Estimate of Impedance due to
Finite Grain Size of copper coating on the beam pipe

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The copper coating on the inner wall of the beam pipe has finite grain size, leading to a non-smooth pipe wall which contributes to some impedance as seen by the beam. In this note, we will give an order of magnitude estimate of this impedance contribution.

We assume the copper coating is infinitely conducting. Let the grain size be g. For an order of magnitude estimate, we model the wall graininess by a collection of cylindrically symmetric cavity-like structures with cavity depth g. The total number of such cavities around the SSC is taken to be

$$m = \frac{2\pi R}{2g} \tag{1}$$

i.e. there is on the average a cavity structure every distance of 2g over the entire circumference of $2\pi R$. We believe this model will over-estimate the impedance.

From the literature (Bane and Ruth, Bisognano and Ng, SSC-SR-1017, Impedance Workshop Report), we obtain the impedance at low frequencies for a single bellows:

$$\begin{aligned} \mathrm{Im}\left(\frac{Z_{\parallel}'}{n}\right) &= \frac{m Z_0 g}{2\pi R} \ln S \\ &\approx \frac{Z_0}{2}\frac{g}{b} \end{aligned} \tag{2}$$

$$\begin{aligned} \mathrm{Im}(Z_{\perp}) &= m Z_0 \frac{g}{\pi b^2} \frac{S^2-1}{S^2+1} \\ &\approx Z_0 \frac{R}{b^2}\frac{g}{b} \end{aligned} \tag{3}$$

where $S = (1+g/b)$ and $Z_0 = 377\ \Omega$, and we have assumed $g \ll b$

A side remark. In case the vacuum chamber consists of a continuum of cavities of the size of the order of the pipe size, i.e. $g \approx b$, this ill-designed storage ring will have a $Z_{||}/n$ of the order of $Z_0/2$, or about 190 Ω according to this picture.

For the SSC, taking $g = 5\ \mu m$, $b = 1.65$ cm, $2\pi R = 83$ km, we obtain $Im(Z_{||}/n) = 0.06\ \Omega$, and $Im(Z_{\perp}) = 6\ M\Omega/m$. These are small compared with the total SSC impedance $Z_{||}/n = 0.2\ \Omega$ and $Z_{\perp} = 40\ M\Omega/m$ estimated in the Conceptual Design Report (page 182).

The above estimate is approximately correct up to a frequency roughly of the order of (Bane and Ruth)

$$\omega_R \approx 1.79\ \frac{c}{b}\left(\frac{g}{b}\right)^{-0.55} \tag{4}$$

which for the SSC parameters is about $3\times10^{12}\ s^{-1}$. Since the bunch length is about 7 cm, corresponding to an angular frequency of $4\times10^{9}\ s^{-1}$, the low-frequency approximation is quite adequate.

To obtain an estimate of parasitic loss, we need to estimate the real part of $Z_{||}$. To do so, we model the impedance as a single resonator with resonant frequency ω_R and quality factor Q. The real part of $Z_{||}$ is then given by, for $\omega \ll \omega_R$,

$$Re\,(Z_{||}) = \frac{\omega}{\omega_R Q}\ Im\,(Z_{||}) = \frac{\omega^2}{\omega_R Q}\ \frac{Rg}{2cb}\ Z_0 \tag{5}$$

The parasitic loss power per beam due to this impedance is

$$P = M\left[\frac{N_B^2 e^2 \omega_0}{2\pi^2}\right]\int_{-\infty}^{+\infty} d\omega\ Re\,(Z_{||})\ \exp\left[-\frac{\omega^2\sigma_z^2}{c^2}\right]$$

$$= M\left[\frac{N_B^2 e^2}{8\pi\sqrt{\pi}\ \omega_R Q}\right]\left[Z_0\ \frac{g\,c^3}{b\,\sigma_z^3}\right] \tag{6}$$

where $\omega_0 = c/R$, $M = 1.7 \times 10^4$ is the number of bunches, $N_B = 7.3 \times 10^9$ is the number of particles per bunch, σ_z = 7 cm is the rms bunch length. Letting ω_R be given by eq.(4) and Q=1, the parasitic loss due to the grainy wall is found to be 1.6 watts, which is negligible compared with the resistive wall parasitic loss heating of 71 watts per beam (CDR, page 177).

Conclusion: Grain size of 5μm on the copper coating is acceptable for the SSC vacuum chamber pipe from the impedance point of view. Substantially larger grain size, however, is to be avoided.