CROSSING TRANSITION IN THE INTERMEDIATE ENERGY BOOSTER

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Abstract

The Intermediate Energy Booster (IEB), considered as substitute of both LEB and MEB, requires transition crossing. In order to minimize the enhancement of the longitudinal emittance, fast γ_{tr}-jump and other RF gymnastics are proposed.

1. Introduction

In a previous note[1] a set of Intermediate Energy Boosters has been analyzed rather throughly: a relevant problem left behind, the transition crossing, is now going to be studied.

As done in Ref. 1, the transition gamma is the same, i.e. $\gamma_{tr} = 8.6$, for the five synchrotrons considered. All the "troubles" afflicting the transition-crossing arise from the fact that the transition parameter

$$\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2} \qquad (1)$$

becomes zero when the Lorentz relativistic factor γ equals γ_{tr} during the accelerating cycle.

Even though the transition-drawbacks will be discussed as soon as they will be met, it is anticipated that an obvious cure[2-5] consists of making as small as possible the time interval during which η is close to zero.

Fig. 1 shows how this can be achieved, just varying γ_{tr} without affecting[5,6] Q_H.

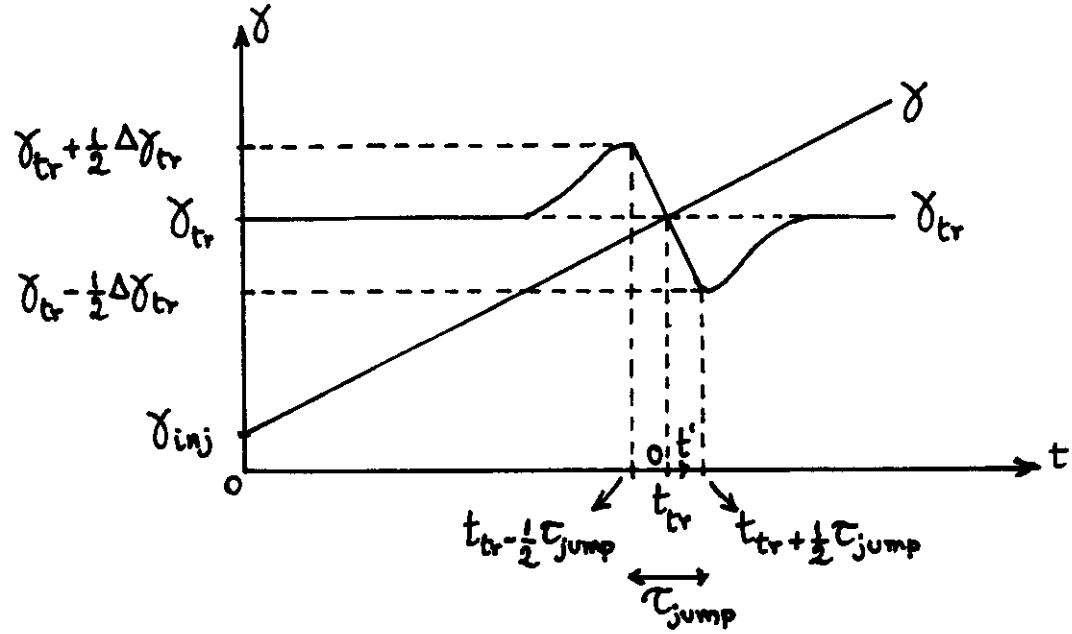

Fig. 1. Speeding up the crossing of transition by a $\gamma_{transition}$-jump.

Considering the linear (negative) ramp of γ_{tr} is much steeper than the steady increase of the proton energy, one has about transition:

$$\gamma_{tr}(t') \simeq \gamma_{tr} - \dot{\gamma}_{tr} t' \qquad (2.a)$$

$$\gamma(t') = \gamma_{inj} + \dot{\gamma}(t_{tr} + t') \qquad (2.b)$$

with

$$t' = t - t_{tr}$$

and

$$\dot{\gamma}_{tr} \simeq \frac{\Delta\gamma_{tr}}{\tau_{jump}} \qquad (3.a)$$

$$\dot{\gamma} = 2\,\frac{\gamma_{Max} - \gamma_{inj}}{\tau_{IEB}} \qquad (3.b)$$

with

$$\dot{\gamma}_{tr} \gg \dot{\gamma} \qquad (4)$$

Indeed the steady ramp should regard $\beta\gamma$ rather than γ ; nevertheless the choice of $\dot{\gamma}$ is correct : in fact $(\beta\gamma)_{tr} \simeq 8.54 \lesssim 8.6 = \gamma_{tr}$, $(\beta\gamma)_{Max} \simeq \gamma_{Max}$, while the sharper difference between $(\beta\gamma)_{inj}$ and γ_{inj} has a smaller weight.

Equations (2) and (3.b) can be worked out by means of data available in Ref. 1 ; instead eq. (2a) needs a few comments.

Successful operations in the CERN PS have been achieved [5,7] with $\Delta\gamma_{tr} \simeq 4$ and $\tau_{jump} \simeq 0.4$ ms , or roughly 200 revolutions : then $\dot{\gamma}_{tr} \simeq 10^4$. Since the microwave stability limit [8] requires a γ_{tr}-jump as fast as possible , as it will be seen in Sect. 2 , for the IEB one can envisage $\Delta\gamma_{tr} = 4$ (as for CERN PS) and $n_{rev} = 100$ revolutions (twice faster than CERN PS), being $n_{rev} = \tau_{jump} / \tau_{rev}$. Then eq. (3a) gives :

$$\dot{\gamma}_{tr} = \frac{0.04}{\tau_{rev}} \qquad [s^{-1}] \qquad (5)$$

Making use of eqs. (2), (3), (5) and bearing in mind all data of interest taken from Ref. 1 , one obtains the following Table I , where the last column demonstrates how much the condition (4) is fulfilled.

Table I

p_M (GeV/c)	$\dot{\gamma}_{tr}$ (s^{-1})	$\dot{\gamma}$ (s^{-1})	$\dot{\gamma}_{tr}/\dot{\gamma}$
40	1.67×10^4	40.2	415
50	1.25×10^4	36.1	347
60	1.04×10^4	40.6	257
75	8.37×10^3	38.3	218
100	6.27×10^3	51.3	122

Taking into account that the slope of the accelerating ramp can be written as $\dot{\gamma} = (\gamma_{tr} - \gamma_{inj})/t_{tr}$, eq.(2b) becomes

$$\gamma(t) = \gamma(t') = \gamma_{tr} + \frac{\gamma_{tr} - \gamma_{inj}}{t_{tr}} t' \qquad (6)$$

Introducing eqs. (2a) and (6) into eq. (1) one obtains for small values of t':

$$\eta \simeq -\frac{2}{\gamma_{tr}^3}(\dot{\gamma}_{tr} - \dot{\gamma})t'$$

which, due to eq. (4), turns into

$$\eta \simeq -\frac{2\dot{\gamma}_{tr}}{\gamma_{tr}^3} t' \qquad (7)$$

2. Microwave Stability Limit

Close to transition energy the longitudinal stability criterion

$$\left|\frac{Z_{\parallel}}{n}\right| \leq \frac{1}{2} F |\eta| \beta^2 \gamma \frac{V_p}{\hat{I}} \left(\frac{\Delta p}{p}\right)^2 \qquad (8)$$

no longer applies because $|\eta| = 0$

On the other hand the rise-time of the instability

$$\tau_s = \tau_{rev} \sqrt{\frac{\beta^2 \gamma (V_p / \hat{I})}{2\pi |\eta| n^2 |Z_{\parallel}/n|}} \qquad (9)$$

becomes infinite for the same reason.

This means that, when the beam becomes unstable as $|\eta|$ approaches zero, the instability grows very slowly.

Note that:

$Z_{\parallel}$ = longitudinal impedence
n = mode number of the μ-wave
F = form factor (often set equal to 1)
V_p = 938 MV
$\hat{I}$ = peak current
$\frac{\Delta p}{p}$ = r.m.s. momentum spread

A good model[8] for the increase of the longitudinal emittance S is given by the equation

$$\frac{1}{S}\frac{dS}{dt'} = \frac{1}{\tau_s(t')} \qquad \left(\frac{d}{dt} = \frac{d}{dt'}\right) \tag{10}$$

with $\tau_s(t')$ given by eq. (9), where $|\eta|$ is replaced by eq. (7); hence:

$$S = S_0\, e^{\alpha_s} \tag{11}$$

with

$$\alpha_s = \int_0^T \frac{dt'}{\tau_s(t')} = \frac{4}{3}\frac{1}{\beta_{tr}\gamma_{tr}^2}\frac{1}{\tau_{rev}}\sqrt{\pi\dot{\gamma}_{tr}\left|\frac{Z_{\parallel}}{n}\right|\frac{\hat{I}}{V_p}}\; T^{3/2} \tag{12}$$

where T is the time during which eq. (8) is satisfied, i.e.

$$T = \frac{1}{F\dot{\gamma}_{tr}}\left(\frac{\gamma_{tr}}{\beta_{tr}}\right)^2\left|\frac{Z_{\parallel}}{n}\right|\frac{\hat{I}}{V_p}\frac{1}{\left(\frac{\Delta p}{p}\right)^2} \tag{13}$$

having again considered eq. (7).

Then combining eqs. (12) and (13) one obtains:

$$\alpha_s = \frac{4}{3}\frac{\pi^{1/2}}{F^{3/2}}\frac{n}{\dot{\gamma}_{tr}\tau_{rev}}\frac{\gamma_{tr}}{\beta_{tr}^4}\left|\frac{Z_{\parallel}}{n}\right|\left(\frac{\hat{I}}{V_p}\right)^2\frac{1}{\left(\frac{\Delta p}{p}\right)^3} \tag{14}$$

The peak current is $\hat{I} = \langle I \rangle / B$, where $B = 0.25$ is the bunching factor and $\langle I \rangle = N_T e \beta c / C$ is the average current ; besides[1] $N_T = N_{ssc} / n_{batches}$ and $C = S_B h$, then :

$$\hat{I} = \frac{N_{ssc}\, e c \beta_{tr}}{B S_B h n_{batches}} \simeq 0.365\ A \qquad (15)$$

having put

$$N_{ssc} = 1.092 \times 10^3 \quad , \quad S_B = 4.8\ m \ , \quad h n_{batches} \simeq 1190$$

$$\beta_{tr} = \sqrt{1 - \frac{1}{(8.6)^2}} = 0.9932$$

Note that $\langle I \rangle = B \hat{I} \simeq 90\ mA$, close to the values shown in Table 4.10.1 of Ref. 9 .

Other quantities are :

$\dot{\gamma}_{tr} \tau_{rev} = 0.04$, as stated before ; $\left| \frac{Z_{\parallel}}{n} \right| = 10\ \Omega$;

$n = \tau_{rev} f_{\mu\text{-wave}} \simeq (\text{a few } \mu s) \times (3\ GHz) \simeq 10^4$;

$\frac{\Delta p}{p} = \pm 10^{-3}$ = momentum spread in the MEB[9].

Inserting all these data into eq. (14) one obtains

$$\alpha_s = 0.081$$

or

$$\frac{S}{S_0} = 1.084$$

i.e. the longitudinal emittance of all the five Intermediate Energy Boosters grows of the 8.4 % only .

3. Lengthening of the Bunch

Longitudinal space-charge forces can induce[10] bunch oscillation during transition crossing, because these forces counteract the RF fields below transition and foster the RF fields above transition.

This dual behaviour makes the bunch longer before transition and shorter above : the matching between the two different lengths can give rise to an oscillation of the bunch-length.

For a parabolic-shaped bunch, of equilibrium length L_B, the longitudinal space-charge force, acting on a particle inside the bunch, is[10,11]

$$F_{//} = \frac{3 N e^2 g}{\pi \varepsilon_0 L_B^3 \gamma^2} s \qquad (16)$$

where

N = number of protons per bunch = 10^{10}

$g = 1 + 2 \ln(\text{vacuum-pipe-radius} / \text{beam-radius}) \simeq 4.5$

$s = R\theta = R\frac{\varphi}{h} = R\frac{\phi - \phi_s}{h}$

with R = average machine radius, θ = machine azimuth, h = harmonic number, $\phi(\phi_s)$ = RF phase (sinchronous)

The modified Hamiltonian for synchrotron oscillations becomes :

$$H(\phi, W) = \frac{h\omega_{rev}^2 \eta}{2\beta^2\gamma mc^2} W^2 - \frac{eV}{2\pi}\left[\cos\phi - \cos\phi_s + (\phi-\phi_s)\sin\phi_s\right] -$$

$$- \frac{3g}{4\pi^2} \frac{Ne^2c^2\beta}{\varepsilon_0 L_B^3 \omega_{rev}^2 h^2\gamma^2} (\phi-\phi_s)^2 \qquad (18)$$

where

$$W = \frac{\Delta E}{\omega_{rev}} = \frac{eV}{\omega_{rev}} (\sin\phi - \sin\phi_s) \qquad (19)$$

$V =$ RF crest-voltage $=$ 600 KV

$\phi_s = 30°, 150°$

The Hamilton equations are

$$\dot{\phi} = \frac{\partial H}{\partial W} = \frac{h\omega_{rev}^2}{\beta^2\gamma mc^2}\eta W \qquad (20,a)$$

$$\dot{W} = -\frac{\partial H}{\partial \phi} = -\frac{eV}{2\pi}(\sin\phi - \sin\phi_s) + \frac{3g}{2\pi^2}\frac{Ne^2c^2\beta(\phi-\phi_s)}{\varepsilon_0 L_B^3\omega_{rev}^2 h^2\gamma^2}$$

or

$$\dot{W} = -\frac{\omega_{rev}}{2\pi} W + \frac{3g}{2\pi^2}\frac{Ne^2c^2\beta}{\varepsilon_0 L_B^3\omega_{rev}^2 h^2\gamma^2}(\phi-\phi_s) \qquad (20,b)$$

Performing a further time-derivation of eq. (20,a) and taking into account eq. (20,a) again, one obtains:

$$\ddot{\phi} = \frac{h\omega_{rev}^2}{\beta^2\gamma mc^2}(W\dot{\eta} + \eta\dot{W}) = \frac{\dot{\eta}}{\eta}\dot{\phi} + \frac{h\omega_{rev}^2}{\beta^2\gamma mc^2}\eta\dot{W} \qquad (21)$$

Since $\frac{d}{dt} = \frac{d'}{dt'}$ and $\frac{d^2}{dt^2} = \frac{d^2}{dt'^2}$, eq. (7) gives $\eta = \dot{\eta}t'$ and eq. (21) becomes:

$$\frac{d^2\varphi}{dt'^2} - \frac{1}{t'}\frac{d\varphi}{dt'} + a t' \varphi = 0 \qquad (22)$$

having considered eq. (20,b) and having assumed $\varphi \ll \phi, \phi_s$; besides

$$a = \frac{12g}{\pi} \frac{N r_p c^2 \dot{\gamma}_{tr}}{L_B^3 h^2 \gamma_{tr}^6} - \omega_{rev}^2 \frac{h \cos\phi_s}{\pi \beta_{tr}^2 \gamma_{tr}^4} \frac{V}{V_p} \dot{\gamma}_{tr} \tag{23}$$

where $V_p = \frac{m_p c^2}{e} = 938$ MV

During the γ_{tr} - jump one can devise to decrease the RF crest-voltage V from the operating value (600 KV) down to that value V_0 which makes $a = 0$ in eq. (22); this V_0 is given by eq. (23):

$$V_0 = \frac{12 g N \beta_{tr}^2}{h^2 \gamma_{tr}^2 |\cos\phi_s|} \frac{r_p c^2}{L_B^3 \omega_{rev}^2} V_p$$

which, bearing in mind that $L = B S_B$ and $h\omega_{rev} = 2\pi \beta_{tr} f_B$ (with $f_B = c/S_B$ as defined in Ref. 1), becomes

$$V_0 = \frac{3g}{\pi^2} \frac{N r_p}{B^3 \gamma_{tr}^2 |\cos\phi_s| S_B} \simeq 4.10 \text{ V} \tag{24}$$

or practilly zero, with the usual values (i.e. $N = 10^{10}$ ppb, $B = 0.25$, $S_B = 4.8$ m, $|\cos\phi_s| = \sqrt{3}/2$).

Under these conditions eq. (22) scales to

$$\frac{d^2\varphi}{dt'^2} - \frac{1}{t'} \frac{d\varphi}{dt'} = 0 \tag{25}$$

whose solution is

$$\varphi = \varphi_M - \frac{1}{2} \kappa t'^2 \tag{26}$$

with its physical implications shown in Fig. 2.

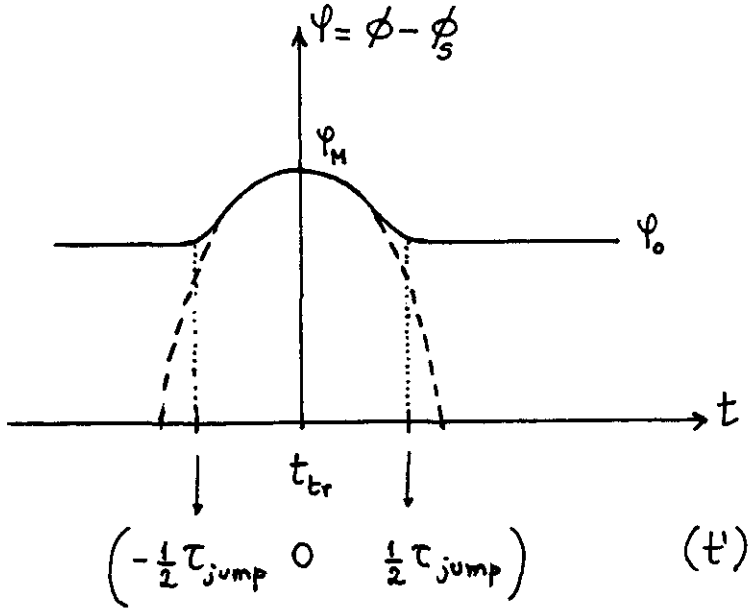

Fig. 2 Bunch dilution over transition

Being $\dot{\phi} = \frac{d\varphi}{dt'}$, eq. (26) yields $K = \frac{\dot{\phi}}{t - t_{tr}}$ wich gives

$K = \frac{2|\dot{\phi}|}{\tau_{jump}}$, for $t = t_{tr} \pm \frac{1}{2}\tau_{jump}$.

On the other hand eqs. (19) and (20a), together with the relation $h\omega_{rev} = 2\pi\beta_{tr} / \tau_B$ ($\tau_B = 16$ ns, as in Ref. 1), give

$$\frac{K}{\varphi_o} = \frac{4\pi\eta_{jump}}{\beta_{tr}\gamma_{tr}\tau_B\tau_{jump}} \frac{V}{V_p} \cos\phi_s \qquad (27)$$

having carried out the approximation $\sin\phi - \sin\phi_s \simeq \varphi\cos\phi_s$.

Assessing $\varphi_o = \varphi(t = t_{tr} \pm \frac{1}{2}\tau_{jump})$ eq. (26) yields :

$$\varphi_M = \varphi_o\left(1 + \frac{\pi\eta_{jump}}{2\beta_{tr}\gamma_{tr}} \frac{\tau_{jump}}{\tau_B} \frac{V}{V_p} \cos\phi_s\right) \qquad (28)$$

Defining $\eta_{jump} \simeq \frac{2\Delta\gamma_{tr}}{\gamma_{tr}^3} \simeq \frac{2 \times 4}{(8.6)^3} = 0.0126$, bearing in mind that $\tau_{jump} = 100\,\tau_{rev}$, eq. (28) yields the following Table II

Table II

p (GeV/c)	τ_{jump} (ms)	φ_M/φ_0
40	0.261	1.010
50	0.311	1.013
60	0.370	1.015
75	0.496	1.020
100	0.652	1.025

Note that the RF crest-value is the average between the operating 600 KV and V_0 given by eq (24) : i.e $V \simeq 300$ KV.

4. Modification of the Guide Magnetic Field

The procedure proposed at the end of Sect. 3 implies a certain amount of inward-spiraling, unless the steady ramp of the guide magnetic field is slightly varied as illustrated in Fig. 3.

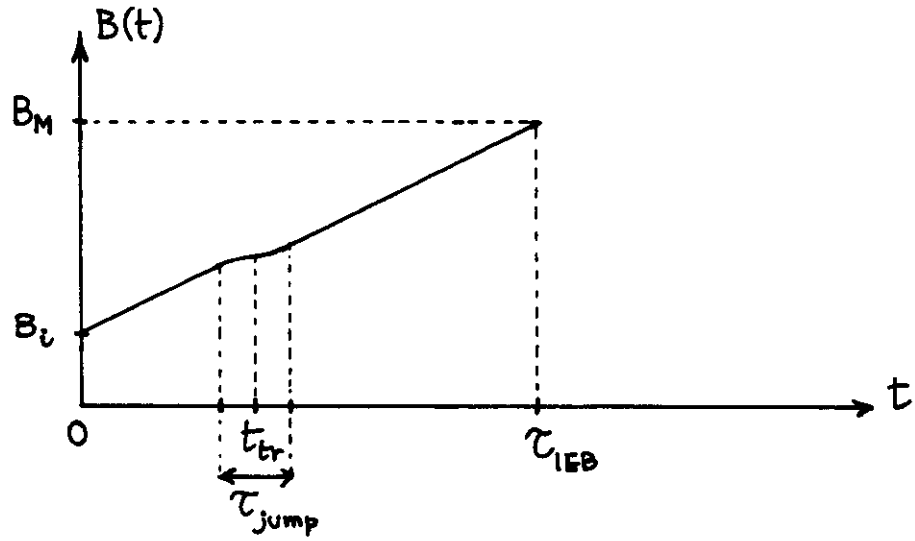

Fig. 3 Modified time-dependence of the IEB main magnetic field.

Namely the magnetic field has to increase slower and slower while the RF crest-voltage is decreasing; when V_{RF} begins to grow again, the magnetic field returns smoothly to the original ramp.

Should the $\dot{B}$ ramp remain steady, the radial displacement can be evaluated by elaborating the relation $p = eRB$

$$\frac{\Delta R}{R} = \alpha \left(\frac{\Delta p}{p} - \frac{\dot{B}}{B} \Delta t \right) \qquad (29)$$

where

$$\alpha = \frac{1}{\gamma_{tr}^2} = 0.0135 = \text{momentum compaction factor} \qquad (30)$$

In normal condition, over a single revolution one has:

$$\frac{\Delta R}{R} = 0 \ , \quad \Delta t = \tau_{rev} \ , \quad \frac{\Delta p}{p} \simeq \frac{1}{\beta_{tr}\gamma_{tr}^2} \frac{V}{V_p} \sin\phi_s$$

having assumed $\phi \simeq \phi_s$.

If the RF crest-voltage decreases linearly from V, for $t = t_{tr} - \frac{1}{2}\tau_{jump}$, to almost zero for $t = t_{tr}$ (and growing again towards V in the interval $t_{tr} \div t_{tr} + \frac{1}{2}\tau_{jump}$), eq. (29) has to be modified into

$$\frac{\Delta R}{R}(t) \simeq \alpha \left[\frac{\sin\phi_s}{\beta_{tr}\gamma_{tr}^2} \frac{V}{V_p} \left(1 - 2\frac{|t-t_{tr}|}{\tau_{jump}} \right) - \frac{\dot{B}}{B} n(t) \tau_{rev} \right]$$

or

$$\frac{\Delta R}{R}(\ ) \simeq \alpha \left[\frac{\sin\phi_s}{\beta_{tr}\gamma_{tr}^2} \frac{V}{V_p} \left(1 - \frac{2n'}{\tau_{jump}} \right) - \frac{\dot{B}}{B} (n'+1) \tau_{rev} \right] \qquad (31)$$

where $n_{rev} = 100$ and n' is the number of revolution

Evaluating $\sigma_H = \frac{1}{2}(\beta_H \varepsilon)^{1/2}$ with $\varepsilon = \varepsilon_N / (\beta\gamma)_{tr}$ ($\varepsilon_N = 0.75$ mm mrad), $\beta_H \simeq C / 2\pi\gamma_{tr}$, one can easily shows in Table III how the radial displacement (31) is comparable to or bigger than the radial r.m.s. dimensions ($2\sigma_H$ containing 95% of the beam).

Table III

p_H (GeV/c)	σ_H (mm)	ΔR (mm)
40	0.54	0.89
50	0.62	1.41
60	0.68	2.29
75	0.76	3.36
100	0.88	8.01

Perhaps a sharp drop of the RF crest-voltage over few turns could diminish the radial spiraling, provided that this process does not have nasty implications on the bunch dilution.

Nevertheless, according to the very good results achieved (2-7) so far, a fast γ_{tr}-jump should secure a limited enhancement of the bunch-length, even without making use of a dent in the RF crest-voltage.

4. Conclusions

All the five Intermediate Energy Boosters, proposed in Ref. 1, seem to be able to undergo transition-crossing without any kind of instability; expecially if a rather quick γ_{tr}-jump is implemented.

Microwave effects are just within the stability limit, while space-charge effects should not be very harmful, even without performing some gymnastics on the RF crest-voltage.

The technical feasibility of the variation of the $\dot{B}$ slope should be further discussed.

The slight worsening of the data pertaining to the last rows of all the Tables can be eliminated simply by increasing γ_{tr} in each IEB, only mantaining the ratio $40\ GeV/(8.6\ m_pc^2) \simeq 4.96$

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