partial list includes the following.

- a. The effects of magnet coil winding errors could be corrected with continuous coils. This type of continuous correction coil will have to correct out normal and skew quadrupole, sextupole and possibly octupole and decapole. The proposed continuous correction coils do not appear to correct for coil fabrication errors.
- b. One might correct for iron saturation or the inverse by a designed in error which goes to zero as the iron saturates. The latter appears in the magnet as presently configured. It should be noted that there are other ways of correcting out this term. The iron saturation induces primarily a normal dipole, sextupole and decapole error. There should be little or no quadrupole, octupole, $N=6$ or any of the skew terms.
- c. Correction might be required for eddy currents in the bore tube. Eddy current error will manifest itself as a normal dipole or sextupole error. Other higher normal odd multipoles might be present.
- d. Correction for magnetization effects in the superconductor will be required. These effects manifest themselves as normal dipole, sextupole, decapole and other higher normal odd multipoles.

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- e. Steering errors might be corrected with a continuous normal and skew dipole.
- f. Tune errors might be corrected with a continuous normal and skew quadrupole or sextupole.

There are probably others which I have not thought of or do not understand. The above list is probably not quite complete.

The next question one should ask is can correction be done with lumped elements every half cell. The answer to this question is at best complex and at worst a nightmare. If one is to eliminate continuous correction, one probably has to meet some of the following conditions.

- a. The lumped corrector probably has to correct up to at least decapole.
- b. The lumped corrector will probably have to be located at a point in the cell where beta is large. (The beam size should be large at this location.)
- c. The iron saturation sextupole or its inverse will probably have to be reduced to the order of two units (2 parts in 10,000) or less.
- d. Passive correction of the sextupole, decapole and $N=7$ terms of the magnetization field must be possible. Preliminary studies suggest that correction is theoretically possible, but the level of decapole correction required has yet to be demonstrated. There are lots of opinions on the subject of passive correction of magnetization effects. Serious consideration should be given to passive correction if it can be shown that the continuous correction is not needed for anything other than magnetization effects.
- e. Field errors due to conductor placement should be small (less than one unit of sextupole).

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Once one has demonstrated that continuous correction is needed, one should know what one is correcting for. Is one correcting for skew as well as normal multipoles? Is one correcting for only normal symmetric multipoles? Is only a normal sextupole correction needed? The answers to these questions can weight the pro and con arguments for the wedge type continuous correction element versus other types of correction.

The advantages of the using the wedges as correction elements are as follows.

- a. The wedges must be accurately located along with the coil block. As a result, the wedge position determines the conductor position with enough accuracy to do a good job of locating the correction windings.
- b. Correction of normal dipole, sextupole and decapole can be done using four leads (three active leads and a ground lead). Correction of the normal and skew terms for $N=1$ (dipole) through $N=6$ (12 pole) can be accomplished with 13 leads (12 active leads and a ground lead).
- c. The correction occurs over the full straight section length of the magnet. End effects can be compensated for by small changes of current in the wedges.
- d. The bore tube does not have to be accurately machined or formed, because it does not carry correction coils.
- e. The cold bore tube can be moved closer to the superconducting coil, increasing beam clearance somewhat.
- f. The outside surface of the bore tube can be helium cooled directly. Thus, hot spots due to synchrotron radiation can be reduced greatly.

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The disadvantages of wedge type correction windings are:

- a. The leads must be designed for 50 to 150 A instead of 20 A. This will probably increase refrigeration requirements.
- b. The routing of correction coil leads in and out of the dipole may prove to be difficult.
- c. Additional insulation on the wedges may be required in order to protect the correction circuit during a quench of the magnet.
- d. Wedges will have to be extruded in continuous lengths up to the full 17-meter magnet length.
- e. Wedge correction will require from 3 to 12 small power supplies per magnet. In addition to the power supplies is the power supply computer control which must be hooked into the central computer system.

3. How One Might Build a System of Wedge Correction

Figure 4 or Figure 5 shows the location of the superconductor on the three inside wedges of the SSC Dipole. The wedges for the dipole will have to be built in continuous lengths of 17 meters. This could be done using extrusions which have a slot for the superconductor already put in it. The next step is to soft solder the superconductor into the slot. The wedge is then insulated over its full 17-meter length.

How the superconductor is brought out at the end of the straight wedge is the problem. How the superconductor is handled at the end is a function of the end design of the magnet, the clearance between the coil and the collars, and the type of corrections the wedge correctors are expected to do. Let us start with the last one first. The wedge correctors can be used to do two

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types of corrections: 1) corrections of normal dipole, sextupole and decapole, or 2) corrections of normal and skew dipole, quadrupole, sextupole, octupole, decapole and $N=6$. The normal dipole, sextupole and decapole will be the largest correction required whether $N=2, 4$ and 6 and whether all the skew terms have to be corrected depends on the quality of the production dipole which can be produced.

If the wedges correct only normal $N=1, 3$ and 5 , only three circuits are needed. A circuit diagram for this case is shown in Figure 6. There are three circuit leads and a ground lead common to the three circuits. All four leads are at one end of the dipole.

If the wedges correct $N=1$ through $N=6$, both normal and skew, twelve separate circuits are needed. Figure 7 shows the circuit diagram for this case, which includes twelve leads and a common ground lead. The ground lead is at one end of the dipole, and the twelve hot leads are at the other end of the dipole. The ground lead in this case does not carry much current because the normal $N=1, 3$ and 9 connections will produce no net current in the ground lead. Therefore, the ground lead is small, and it can be brought back in the dipole lead slot or a small slot in the collar along the midplane. The field generated by the ground lead can be corrected for by the currents in the twelve powered leads.

I have seen the Brookhaven drawing of the magnet ends. The drawing offers no clue as to the exact location of the inner layer with respect to the outer layer. It appears that the outer layer is shorter than the inner layer, so there should be room to bring the wedge leads out of the inner layer and around and out of the ends of the magnet.

CIRCUIT DIAGRAM FOR WEDGE CORRECTORS

CORRECTS 1N, 3N, AND 5N

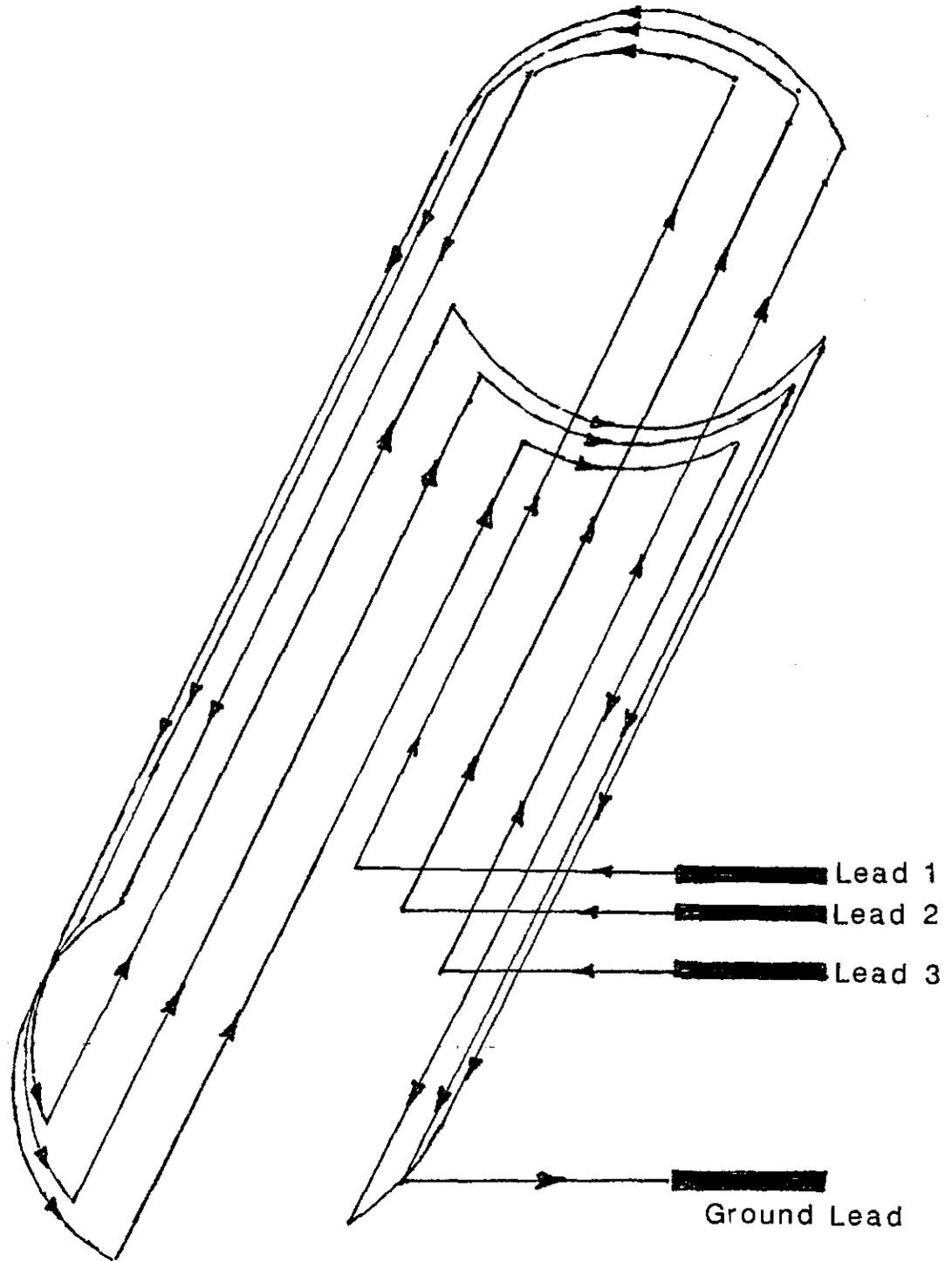


Figure 6 A Four-Lead Circuit for the Correction of Normal $N = 1, 3$ and 5

CIRCUIT DIAGRAM FOR WEDGE CORRECTORS

CORRECTS 1N, 2N, 3N, 4N, 5N, AND 6N
CORRECTS 1S, 2S, 3S, 4S, 5S, AND 6S

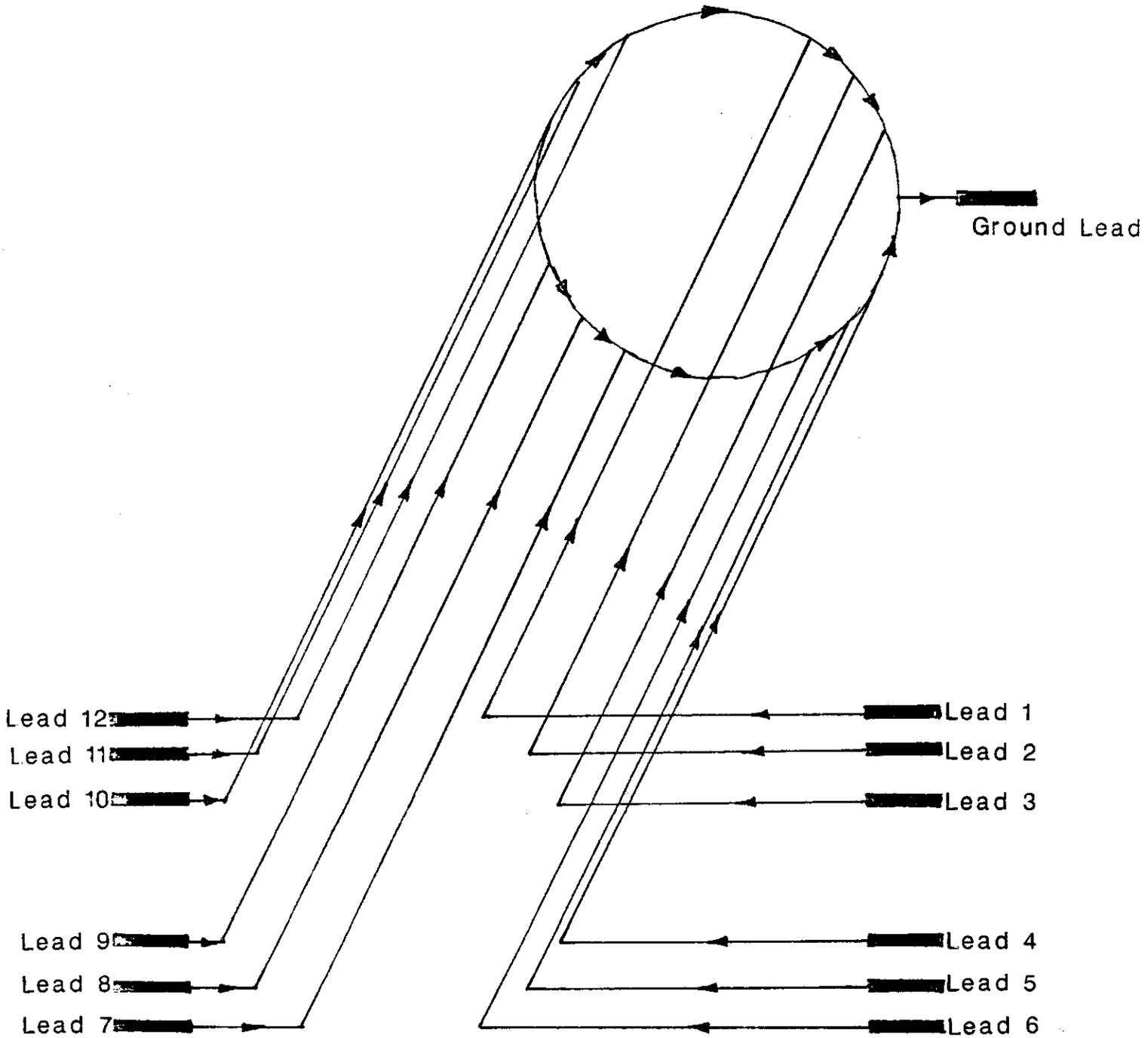


Figure 7 A Thirteen-Lead Circuit for the Correction of Normal and Skew $N = 1, 2, 3, 4, 5$ and 6

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The leads for the wedge correctors will probably have to be gas cooled. One might be able to design all of the leads into common gas-cooled leads with 4 to 13 current paths which are insulated from one another. If the case shown in Figure 4 and Table 3 applies, the highest current which can be carried by each of the wedges is as follows: wedge one can carry 90 A; wedge two can carry 140 A; and wedge three can carry 115 A. Using these values of the current, one can estimate the refrigeration needed to cool the gas-cooled leads. For the three-circuit correctors, 1.5 W of refrigeration is required (this is not direct refrigeration, it is the liquification equivalent of 1.5 W of refrigeration) for the leads in the worst case (about 0.09 W per meter of magnet length). If one must use the 12 circuit correction coil system, the lead gas cooling equivalent refrigeration goes up to between 5 and 6 W per magnet (between 0.29 and 0.35 W per meter of magnet length). These numbers should be compared to the lead equivalent refrigeration requirements for the continuous correction coil and the lumped correctors. A careful analysis of what corrections are needed and not needed will undoubtedly reduce the lead current requirements below the estimates given here.

4. Concluding Comments

It appears that superconducting wedges can be used to correct the field along the SSC dipole. It is proposed that these correctors could be used in place of bore tube correction coils. Wedge correction will correct out more multipoles than one or two bore tube correction coils. The three inner wedges can be used to correct all terms from $N=1$ (dipole) to $N=6$ both normal and skew. The price one pays for this is a probable increase in lead refrigeration,

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additional insulation on the wedges, additional complication in the fabrication of the wedges and the problem associated with getting the currents in and out of the ends. In the process, one eliminates the bore tube correction coils and perhaps one simplifies the lumped correction system. The power supplies required for the wedge correction system pose an additional problem area.

Acknowledgment

The author would like to thank Tom Chan for producing the drawing of the coil cross-section. The author also acknowledges his interaction with J. M. Peterson and R. Talman.

References

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2. Brookhaven National Laboratory Drawing 22.273.01-5A, March 28, 1986.
3. K. Halbach, Nuclear Instruments and Methods 78 (1970), pp 185-198.
4. J. D. Jackson, "Conduction of Heat of Synchrotron Radiation Through the Wrapped Beam Tube Wall", SSC-N-235, September 1986.

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APPENDIX

The field generated by a single current I traveling perpendicular to the x - y plane can be represented in complex form $z = x + iy$ as follows.

$$H^*(Z) = \frac{I}{2\pi i} \frac{1}{Z - Z_c} \quad (1)$$

Where $H^*(Z)$ is the complex conjugate of the field $H(Z) = H_x - iH_y$ of the field generated at complex location Z by the current I at complex location Z_c .

The field in Equation 1 can be expanded in a Taylor series. The expansion of equation 1 is as follows:

$$H^*(Z) = \sum_{n=1}^{\infty} a_n Z^{n-1} \quad (2)$$

Where Z_c is the radius of convergence of the series, and $|Z| \leq |Z_c|$ such that the point Z is inside the radius of convergence of the series which has its origin at $Z = 0$.

For the general case of a single current point I at Z_c , the expansion coefficient a_n takes the following general form:

$$a_n = \frac{-I}{2\pi i} Z_c^{-n} \quad (3)$$

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where $n = 1$ is a dipole, $n = 2$ is a quadrupole, $n = 3$ is a sextupole, and so on. a_n has both a real and imaginary part. The imaginary part of $a_n = \text{Im} |a_n|$ is the normal term. The real part of $a_n = \text{Re} |a_n|$ is the skew term. A normal dipole is $\text{Im} |a_1|$; a skew dipole is $\text{Re} |a_1|$; a normal quadrupole is $\text{Im} |a_2|$; a skew quadrupole is $\text{Re} |a_2|$, and so on.

In the design of a dipole, symmetry is usually involved. A current I at θ has a current I at $-\theta$ and currents $-I$ at $\pi + \theta$ and $\pi - \theta$. This is the so-called dipole symmetry. Correction superconductor in the wedges is located symmetrically because the wedge location is symmetric. If dipole symmetry is invoked, a_n will take the following general form when θ_c is the angle of the symmetric current and r_c is the radius of the symmetric current:

$$a_n = -\frac{2I}{\pi i} \cos(N\theta_c) r_c^{-n} \quad (4a)$$

A

for $n = 1, 3, 5$, and so on.

$$a_n = 0 \quad (4b)$$

for $n = 2, 4, 6$, and so on. One can note that the non-zero term for a_n is imaginary, so symmetry not only eliminates the even multipoles $n = 2$ (quadrupole), $n = 4$ (octupole), $n = 6$, and so on, but it also eliminates all of the skew terms as well. (There is no non-zero real part.)

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If one introduces the concept of unsaturated circular iron $\mu = \infty$ with the center of the iron at the origin coincident with the center of the series at $Z = 0$, one can invoke the method of images. (The image of the current is within the iron shell.) With circular iron of radius R , and with $\mu = \infty$ and its center coincident with the center of the Taylor series, the Taylor series takes the following form:

$$H^*(Z) = \sum_{n=1}^{\infty} c_n Z^{n-1} \quad (5)$$

where

$$c_n = a_n + b_n \quad (6)$$

The a_n term (the term due to the current I) is defined by Equations 3 or 4a - 4b. The b_n term (the term due to the image current I in the iron shell) is defined as follows when symmetry is not invoked:

$$b_n = \frac{-I}{2\pi i} \frac{(Z_c^*)^n}{R^{2n}} \quad (7)$$

Where Z_c^* is the complex conjugate of Z_c .

The value of b_n in Equation 7 has both an imaginary term for normal multipoles and a real term for the skew multipoles.

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Dipole symmetry can be invoked in the same way it was for Equation 3 to yield a symmetrical form for the image currents which have a value I at $+\theta_c$ and $-\theta_c$ and a value $-I$ at $\pi + \theta_c$ and $\pi - \theta_c$. When dipole symmetry is used, the image current expansion coefficient takes the following form for symmetrical current a : radius r_c and symmetry angles θ_c , $-\theta_c$, $\pi + \theta_c$ and $\pi - \theta_c$:

$$b_n = -\frac{2I}{\pi i} \cos(n\theta_c) \frac{r_c^n}{R^{2n}} \quad (8a)$$

When $n = 1, 3, 5$, and so on; and

$$b_n = 0 \quad (8b)$$

When $n = 2, 4, 6$, and so on. Like the symmetrical a_n term, there is no non-zero real part, so the Taylor series of the symmetrical dipole can only have normal odd multipole $n = 1, 3, 5 \dots$. For more information see Reference 3.