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LIMITS ON SYSTEMATIC SKEW MULTIPOLE COMPONENTS IN SSC DIPOLES OF THE 90 DEGREE LATTICE FROM LINEAR APERTURE REQUIREMENTS

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ABSTRACT

Limits on the systematic skew multipole components in the SSC dipoles were derived by applying the linear aperture criteria to an analysis of the 90 degree standard lattice.

INTRODUCTION

Limits on the allowable systematic b_n in the SSC dipoles have been calculated from the tune shift criterion (which requires that the tune shift be less than 0.005) for the 90 degree lattice (Garren/Johnson L90). The procedure for determining the tolerances are the same as in Reference 1, i.e. the limits on the systematic skew multipoles are determined from both the linear aperture criteria and the demand that the coupling constant (defined below) be less than 0.005.

ANALYSIS AND RESULTS

In this study, the tunes of the 90 degree standard lattice (Reference 2) were adjusted to 96.265 in x, and 95.285 in y. As previously (Reference 1), the integer tune split was necessary to reduce the effects of coupling. The lattice was studied with TEAPOT (Reference 3) without random multipole errors in the dipoles.

First the coupling constant and tune shifts at zero amplitude were computed for systematic a_n. The coupling constant is a measure of the coupling strength which is comparable to the tune shift and is defined

$$|C| = \left| \frac{1}{4\pi} \sum q_s \sqrt{\beta_x \beta_y} e^{-i(\phi_x - \phi_y)} e^{-i\theta(k - \nu_x + \nu_y)} \right|$$

$$\approx \left| \frac{2 det^{1/2} m}{4\pi(sin\mu_x + sin\mu_y)} \right|$$

where m is the lower 2×2 block of the usual 4×4 transfer matrix for the machine (Reference 4), and the sum is over all the skew elements in the machine. k is the integer closest to $\nu_x - \nu_y$ and $\theta = \int \frac{ds}{R}$. $2|C|$ is the distance of closest approach of the perturbed tunes in a coupled machine. It is assumed that the x-y coupling is approximately the same as the y-x coupling.

Table 1. summarizes the results. The upper limits on the systematic a_n are determined by the criteria that the tune shifts in x and y be less than 0.005

and the coupling constant be less than 0.005 (at $\delta = 0.001$ for $n \geq 2$). In the next step a more stringent limit using the linear aperture criteria is set. After the scan in a_1 the values for a_2, a_3 and a_4 for which the tune shifts were the maximum allowed could be predicted using the fact that the coupling constant is linear in the strength and the tune shift goes like the strength squared. For a_1, a_2 the limits are set by the tune shift criterion, while for a_3 and a_4, a clear cut distinction of the limiting criterion between the tune shift and coupling constant cannot be made. The limiting values are $a_1 \leq 0.090 \times 10^{-2} m^{-1}$, $a_2 \leq 0.20 m^{-2}$, $a_3 \leq 0.65 \times 10^2 m^{-3}$, and $a_4 \leq 2.2 \times 10^4 m^{-4}$. Compared with the corresponding results of the 60 degree $(2,4)_b$ lattice (reference 1), the values lie noticeable higher.

		C	ΔQ_x	ΔQ_y
$a_1(m^{-1})$	0.070×10^{-2}	0.0003	0.003	-0.003
	0.080×10^{-2}	0.0003	0.004	-0.004
	0.100×10^{-2}	0.0007	0.006	-0.006
$a_2(m^{-2})$	0.100	0.0023	0.001	-0.001
	0.150	0.0034	0.003	-0.002
	0.200	0.0046	0.004	-0.004
$a_3(m^{-3})$	0.450×10^2	0.0034	0.002	-0.002
	0.600×10^2	0.0044	0.004	-0.004
	0.700×10^2	0.0052	0.006	-0.005
$a_4(m^{-4})$	0.900×10^4	0.0020	0.001	-0.001
	2.000×10^4	0.0044	0.004	-0.004
	2.250×10^4	0.0050	0.005	-0.005

Table 1. Coupling constant and zero amplitude tune shifts at $\delta = 0.001$ ($\delta = 0.000$ for a_1) for various values of a_n.

These parameters are subjected now to the more stringent linear aperture criteria, which require that the tune shift be 0.005 or less for an amplitude of $7mm$ at $\hat{\beta}$ in the arcs on-momentum, and $5mm$ at $\delta = 0.001$. Under the same conditions, the smear is required to be 10% or less. These conditions give the limits on the systematic a_{1-4} in Table 2. These limits are set in general by the off-momentum amplitude dependent tune shift.

		δ	smear(%)	ΔQ_x	ΔQ_y
$a_1(m^{-1})$	0.090×10^{-2}	0.000	0.9	-0.0006	-0.0005
		0.001	0.5	-0.0003	-0.0002
		-0.001	0.9	-0.0003	-0.0002
$a_2(m^{-2})$	0.20	0.000	1.0	-0.0006	-0.0005
		0.001	0.5	-0.0004	-0.0002
		-0.001	0.9	-0.0003	-0.0003
$a_3(m^{-3})$	0.45×10^2	0.000	1.4	-0.0047	-0.0033
		0.001	4.2	-0.0047	-0.0016
		-0.001	2.3	-0.0043	-0.0027
$a_4(m^{-4})$	0.70×10^4	0.000	0.9	-0.0006	-0.0005
		0.001	7.7	-0.0043	-0.0009
		-0.001	0.9	-0.0045	-0.0012

Table 2. Smear and tune shifts for $7mm$ (at $\delta = 0.0$) and $5mm$ (at $\delta = 0.001$) particles for maximum permissible values of a_n.

CONCLUSIONS

Limits on the systematic skew multipole errors in the SSC dipoles have been arrived at from linear aperture considerations for the 90 degree standard lattice. Those limits are $a_1 \leq 0.09 \times 10^{-2} m^{-1}$, $a_2 \leq 0.20 m^{-2}$, $a_3 \leq 0.45 \times 10^2 m^{-3}$,

$a_4 \leq 0.70 \times 10^4 m^{-4}$. These limits when expressed in the units used by the magnet errors group (parts per 10^4 at $1cm$) are 0.09, 0.20, 0.45, and 0.70 and are in general a factor of 2 larger than those for the 60 degree lattice.

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