SSC-N-284
January 1987

# IR Quadrupole Multipoles and the SSC Linear Aperture

David Neuffer
Texas Accelerator Center
2319 Timberloch Place
The Woodlands TX 77380

Previous studies have noted the importance of the multipole components of the SSC dipoles in limiting the linear aperture of the SSC.[1] The multipole components in the cell quadrupoles set somewhat less serious constraints, because of the relatively small fraction of the SSC circumference covered by these quads. However, for high luminosity operation the SSC requires focusing of the beam down to very small sizes at the interaction region (IR) centers and this focusing is obtained by relatively strong IR quadrupoles. The strengths of the IR quads and the enlarged size of the beam passing through them magnifies the nonlinear effects of the quad multipole components. The effects have already been noted in tracking studies, which have demonstrated great reductions in the dynamic aperture due to IR quad multipoles.[2] In this note we explore the effects of these multipoles on amplitude dependent tune shifts and estimate the resulting constraints on the SSC linear aperture.

The linear aperture of the SSC is defined by the constraint that the motion be substantially linear. This implies a limitation on the amplitude and momentum dependent tune shifts of the form:

$$\Delta\nu \lesssim 0.005 \qquad (1)$$

over the entire aperture. In the IR quads the dispersion function $\eta(s)$ is nearly zero, and will therefore be ignored. To lowest order, there is consequently no momentum dependent tune shift due to the IR quad multipoles and only amplitude dependence is included. We simplify the dynamics to 1-D motion of the form:

$$x = A_x \cos(\varphi(s))$$

where $\varphi(s)$ is the betatron phase and $A_x(s)$ is the amplitude of the motion ($A_x(s) \propto \sqrt{\beta(s)}$ ). In this approximation, the tune shift due to a multipole of order n with strength given by $B_0 b_n(s)$ is given by

$$\Delta\nu_x = \frac{1}{2\pi} \int \frac{\beta_x(s)\ B_0 b_n(s)}{A_x(s)\ B\rho} \left(A_x(s)\ \cos(\varphi(s))\right)^n \cos(\varphi(s))\ ds \qquad (2)$$

where in the present case we only include contributions from the IR quads. This expression may be estimated by inserting averaged IR values for the various quantities:

$$\Delta\nu_x \cong \frac{\beta_{max} B_0 \bar{b}_n\ L_q\ A_x^{(n-1)} \sqrt{N_q}\ F}{2\ \pi\ B\rho}$$

where

$$F = \frac{1 \cdot 3 \cdot \ldots \cdot\ n}{2 \cdot 4 \cdot \ldots \cdot (n+1)}$$

and we note that $A_x$ is evaluated in the IR quads and is, approximately, $\sqrt{(\beta_{max}/\beta_{arc})}$ greater than in the arcs. For the SSC with an IR low-$\beta$ value of 0.5 m, $\beta_{max} \cong 8000$ m. Typical values of the other parameters are $B_0 \cong 2$ T, $L_q \cong 15$ m, $N_q \cong 16$, and $B\rho \cong 66700$ T-m.

The above expression assumes the multipole strengths are randomly distributed from quad to quad with rms strength of $b_n$ per quad. Tune shifts due to systematic multipoles are $\sqrt{N_q}$ greater. Because of the statistically small number of IR quads, the difference between systematic and random multipole tune shifts is not as marked as in the case of the cell dipoles, and random IR quad multipoles can cause large tune shifts.

Equations 1 and 2 maybe combined to set a limit on the allowable random $b_n$ component as a function of $A_x$ by solving:

$$\Delta\nu(A_x, b_n) = .005$$

using SSC parameters. Results are displayed in Table 1. The amplitudes in the IR quads can be converted to their reference values in the SSC arcs ($\beta_{arc} \cong 330$ m) which are a factor of 4.9 smaller; see Table 1. The results are to be compared with current estimates of SSC quad multipole content; these obtain $b_n \sim 10^{-4}$ to $10^{-5}$ $cm^{-n}$.

The SSC design goal is a linear aperture of ~ 0.7 cm in the arcs for injection operation ,[3] a goal which is extremely difficult with low-$\beta$ IR optics. A linear aperture up to $A_x$= 3.5 cm would be required in the IR quads and the resulting multipole constraints are unrealistic (see Table 1). This is particularly unrealistic since the physical aperture of the IR quads is not expected to be greater than ~ 2 cm, and must include sufficient space for beam separation (~ 4 mm) and closed orbit error. Linear aperture in the IR quads will be limited to the order of 1 cm (0.2 cm in the arcs). This number may be compared with the rms beam size, which is $\sigma \sim .063$ cm at 20 TeV (.013 in the arcs) and, with low-$\beta$ optics, $\sigma \sim .28$ cm at 1 TeV (.057 cm in the arcs). Linearity up to 6 $\sigma$ with an allowance for closed orbit error is a minimum SSC requirement, and this goal cannot be reached at injection energy with low-$\beta$ optics. The SSC is therefore required to inject and accelerate without low-beta optics, and to switch on low-$\beta$ optics only at high energy after a stable small amplitude closed orbit is established. The multipole content in the IR quads then becomes the dominant constraint in determining the linear aperture, which becomes ~ 1.0 cm (0.2 cm in the arcs) with $b_n \sim 10^{-4}$ $cm^{-n}$ in the IR quads. These conclusions are in good agreement with tracking studies.[2]

The nonlinear multipole content of the IR quads is found to severely restrict the SSC linear aperture in the low-$\beta$ optics configuration. The restrictions are probably not intolerable in a tuned SSC with the low-$\beta$ optics only on at high energy. The Conceptual Design includes plans to compensate IR quad mutipoles to $\lesssim$ .1 of uncorrected values which should place $b_n \lesssim 10^{-5}$ $cm^{-n}$,[3] which should permit tolerable colliding aperture.

## References

1. D. Neuffer, SSC-N-113, August 1985.

2. F. Dell, SSC-n-132, 1986.

3. Supercollider Conceptual Design Report, April 1, 1986.

Table: Random Multipole Strength Tolerances for the SSC IR Quads for the Colliding Beam Lattice

| | | | | | | |
|---|---|---|---|---|---|---|
| $A_x$ amplitude (IR quads) | 3.44 | 2.46 | 1.97 | 1.48 | 0.985 | cm |
| $A_x$ amplitude (Arc Cells) | 0.7 | 0.5 | 0.4 | 0.3 | 0.2 | cm |
| | Multipole Tolerances ($\times 10^{-4}$ $cm^{-n}$) | | | | | |
| $b_3$ | 0.049 | 0.096 | 0.15 | 0.27 | 0.60 | |
| $b_5$ | 0.005 | 0.019 | 0.046 | 0.15 | 0.75 | |
| $b_7$ | 0.00047 | 0.0036 | 0.014 | 0.077 | 0.88 | |
| $b_9$ | 0.000044 | 0.00066 | 0.0039 | 0.039 | 1.00 | |