SSC-MAG-125
SSC-N-300

EFFECTIVE MODULUS OF COLLARS WITH UNIFORM INTERNAL PRESSURE

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February 11, 1987

Some two and a half years ago, Ken Mirk and Craig Peters conspired to test some aluminum, 25-mm-thick collars*. A uniform internal pressure was applied by poking a plunger into a rubber plug; the changes in horizontal and vertical diameters were measured as a function of the pressure.

Using their data I have calculated an effective modulus for the collars. The number I get is 3.37 Mpsi, compared with 10.0 Mpsi for solid aluminum. Because of the way the collars are laminated, one would expect to get 5 Mpsi at the very most; the reduction to 3.37 Mpsi can easily be accounted for by local deformations and the simplisticity of the analytical model.

There is no assurance that the actual/solid factor of .337 can be applied to the current 15-mm collars, or to stainless steel collars. It would seem prudent to find out, as this number is crucial to any prediction of coil cooldown behavior. I'd be pretty surprised if the .337 didn't turn out to be a universal constant.

But occasionally, even I get surprised!

The uniform-pressure loading applied by the rubber plug is not the same kind of loading applied by the prestressed coil. If I knew of a better method than the rubber-plug method, I'd have mentioned it. Simply clamping a real coil, and measuring the coil hoop stress with our little strain-gage devices presents its own difficulties.

My analysis follows.

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* Report LBL-18482, SSC MAG Note 25, October 11, 1984.

σ = circumferential stress at outer surface

ϵ = circumferential strain at outer surface

p = internal pressure

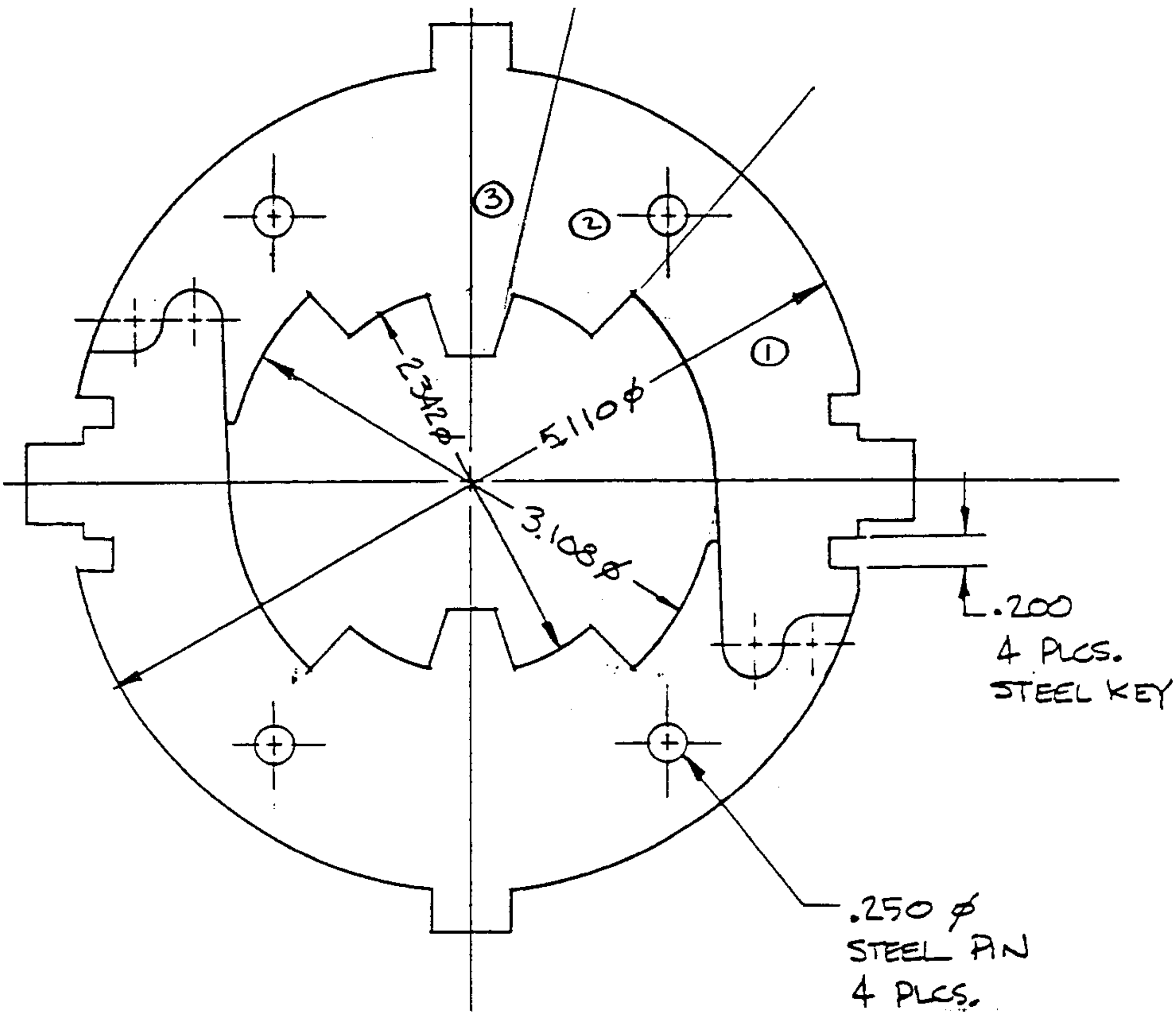
E = elastic modulus

θ = azimuthal extent of section

a = inside radius

b = outside radius

D = outside diameter



The collar is treated as if it consists of four sets (one per quadrant) of sections, three sections per quadrant, and each acting as if it is part of a complete ring; that is, there is no shear on radial planes, and the radial and hoop stresses are independent of azimuth. That's a big lie, but it will have to do.

First, however, we'll treat the collar as if it consisted entirely of Section 1, the thin part. Then we'll fudge the answer to account for the added stiffness of Sections 2 and 3.

For a complete ring, the ratio of hoop stress on the outer surface to internal pressure is

$$\frac{\sigma}{p} = \frac{2a^2}{b^2-a^2} = \frac{2\times 1.554^2}{2.555^2-1.554^2} = 1.1743$$

The strain at the outside is

$$\frac{\epsilon}{p} = \frac{\sigma/p}{E}$$

and so the change in diameter is

$$\Delta D = \epsilon D,$$

$$\frac{\Delta D}{p} = \frac{(\sigma/p)D}{E}$$

Solving for the modulus we get

$$E = \frac{(\sigma/p)\,D}{\Delta D}\,p$$

$$= \frac{1.1743\times 5.110}{.0071^*}\times 5600^* = 4.733\times 10^6$$

For the three-section actual collar, we calculate the stress/pressure ratio of each section as above, and multiply it by the fraction of the circumference occupied by each kind of section. This gives a greater stiffness for the same modulus, so we must multiply the modulus calculated above for the ratio of the real stiffness to that of the uniform-thickness collar.

a	b	$\sigma/p = 2a^2/(b^2-a^2)$	θ	$\frac{\sigma}{p}\frac{\theta}{90}$
1.554	2.555	1.1743	49.0	.6393
1.171	"	0.5318	28.4	.1678
0.775	"	0.2027	12.6	.0284
			90.0	0.8355

Finally,

$$E = 4.773\times 10^6\,\frac{0.8355}{1.1743} = 3.37\times 10^6 \text{ psi}$$

* From Fig. 10 of LBL 18482